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Robust Fault-Tolerant Control for Robotic Systems

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Abstract

In this paper, a robust fault-tolerant control scheme for robot manipulators overcoming actuator failures is presented. The joint(or actuator) fault considered in this paper is the free-swinging joint failure and causes the loss of torque on a joint. The presented fault-tolerant control framework includes a normal control with normal(non-failed) operation, a fault detection and a fault-tolerant control to achieve task completion. For both no uncertainty case and uncertainty case, a stable normal controller and an on-line fault detection scheme are presented. After the detection and identification of joint failures, the robot manipulator becomes the underactuated robot system with failed actuators. A robust adaptive control scheme of robot manipulators with the detected failed-actuators using the brakes equipped at the failed(passive) joints is proposed in the presence of parametric uncertainty and external disturbances. To illustrate the feasibility and validity of the proposed fault-tolerant control scheme, simulation results for a three-link planar robot arm with a failed joint are presented.

1 Introduction

In modern robotics, the reliability and safety based on fault detection and accommodation (FDA) play a key role in the operation of autonomous and intelligent robotic systems. Fault tolerance is increasingly important for robots, especially those in remote or hazardous environments such as space, underwater, nuclear, and medical environments. Robots need the ability to effectively detect and tolerate internal failures in order to continue performing their tasks without need for immediate human intervention [7] [8].

In this paper, a robust fault-tolerant control scheme for robotic systems is developed in the presence of parametric uncertainty and disturbances. A fault detection method for joint failures in robotic systems is proposed for both no uncertainty case and uncertainty case. After system faults in a robot are detected by the presented fault detector, a robust fault-tolerant control overcoming the uncertainties and actuator failures is presented to achieve task completion for the robot. A robot manipulator with failed actuators can be considered an underactuated robot manipulator with less actuators than total joints. To show the feasibility and robustness of the proposed fault-tolerant control scheme, simulation results for a three-link planar robot arm are presented.

2 Dynamics of Robot Manipulators

Using the Lagrangian formulation, the dynamic equation of an n-link rigid robot manipulator can be written in joint space as follows:

$$M(q)\ddot{q} + F(q,\dot{q}) = u + d(t) \quad (1)$$

where $q \in \Re^n$ is the joint coordinates, $M(q) \in \Re^{n\times n}$ is the symmetric, bounded, and positive definite inertial matrix, $F(q,\dot{q}) = C(q,\dot{q})\dot{q} + G(q)$, $C(q,\dot{q})\dot{q} \in \Re^n$ represents the centrifugal and Coriolis torques, $\dot{M}(q) - 2C(q,\dot{q})$ is skew-symmetric, $G(q) \in \Re^n$ is the vector of gravitational torques, $d(t) \in \Re^n$ is a bounded external disturbance vector, $\|d(t)\| \le d_{max}$ where d_{max} are *unknown* positive constants, and $u \in \Re^n$ is the control torques.

Property 1: *There exist positive constants m_{min}, m_{max}, c_{max}, g_{max}, f_g and f_c such that* $m_{min} \le \|M(q)\| \le m_{max}$, $\|C(q,\dot{q})\| \le c_{max}\|\dot{q}\|$, $\|G(q)\| \le g_{max}$, $\|F(q,\dot{q})\| \le f_g + f_c\|\dot{q}\|^2$.

3 Fault-Tolerant Control Framework for Joint Failures

The term *free-swinging failure* refers to a hardware or software fault in a robot manipulator that causes the loss of torque (or force) on a joint. Examples include a ruptured seal on a hydraulic actuator, the loss of electric power, and a mechanical failure in a drive system. The joint(or actuator) failure considered in this work is the free-swinging failure rather than the locked-joint failure having an inability to move. After a free-swinging failure, the failed joint moves freely under the influence of external forces and gravity.

In this section, a fault-tolerant control framework for joint failures of robot manipulators is shown in Fig. 1.

4 Fault Detection for Joint Failures

The procedure of the presented on-line fault detection is shown as follows:

1. *Detect_Fault* : Detection of a joint(or actuator) fault.
2. *ID_Fault* : Identification of the joint location of that fault.

In this section, a fault detection method for joint failures is presented for two cases of no uncertainty case and uncertainty case.

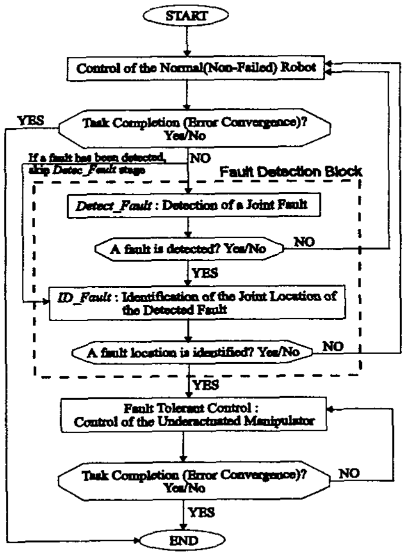


Fig. 1. A Fault tolerant control framework for joint(actuator) failures of robot manipulators.

