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Hydraulic Pumps Driven by Multilayered Piezoelectric Elements

—Mathematical Model and Application to Brake Device—

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Abstract

In this paper, we present a mathematical model of the piezoelectric pump and its application to the automobile brake system. The piezoelectric pump consists of a multi-layered piezoelectric element, a diaphragm, pumping values, resonant pipes and accumulators, and the maximum pumping power of 62W was obtained in the previous experiments by using the piezoelectric element of 22mm diameter and 55.5mm length. A detailed mathematical model of the pump is derived here by considering the compressibility of the working oil , nonlinear characteristics of piezoelectric element , the time delay of pumping values' action and so on. The validity of the model is illustrated by comparing the experimental data and the simulation results.

Using the mathematical model of the piezoelectric pump, a brake system for automobile disk brake is also simulated in this paper. The brake system consists of a piezoelectric pump as a power source, calipers and its piston to generate brake force, and a three position solenoid value to change the brake situation. It is shown that 15mm/sec of piston speed and 20kN of piston force are obtained by using the piezoelectric element of 33mm diameter and 55.5mm length.

1. Introduction

Recently, a new type of automobile brake system called brake-by-wire has attracted the attention of researchers. Although the conventional brake system consists of a central master cylinder and hydraulic hose lines connecting individual brakes, the brake-by-wire-system consists of independent electrically-powered self-contained brake devices attached to each wheel. Since the each brake device is independent, the reliability of the total brake system for failures is improved, and the development of new braking scheme becomes easier.

In this paper, we try to design a brake device as an application of the piezoelectric pump that we designed and experimented in the previous work [1]. The schematic diagram of the brake device is shown in Fig.1, where the main components are the pump ①+②+③, the solenoid valve ⑪, the calipers ⑥+⑦+⑧, the accumulator ⑤, and the reservoir ④. When the solenoid coil SOL-b is excited, the pump generates the pressure $P_4 - P_{ACC}$ which is approximately proportional to the driving voltage,

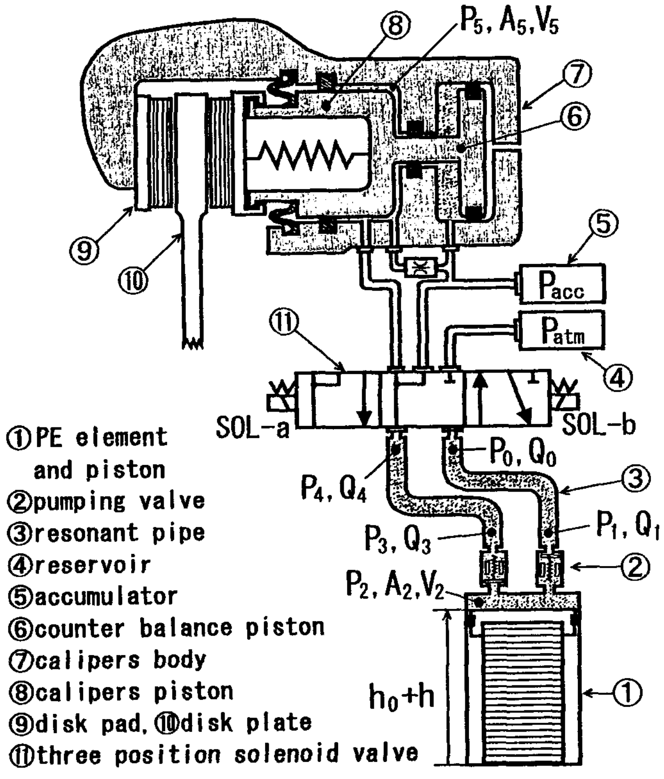


Fig.1 Schematic diagram of the brake device

and then the caliper piston presses the disk pads to the disk plate. The throttle valve connecting the piston chamber to the accumulator is needed to decrease the pressure $P_4 - P_{ACC}$ as the driving voltage becomes smaller. The accumulator pressure P_{ACC} is also required to prevent the cavitation in the pumping chamber. On the other hand, when the solenoid SOL-a is excited, the pump scoops the reservoir oil into the accumulator to supply the leakage oil.

In order to simulate the above brake device, a detailed mathematical model of the piezoelectric pump is derived here by considering the compressibility of the working oil, the nonlinear characteristics of the piezoelectric element, the time delayed motion of pumping valves and so on. Using this model and some additive equations about the motion of the calipers piston, the simulation of the brake device is carried out, and its results show that the piston speed and the piston force larger than 15mm/sec and 20kN, respectively, can be achieved by using the piezoelectric element of 33mm diameter and 55.5mm length.

2. Mathematical Model of Piezoelectric Pump

Fig.2 shows the schematic diagram of the piezoelectric pump which we experimented in the previous work [1], and experimental results are shown in Figs.3-5. In these figures, $P_{out} = P_4 - P_0$ is the output pressure of the pump, Q_p is the mean flow rate, $W_{out} = Q_p P_{out}$ is the output power, $\tilde{E}$ is the alternative voltage applied to the piezoelectric element, f is the frequency of $\tilde{E}$, and η is the pump efficiency defined by $\eta = W_{out} / (f\ W)$, where W is the work done by the piezoelectric element in one cycle of $\tilde{E}$. W is equal to the area inside the $P_2 - h$ curve shown in Fig.5.

In order to explain these experimental data, we propose the following mathematical model of the pump ;

(A)pumping chamber :

$$A_2 \frac{dh}{dt} = Q_1 + Q_3 + \frac{V_2}{B_2}\frac{dP_2}{dt} \tag{1}$$

$$h = \bar{\beta}\tilde{E} - \alpha_1\sigma + \alpha_2\left[\exp\left(-\alpha_3\sigma - \alpha_4\sigma^2\right) - 1\right] \tag{2}$$

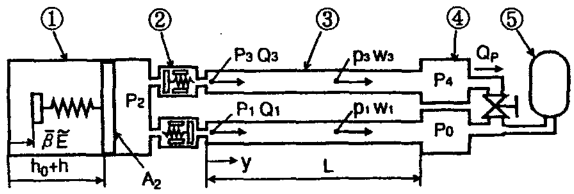
$$\tilde{E} = \frac{1}{1+RCS} E\varsigma(t) - 100 \tag{3}$$

$$\sigma = A_2 P_2 / A_e \tag{4}$$

where V_2 is the volume of the working oil in the diaphragm, B_2 is the equivalent bulk modulus in the diaphragm, $\bar{\beta}$ is the piezoelectric constant dependent on the driving voltage E as shown in Fig.6, R is the output resistance of the power amplifier, C is the capacitance of the piezoelectric element, $\varsigma(t)$ is the rectangular wave with two values of 1 and 0, A_e is the cross-sectional area of the piezoelectric element, $\alpha_1 \sim \alpha_4$ are the constant to express the nonlinear compliance of the piezoelectric element.

(B)pumping values :

$$\left.\begin{aligned} Q_1 &= C_v A_v R_1 \operatorname{sgn}(P_2 - P_1)\sqrt{2|P_2 - P_1|/\rho} \\ R_1 &= \left[1/(1+T_v S)\right]\xi(P_1 - P_2) \end{aligned}\right\} \tag{5}$$



①Piezoelectric Device and Diaphragm, ②Pumping Valve ③Resonant Pipe, ④Pressure Chamber, ⑤Accumulator

Fig.2 Schematic diagram of piezoelectric pump

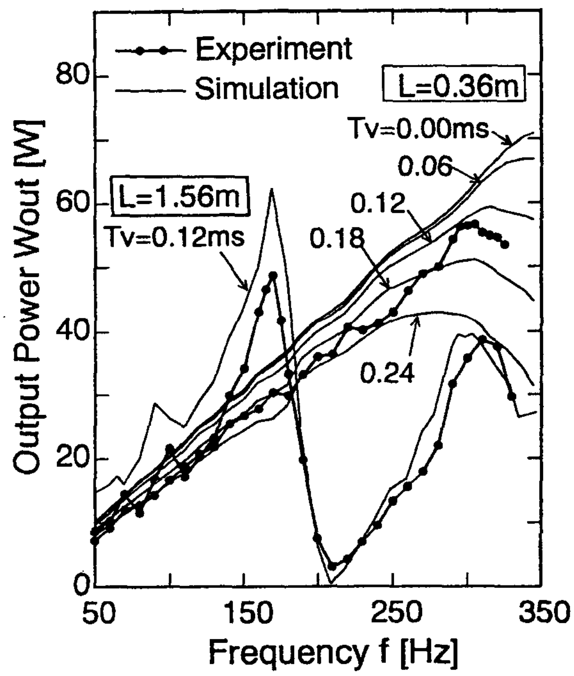


Fig.3 Frequency characteristics of pumping power

$$\left.\begin{aligned} Q_3 &= C_v A_v R_3 \,\mathrm{sgn}(P_2 - P_3)\sqrt{2|P_2 - P_3|/\rho} \\ R_3 &= \left[\frac{1}{(1+T_v S)} \right] \xi(P_2 - P_3) \end{aligned}\right\} \qquad (6)$$