4.1 No Uncertainty Case

The controller used for the normal operation of a robot manipulator with no parametric uncertainty and no disturbances is the Computed Torque Controller (CTC) with PD feedback control. The well-known CTC for the robot manipulator is as follows.

$$u = M(q)(\ddot{q}_d - K_v\dot{e} - K_p e) + F(q,\dot{q}) \qquad (2)$$

where $e = q - q_d \in \Re^n$ is the joint tracking error, and K_v and K_p are $n \times n$ positive definite constant diagonal gain matrices.

Substituting (2) into (1) with no disturbances ($d(t) = 0$), the closed-loop stable error dynamics is obtained as follows:

$$\ddot{e} + K_v\dot{e} + K_p e = 0. \qquad (3)$$

Therefore, the tracking errors e and $\dot{e}$ are globally exponentially stable.

At first, the reference normal joint position signal to compare with the actual joint position signal of the manipulator operating currently is needed to detect a joint failure. The reference normal joint position signal is obtained numerically by updating the known robot model when the computed torque controller (2) is applied. The actual joint position signals from the real manipulator's joints are measured by the encoders equipped at the joints when the same computed torque controller (2) is appled.

The criterion for detecting a joint failure is as follows. Let q_{c_0} be the reference normal joint position vector representing the no-fault state.

$$e_{c_0} = q - q_{c_0} \in \Re^n \qquad (4)$$

where e_{c_0} is the joint position error between the actual joint position and the reference normal joint position. The first detection stage, that is, *Detect_Fault* condition is as follows:

- No fault : continuing the control loop if $\|e_{c_0}\| = 0$
- Occurrence of a fault : go to *ID_Fault* stage if $\|e_{c_0}\| \neq 0$.

The next detection stage, that is, *ID_Fault* stage is the stage for identifying the location of the failed joint immediately as soon as a joint fault occurs. In this step, the several reference joint position signals are needed to compare with the actual joint position signal after the fault is found. Because the actuator fault dealt with in this work is the free-swinging failure, it means the loss of a torque. If an n-joint robot manipulator has p failed joints, the number of the reference signals are as follows:

$$\sum_{p=1}^{int(n/2)} \frac{n!}{p!(n-p)!} \qquad (5)$$

where it is assumed that the number p of the failed joints is the maximum integer less than $n/2$. Thus, '$int(x)$' means the greatest integer less than or equal to the argument x.

In this stage, the initial values of the reference signals are set as the same values of the actual signals at the fault occurrence time. The reference joint position signals are updated numerically by the known robot dynamics with the failed joint torques. For the i-th joint failure, the i-th joint torque is $u_i = 0$. The remaining normal joint torques without any fault are obtained from the values given by the computed torque controller.

The criterion for identifying the location of the failed joints is as follows. Without loss of generality, let's consider a 3-joint planar robot manipulator to simplify the problem. For this 3-joint manipulator, the number of the needed reference signals is $\frac{3!}{1!2!} = 3$. Let q_{c_i} be the reference joint position vector representing the occurrence of the i-th joint failure.

$$e_{c_i} = q - q_{c_i} \quad \text{for } i = 1,2,3 \qquad (6)$$

where e_{c_i} is the reference joint position error. For this 3-joint manipulator, the second detection stage, that is, *ID_Fault* condition is as follows:

1. Case 1 : Failure at Joint 1 if $\|e_{c_1}\| = 0$
2. Case 2 : Failure at Joint 2 if $\|e_{c_2}\| = 0$
3. Case 3 : Failure at Joint 3 if $\|e_{c_3}\| = 0$.

Now, under the uncertainty such as parametric uncertainty and external disturbances, a fault detection strategy is discussed in the next subsection.

4.2 Uncertainty Case

The computed torque controller (2) with PD feedback control cannot be used in the presence of uncertainty.

Therefore, we use a robust adaptive control scheme for the successful tracking control under the uncertainty.

A robust control law overcoming the uncertainty can be summarized as follows:

$$u = -\hat{M}(-\ddot{q}_d + \Lambda\dot{e}) + \hat{F} + u_r \in \Re^n \quad (7)$$

$$u_r = -Ks - \hat{\rho}\frac{s}{\|s\| + \epsilon}, \quad \epsilon > 0, \quad (8)$$

$$\hat{\rho} = \hat{\theta}^T\psi, \quad \psi = (1 \ \|q\|^2 \ \|\ddot{q}_d\| \ \|\dot{e}\| \ \|\dot{q}\|\|s\|)^T \quad (9)$$

$$\dot{\hat{\theta}} = \Gamma(\frac{\psi\|s\|^2}{\|s\| + \epsilon} - \sigma\hat{\theta}) \in \Re^5, \quad \sigma > 0 \quad (10)$$

where $q_d(t)$ is a twice differentiable desired trajectory, $e = q - q_d$ is the joint position error, $s = \dot{e} + \Lambda e$ is the augmented error, and Λ, K and Γ are positive definite constant diagonal gain matrices.