where C_v is the coefficient of the value, A_v is the cross-sectional area, ρ is the density of working oil, T_v is the time constant expressing the time delayed motion of the value. $\xi(\bullet)$ is the saturation function shown in Fig.7 where P_v denotes the pressure difference corresponding to the lift of the value plate.

(C) resonant pipes :

$$\left.\begin{aligned} &\rho\frac{\partial w_i}{\partial t} + \frac{\partial P_i}{\partial y} + f_i = 0 \\ &\frac{\partial P_i}{\partial t} + \rho\gamma^2\frac{\partial w_i}{\partial y} = 0 \\ &0 < y < L \ , \ i = 1,3 \end{aligned}\right\} \qquad (7)$$

$$\left.\begin{aligned} &A_P w_1(t,0) = Q_1 \ , \ A_P w_3(t,0) = Q_3 \\ &P_1(t,0) = P_1 \ , \ p_1(t,L) = P_0 \\ &P_3(t,0) = P_3 \ , \ p_3(t,L) = P_4 \end{aligned}\right\} \qquad (8)$$

where P_i and w_i $(i = 1,3)$ are the pressure and the velocity in the resonant pipes, f_i is the frequency-dependent friction [2], γ is the sound speed in the working oil, and A_P is the cross-sectional area of the resonant pipes.

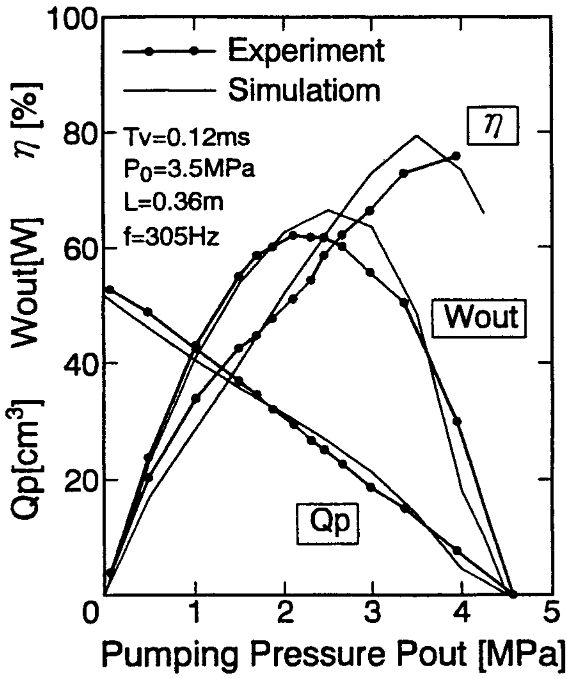


Fig.4 Characteristics of the pump at frequency 305Hz

Simulation results are also shown in Figs.3-5, and good agreement with the experimental results are obtained, where parameters used in the simulation are

$$\left.\begin{aligned} &A_2 = 16.2\,cm^2 \ , \ A_P = 56.7\,mm^2 \ , \ A_v = 2.17\,mm^2 \\ &C = 3.0\,\mu F \ , \ E = 700\,V \ , \ L = 0.36\,m \text{ or } 1.56\,m \\ &P_v = 0.2\,MPa \ , \ R = 64\,\Omega \ , \ V_2 = 1.3\,cm^3 \\ &\gamma = 1250\,m/s \ , \ \rho = 814\,kg/m^3 \end{aligned}\right\} \qquad (9)$$