The stability and convergence are shown by the following Lyapunov function approach. Let's consider a Lyapunov function as follows,

$$V = \frac{1}{2}s^T M(q)s + \frac{1}{2}\tilde{\theta}^T\Gamma^{-1}\tilde{\theta} \quad (11)$$

where $\tilde{\theta} = \hat{\theta} - \theta \in \Re^5$.

Consequently, the derivative of the above Lyapunov function can be obtained as follows:

$$\dot{V} \leq -\frac{1}{2}\lambda_{min}(Q)\|z\|^2 + w(\rho, \|s\|) \quad (12)$$

where $z = (s^T \ \tilde{\theta}^T)^T$, $Q = \begin{pmatrix} 2K & 0 \\ 0 & \sigma \end{pmatrix}$, and $w(\rho, \|s\|) = \frac{1}{2}\theta^T\sigma\theta + \rho\frac{\|s\|\epsilon}{\|s\|+\epsilon}$. Therefore, the errors e and $\dot{e}$ are globally uniformly ultimately bounded.

In the presence of uncertainty, it is very difficult to detect a joint failure because it is hard to obtain the accurate reference normal signal. Therefore, in the first stage, that is, *Detect_Fault*, the tracking position error e and velocity error $\dot{e}$ are used, instead of the reference joint position error e_{c_0}.

The actual joint position signals from the real manipulator's joints are measured by the encoders equipped at the joints when the presented robust adaptive controller (7) $\sim$ (10) is appled.

The criterion for detecting a joint failure is as follows. The first detection stage, that is, *Detect_Fault* condition is as follows:

- No fault : continuing the control loop if $\|e\|^2 + \|\dot{e}\|^2 \leq B_e$
- Occurrence of a fault : go to *ID_Fault* stage if $\|e\|^2 + \|\dot{e}\|^2 > B_e$.

Here, the error bound B_e can be set as follows.

- Regulation Problem : $B_e = [\|e_f\| + (\|e_i\| + e_{io} - \|e_f\|)exp(-\beta_1 t)]^2 + [\|\dot{e}_f\| + (\|\dot{e}_i\| + \dot{e}_{io} - \|\dot{e}_f\|)exp(-\beta_2 t)]^2$
- Tracking Problem :
  - When $\|e_i\| \neq 0$, $B_e = [\|e_f\| + (\|e_i\| + e_{io} - \|e_f\|)exp(-\beta_1 t)]^2 + [\|\dot{e}_f\| + (\|\dot{e}_i\| + \dot{e}_{io} - \|\dot{e}_f\|)exp(-\beta_2 t)]^2$
  - When $\|e_i\| = 0$, $B_e = \|e_f\|^2 + \|\dot{e}_f\|^2$

where e_i is the initial value of the position error e, e_{io} is a user-defined initial offset value of the position error e, $\dot{e}_{io}$ is a user-defined initial offset value of the velocity error $\dot{e}$, e_f is a user-defined final value of the position error e, $\beta_1 > 0$, and $\beta_2 > 0$.

As mentioned above, when an n-joint robot manipulator has p failed joints, the number of the reference signals needed in the second stage *ID_Fault* are $\sum_{p=1}^{int(n/2)} \frac{n!}{p!(n-p)!}$.

In this stage, the initial values of the reference signals are set as the same values as the actual signals at the fault occurrence time. For the i-th joint failure, the i-th joint torque is $u_i = 0$. The remaining normal joint torques without any fault are obtained from the values given by the presented robust adaptive controller. The reference joint position signals cannot be accurately updated numerically by the robot dynamics with the failed joint torques because we do not know the accurate robot parameters. Therefore, the strategy for identifying the location of the failed joints under the uncertainty can be shown as follows.

Without loss of generality, we consider the same 3-joint planar robot manipulator as the above case to simplify the problem. For this 3-joint manipulator, the number of the needed reference signals is $\frac{3!}{1!2!} = 3$. When the reference joint position error e_{c_i} is denoted as (6), the second detection stage, that is, *ID_Fault* condition for this 3-joint manipulator is as follows:

1. Case 1 : Failure at Joint 1 if $\|e_{c_1}\|^2 + \|\dot{e}_{c_1}\|^2 \leq B_c$ and $\|e_{c_2}\|^2 + \|\dot{e}_{c_2}\|^2 > B_c$ and $\|e_{c_3}\|^2 + \|\dot{e}_{c_3}\|^2 > B_c$.
2. Case 2 : Failure at Joint 2 if $\|e_{c_1}\|^2 + \|\dot{e}_{c_1}\|^2 > B_c$ and $\|e_{c_2}\|^2 + \|\dot{e}_{c_2}\|^2 \leq B_c$ and $\|e_{c_3}\|^2 + \|\dot{e}_{c_3}\|^2 > B_c$.
3. Case 3 : Failure at Joint 3 if $\|e_{c_1}\|^2 + \|\dot{e}_{c_1}\|^2 > B_c$ and $\|e_{c_2}\|^2 + \|\dot{e}_{c_2}\|^2 > B_c$ and $\|e_{c_3}\|^2 + \|\dot{e}_{c_3}\|^2 \leq B_c$.
4. Else : Calculation of $e_{min} = \min(\|e_{c_1}\|, \|e_{c_2}\|, \|e_{c_3}\|)$ and $\dot{e}_{min} = \min(\|\dot{e}_{c_1}\|, \|\dot{e}_{c_2}\|, \|\dot{e}_{c_3}\|)$
   (a) Case 1 : Failure at Joint 1 if $e_{min} = \|e_{c_1}\|$ and $\dot{e}_{min} = \|\dot{e}_{c_1}\|$.
   (b) Case 2 : Failure at Joint 2 if $e_{min} = \|e_{c_2}\|$ and $\dot{e}_{min} = \|\dot{e}_{c_2}\|$.
   (c) Case 3 : Failure at Joint 3 if $e_{min} = \|e_{c_3}\|$ and $\dot{e}_{min} = \|\dot{e}_{c_3}\|$.
   (d) Else : No decision for a joint failure : continuing the control loop

Here, B_c is a user-defined small positive constant selected appropriately.

5 Robust Fault-Tolerant Control : Robust Control of Underactuated Manipulators

The failed joints are called the *passive joints*. The remaining normally operating joints are called the *active joints*.

The passive joints has *brakes* instead of failed actuators. It is assumed that the brakes equipped at passive joints operate normally.

After the failed joints are detected, the robot manipulator system behaves as the underactuated manipulator with less actuators than total joints. Therefore, the control of the underactuated manipulator with the failed actuators is presented as follows to achieve task completion thereafter even if a joint fails.

The control objective considered here is the regulation that all joints get converged to their desired set-points. The control procedure of an underactuated robot manipulator in joint space is as follows.

1. **Mode 1 : Control of all passive joints**; *Control all passive joints using the dynamic coupling between the active joints and the passive ones.*
2. **Mode 2 : Braking of all passive joints**; *Brake each passive joint as soon as they reach their set-points with zero velocity. Practically, the desired small position and velocity error bounds of the passive joints are a priori defined. Wait until all passive joints are locked.*
3. **Mode 3 : Control of all active joints**; *Control all active joints by a new control law.*

From the above control procedure, two control stages are needed to control all joints. These control laws are developed in the following subsections.

5.1 Control of Passive Joints

This equation (1) can be partitioned as follows.

$$\begin{pmatrix} M_{aa} & M_{ap} \\ M_{pa} & M_{pp} \end{pmatrix}\begin{pmatrix} \ddot{q}_a \\ \ddot{q}_p \end{pmatrix} + \begin{pmatrix} F_a \\ F_p \end{pmatrix} = \begin{pmatrix} \tau_a + d_a(t) \\ O_p + d_p(t) \end{pmatrix} \tag{13}$$

where $q_a \in \Re^r$ is the position vector of active joints, $q_p \in \Re^p$ is the position vector of passive(failed) joints, $u = (\tau_a^T \; O_p^T)^T$, $\tau_a \in \Re^r$ is the actual control torque input vector applied to the active joints, $O_p \in \Re^p$ is the zero vector at the passive joints, and $d(t) = (d_a^T \; d_p^T)^T$.

The dynamic equation for the passive joints using the partitioned dynamics (13) can be obtained as

$$\ddot{q}_p = M_{p\tau_a}\tau_a + H_p \;\in \Re^p \tag{14}$$

where $M_{p\tau_a} = -\tilde{M}_{pp}^{-1} M_{pa} M_{aa}^{-1}$, $\tilde{M}_{pp} = M_{pp} - M_{pa}M_{aa}^{-1}M_{ap}$, and $H_p = -M_{p\tau_a}F_a - \tilde{M}_{pp}^{-1}F_p + M_{p\tau_a}d_a + \tilde{M}_{pp}^{-1}d_p$.

The position error and the augmented error of the passive joints are denoted by $e_p = q_p - q_{p_d}$ and $s_p = \dot{e}_p + \Lambda_p e_p$ where q_{p_d} is a desired position vector of the passive joints and $\Lambda_p = \Lambda_p^T > 0$.

A robust passive joint controller is presented as follows:

$$\tau_a = \hat{M}_{p\tau_a}^{\#}\left(V_{p_r} - \hat{H}_p\right) \in \Re^r,\; V_{p_r} = V_p + \Delta V_p, \tag{15}$$

$$V_p = \ddot{q}_{p_d} - (K_p + \Lambda_p)\dot{e}_p - K_p\Lambda_p e_p \in \Re^p, \tag{16}$$

$$\Delta V_p = -\hat{\rho}_p \frac{\alpha_p}{\gamma_p(\|\alpha_p\|)} \in \Re^p, \tag{17}$$

$$\hat{\rho}_p = \hat{\theta}_p^T\psi_p,\; \psi_p = \left(1 \; \|\dot{q}\|^2 \; \|\ddot{q}_{p_d}\| \; \|\dot{e}_p\| \; \|e_p\|\right)^T \in \Re^5, \tag{18}$$

$$\dot{\hat{\theta}}_p = \Gamma_p\left(\frac{\psi_p\|\alpha_p\|^2}{\gamma_p(\|\alpha_p\|)} - \sigma_p\hat{\theta}_p\right),\; \sigma_p > 0,\; \alpha_p = R_p s_p, \tag{19}$$

where $\hat{M}_{p\tau_a}^{\#} \in \Re^{r\times p}$ is a pseudoinverse matrix of $\hat{M}_{p\tau_a}$ with nominal dynamic parameters obtained as $\hat{M}_{p\tau_a}^{\#} = \hat{M}_{p\tau_a}^T(\hat{M}_{p\tau_a}\hat{M}_{p\tau_a}^T)^{-1}$ under the assumption of $r \geq p$ and the full-rankness of $\hat{M}_{pa}$. $\hat{H}_p = -\hat{M}_{p\tau_a}\hat{F}_a - \hat{\tilde{M}}_{pp}^{-1}\hat{F}_p$, K_p, R_p and Γ_p are positive definite constant diagonal matrices, and $\gamma_p(\|\alpha_p\|)$ is a chattering alleviation function such as $\gamma_p(\|\alpha_p\|) = \|\alpha_p\| + \epsilon_p$ with $\epsilon_p > 0$.

The closed-loop error dynamics for s_p becomes $\dot{s}_p = -K_p s_p + \Delta V_p + \eta_p$ where $\eta_p = \left(M_{p\tau_a}\hat{M}_{p\tau_a}^{\#} - I_p\right)V_{p_r} + \left(H_p - M_{p\tau_a}\hat{M}_{p\tau_a}^{\#}\hat{H}_p\right)$ is the *lumped uncertainty* term and I_p is a $p \times p$ identity matrix.

Assumption 1: There exists $c_0 > 0$ such that $\|M_{p\tau_a}\hat{M}_{p\tau}^{\#} - I_p\| \leq c_0 < 1$.

Property 2: *By Property 1, there exist* $c_1 > 0$ *and* $c_2 > 0$ *such that* $\|H_p - M_{p\tau_a}\hat{M}_{p\tau_a}^{\#}\hat{H}_p\| \leq c_1 + c_2\|\dot{q}\|^2$.

Property 3: *By the definition of the controller and when* $\hat{\theta}_p(0) \geq 0$, *there exist* $c_3 > 0$ *and* $c_4 > 0$ *such that* $\|V_{p_r}\| \leq \|\ddot{q}_{p_d}\| + c_3\|\dot{e}_p\| + c_4\|e_p\| + \hat{\rho}_p$.

$$\|\eta_p\| \leq \bar{\theta}_{p_1} + \bar{\theta}_{p_2}\|\dot{q}\|^2 + \bar{\theta}_{p_3}(\|\ddot{q}_{p_d}\| + \hat{\rho}_p) + \bar{\theta}_{p_4}\|\dot{e}_p\| + \bar{\theta}_{p_5}\|e_p\|$$

where $\bar{\theta}_{p_1} = c_1$, $\bar{\theta}_{p_2} = c_2$, $\bar{\theta}_{p_3} = c_0$, $\bar{\theta}_{p_4} = c_0c_3$ and $\bar{\theta}_{p_5} = c_0c_4$.

Theorem 1: *Under Assumption 1, the proposed control system (15) ∼ (19) guarantees that the errors of the passive joints are globally uniformly ultimately bounded (GUUB).*

Proof: Let's consider a Lyapunov function candidate,

$$V = (1/2)\{s_p^T R_p s_p + (1 - \bar{\theta}_{p_3})\tilde{\theta}_p^T\Gamma_p^{-1}\tilde{\theta}_p\} = (1/2)z_p^T P_p z_p \tag{20}$$

where $\tilde{\theta}_p = \hat{\theta}_p - \theta_p$, $\theta_{p_i} = \bar{\theta}_{p_i}/(1 - \bar{\theta}_{p_3})$ and $z_p = (s_p^T \; \tilde{\theta}_p^T)^T$.

By some effective manipulations, $\dot{V} \leq -z_p^T Q_p z_p/2 + w_p(\rho_p, \hat{\rho}_p, \|\alpha_p\|)$ where $Q_p = block\_diag(2R_pK_p, (1 - \bar{\theta}_{p_3})\sigma_p)$ and $w_p(\rho_p, \hat{\rho}_p, \|\alpha_p\|) = (1 - \bar{\theta}_{p_3})\theta_p^T\sigma_p\theta_p/2 + \|\alpha_p\|[\gamma_p(\|\alpha_p\|) - \|\alpha_p\|][\hat{\rho}_p\bar{\theta}_{p_3} + \rho_p(1 - \bar{\theta}_{p_3})]/\gamma_p(\|\alpha_p\|)$.

Since both s_p and $\tilde{\theta}_p$ are GUUB as $\|s_p\| \leq [2V/\lambda_{min}(R_p)]^{1/2}$ and $\|\tilde{\theta}_p\| \leq [2V/((1 - \bar{\theta}_{p_3})\lambda_{min}(\Gamma_p^{-1}))]^{1/2}$, the errors e_p and $\dot{e}_p$ are also GUUB. See [9] for the details. ■

5.2 Control of Active Joints

Since all passive joints q_p are locked by their brakes, $\dot{q}_p = \ddot{q}_p = 0$. Thus the dynamic equation for the active joints of the robot with the locked passive joints is

$$\ddot{q}_a = \tilde{M}_{aa}^{-1}\tau_a + H_a \;\in \Re^r \tag{21}$$

where $H_a = -\tilde{M}_{aa}^{-1}(\tilde{F}_a - \tilde{d}_a)$, $\tilde{M}_{aa} = M_{aa} - M_{ap}M_{pp}^{-1}M_{pa}$, $\tilde{F}_a = F_a - M_{ap}M_{pp}^{-1}F_p$ and $\tilde{d}_a = d_a - M_{ap}M_{pp}^{-1}d_p$.

The position error and the augmented error of the active joints are denoted by $e_a = q_a - q_{a_d}$ and $s_a = \dot{e}_a + \Lambda_a e_a$ where q_{a_d} is a desired position vector of the active joints and $\Lambda_a = \Lambda_a^T > 0$.

A robust active joint controller is presented as follows:

$$\tau_a = -\hat{\tilde{M}}_{aa}(-\ddot{q}_{a_d} + \Lambda_a e_a) + \hat{\tilde{F}}_a + \tau_r \in \Re^r, \tag{22}$$

$$\tau_r = -K_a s_a + \Delta V_a \tag{23}$$

$$\Delta V_a = -\hat{\rho}_a \frac{s_a}{\gamma_a(\|s_a\|)}, \quad (24)$$

$$\hat{\rho}_a = \hat{\theta}_a^T \psi_a, \;\; \psi_a = \left(1 \; \|\dot{q}\|^2 \; \|\ddot{q}_{a_d}\| \; \|\dot{e}_a\| \; \|\dot{q}\|\|s_a\|\right)^T, \quad (25)$$

$$\dot{\hat{\theta}}_a = \Gamma_a \left(\frac{\psi_a \|s_a\|^2}{\gamma_a(\|s_a\|)} - \sigma_a \hat{\theta}_a \right) \in \Re^5, \; \sigma_a > 0, \quad (26)$$

where K_a and Γ_a are positive definite constant diagonal matrices, $\gamma_a(\|s_a\|) = \|s_a\| + \epsilon_a$ with $\epsilon_a > 0$ such as $\gamma_p(\|\alpha_p\|)$.

The closed-loop error dynamics for s_a becomes $\tilde{M}_{aa}\dot{s}_a = -(K_a + \frac{1}{2}\dot{\hat{M}}_{aa})s_a + \Delta V_a + \eta_a$ where $\eta_a = (\tilde{M}_{aa} - \hat{\tilde{M}}_{aa})(-\ddot{q}_{a_d} + \Lambda_a \dot{e}_a) - (\tilde{F}_a - \hat{\tilde{F}}_a) + \tilde{d}_a + \frac{1}{2}\dot{\tilde{M}}_{aa}s_a$ is called *lumped uncertainty*.

The norm of lumped uncertainty can be bounded as $\|\eta_a\| \le \theta_{a_1} + \theta_{a_2}\|\dot{q}\|^2 + \theta_{a_3}\|\ddot{q}_{a_d}\| + \theta_{a_4}\|\dot{e}_a\| + \theta_{a_5}\|\dot{q}\|\|s_a\| = \theta_a^T \psi_a = \rho_a$.

Theorem 2: *If we apply the proposed control law (22) $\sim$ (26) to the underactuated manipulator with the locked passive joints, then the errors of the active joints are globally uniformly ultimately bounded (GUUB).*

Proof: Consider a following Lyapunov function candidate,

$$V = (1/2)\{s_a^T \tilde{M}_{aa} s_a + \tilde{\theta}_a^T \Gamma_a^{-1} \tilde{\theta}_a\} = (1/2) z_a^T P_a z_a \quad (27)$$

where $\tilde{\theta}_a = \hat{\theta}_a - \theta_a$ and $z_a = (s_a^T \; \tilde{\theta}_a^T)^T$.

By some manipulations, $\dot{V} \le -z_a^T Q_a z_a/2 + w_a(\rho_a, \|s_a\|)$ where $Q_a = block\_diag(2K_a, \sigma_a)$ and $w_a(\rho_a, \|s_a\|) = \theta_a^T \sigma_a \theta_a / 2 + \rho_a \|s_a\| [\gamma_a(\|s_a\|) - \|s_a\|] / \gamma_a(\|s_a\|)$.

Since s_a and $\tilde{\theta}_a$ are GUUB as $\|s_a\| \le \left[2V/\lambda_{min}(\tilde{M}_{aa})\right]^{1/2}$ and $\|\tilde{\theta}_a\| \le \left[2V/\lambda_{min}(\Gamma_a^{-1})\right]^{1/2}$, e_a and $\dot{e}_a$ are GUUB. ■

Remark 1: If ($\epsilon_p = \sigma_p = 0$) and ($\epsilon_a = \sigma_a = 0$) in the controllers, then the closed-loop control system is globally asymptotically stable.

6 Simulation Study

The robot manipulator to be simulated is a three-link planar robot arm ($n = 3$).

The simulated three-link planar robot manipulator is shown in Fig. 2.

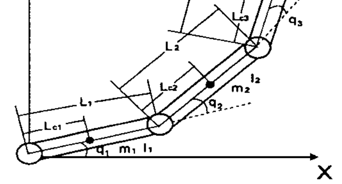


Fig. 2. A three-link planar robot manipulator.

The simulations are performed for two cases: 1. Case 1: *No uncertainty case*; 2. Case 2: *Uncertainty case.*

In order to achieve the normal operation of robot control, the computed torque control (2) with PD feedback and the presented robust control (7) $\sim$ (10) are used for Case 1 and Case 2, respectively.

The numerical real and nominal parameters of the simulated manipulator are given in Table I. It is assumed that the lengths of each link are exactly known. The nominal dynamic parameters used in the robust controller are set to 70% of the real dynamic parameters.

Table I. Numerical parameter values of the simulated three-link manipulator:
$[(L_1, m_1, I_1, L_{c1}) = (L_2, m_2, I_2, L_{c2}) = (L_3, m_3, I_3, L_{c3})]$.

Parameters	Values	Link i (i=1,2,3)
Length	Real	0.5
$[L_i(m)]$	Nominal	
Mass	Real	1
$[m_i(kg)]$	Nominal	0.7
Moment of inertia	Real	0.1
$[I_i(kgm^2)]$	Nominal	0.07
COM position	Real	0.25
$[L_{ci}(m)]$	Nominal	0.175

* 'COM' : Center Of Mass

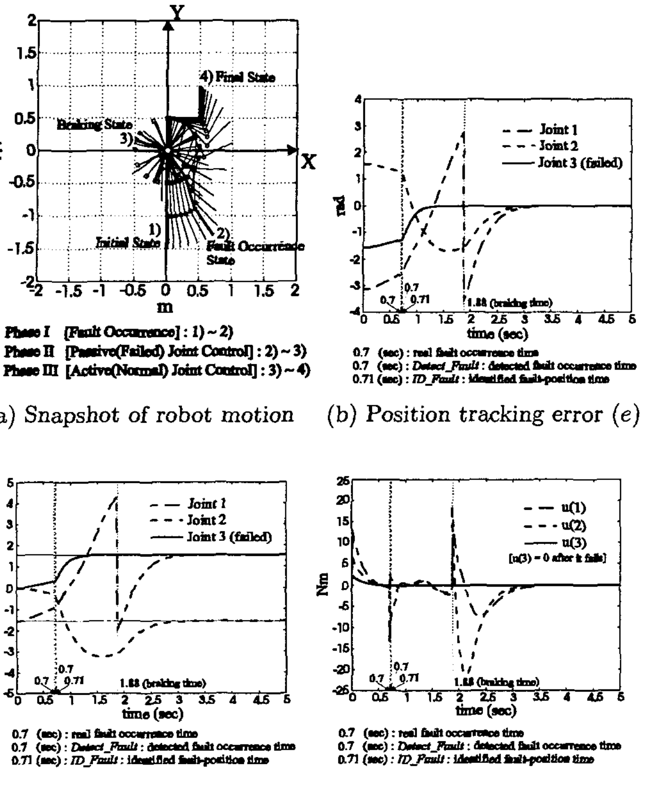
Each external disturbance to be inserted into each joint is the random noise whose each magnitude is bounded by the value of 0.5, that is, $|d_i(t)| \le 0.5$, for i=1,2,3.

The initial positions of each joint are $q_1(0) = -\frac{\pi}{2}(rad)$, $q_2(0) = q_3(0) = 0(rad)$. The initial velocities of each joint are $\dot{q}_1(0) = \dot{q}_2(0) = \dot{q}_3(0) = 0(rad/sec)$. The final desired set-points of each joint are $q_{1_d} = \frac{\pi}{2}(rad)$, $q_{2_d} = -\frac{\pi}{2}(rad)$ and $q_{3_d} = \frac{\pi}{2}(rad)$.

In this simulation, it is assumed that a joint(actuator) fault at the third joint (q_3) occurs at 0.7 (sec). Thus, the torque applied at the third joint is zero after a joint fault occurs. The robot manipulator after an actuator failure at the third joint becomes the underactuated manipulator with the third joint passive. It is assumed that the failed joint has a normal brake. It is assumed that there are no frictions and no joint limits in the manipulator's joints. The sampling time is 0.01 (sec).

The fault-tolerant control results for Case 1 and Case 2 are shown in Fig. 3 and Fig. 4, respectively. For Case 1, the detected fault occurrence time by *Detect_Fault* stage is 0.7 (sec), that is, it is equal to the real fault occurrence time. The identified joint location of the fault by *ID_Fault* stage is the third joint and the time to find out the fault location is thereafter the next sampling time, 0.71 (sec). After 0.71 (sec), the manipulator is controlled by the presented control method for the underactuated manipulator. The braking time of the failed joint is 1.88 (sec). After 1.88 (sec), the failed(third) joint is locked and the remaining normal active joints are controlled to desired set-points. The angle of the passive joint is transformed into the same angle having the value between $-\pi(rad)$ and $\pi(rad)$ after it is locked by its own brake. For example, it is the fact that $\frac{5}{2}\pi(rad)$ is the same as $\frac{1}{2}\pi(rad)$ in viewpoint of the angle. For Case 2, the detected fault occurrence time is 0.82 (sec) and the time to find out the fault location is 0.83 (sec). The braking time of the failed joint is 2.06 (sec).

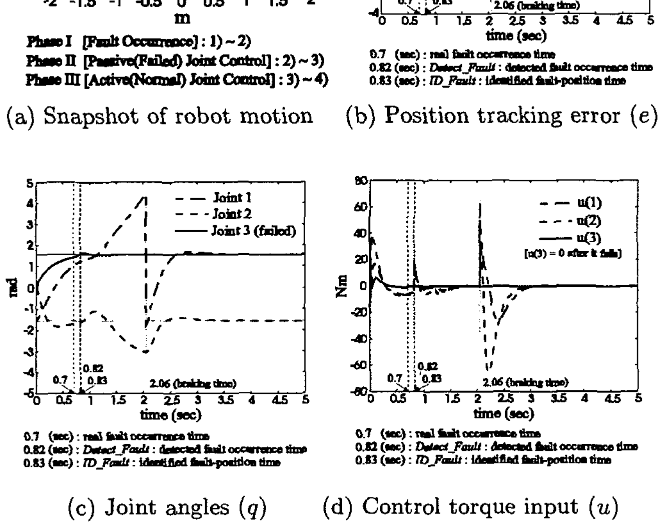
From the simulation results, a joint failure at the robot's joint has been successfully detected and recovered, and the original control objective has been achieved. It has been shown that the proposed robust fault-tolerant control scheme is feasible.



(a) Snapshot of robot motion (b) Position tracking error (e)

(c) Joint angles (q) (d) Control torque input (u)

Fig. 3. Control results for Case 1 (without uncertainty).



(a) Snapshot of robot motion (b) Position tracking error (e)

(c) Joint angles (q) (d) Control torque input (u)

Fig. 4. Control results for Case 2 (with uncertainty).

7 Conclusions

In this paper, a study on the robust fault-tolerant control of robot manipulators overcoming an actuator failure has been performed. A fault detection scheme for both no uncertainty case and uncertainty case has been proposed. The proposed fault detection scheme uses only encoders and tachometers to measure the position error and velocity error and does not require any other special hardware for detecting a joint failure. A robust adaptive control scheme for underactuated manipulators with failed actuators has been proposed using the brakes equipped at passive joints. The proposed control scheme does not need *a priori* knowledge of the accurate dynamic parameters and the exact uncertainty bounds.

It has been observed that the proposed fault-tolerant control scheme is feasible and robust through simulation results.

The robust fault-tolerant control for robotic systems is more useful in remote or hazardous areas such as space, underwater, nuclear power plants, etc. where the repair or replacement of failed actuators is very difficult.

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