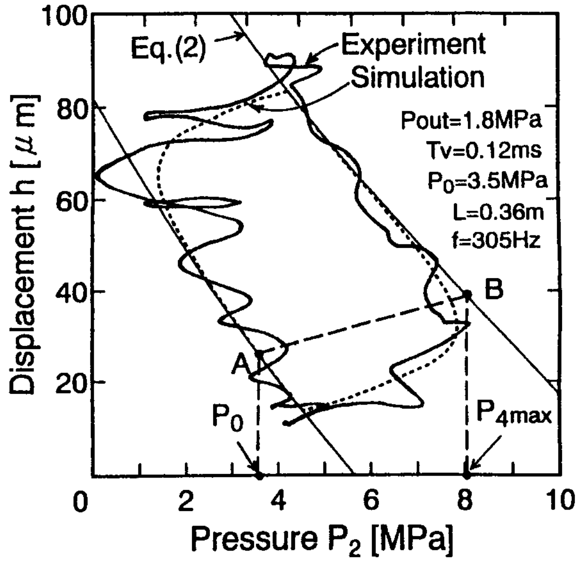


Fig.5 Relation between pressure and displacement in the pumping chamber

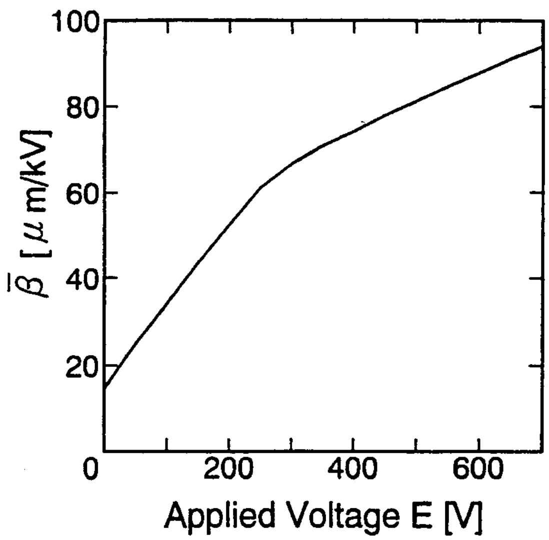


Fig.6 Piezoelectric constant estimated from the experimental data

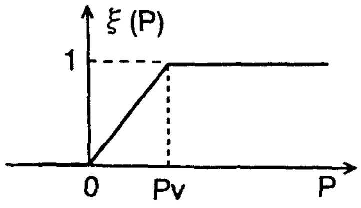


Fig.7 Shape of function $\xi(P)$

and

$$\left.\begin{array}{l} B_2 = 300\,MPa\ ,\ C_v = 0.99\ ,\ T_v = 0.12\,ms \\ \alpha_1 = 2.58\,\mu m/MPa\ ,\ \alpha_2 = 20.6\,\mu m \\ \alpha_2 = 0.065\left(MPa\right)^{-1}\ ,\ \alpha_4 = 0.003\left(MPa\right)^{-2} \end{array}\right\} \qquad (10)$$

Parameters in eq.(9) are the measurements obtained from the experimental device, and the parameters in eq.(10) are the estimates in order to obtain the agreement between the experimental data and simulation results.

3. Brake Device

3.1 Equations of motion Let consider the equation of motion with respect to the calipers for the situation that the solenoid SOL-b is excited. Since the inertial force of calipers piston and body are relatively small compared with the hydraulic force, the equation of motion can be written as follows;

$$C_5 \frac{dx}{dt} + k_{SEAL} x = A_5\left(P_4 - P_0\right) - F \qquad (11)$$

$$V_5 \frac{dP_4}{dt} = B_5\left(Q_4 - Q_T - A_5 \frac{dx}{dt}\right) \qquad (12)$$

$$F = \begin{cases} 0 & \left(x < d_1\right) \\ F_0 & \left(d_1 \le x < d_1 + d_2\right) \\ k_{PAD}\left(x - d_1 - d_2\right) + F_0 & \left(x \ge d_1 + d_2\right) \end{cases} \qquad (13)$$

$$Q^T \Big/ Q^* = C_T \,\mathrm{sgn}\left(P_4 - P_0\right)\sqrt{\left|P_4 - P_0\right| / P^*} \qquad (14)$$

$$P_0 = P_{ACC} \qquad (15)$$

where x is the displacement of calipers piston, F is the press force to the disk pads given by calipers body and piston, Q_T is the flow rate passing through the throttle value, C is the damping coefficient, k_{SEAL} is the equivalent spring constant of oil seal using in the calipers, A_5 is the area pushed by the difference pressure $P_4 - P_0$, V_5 is the volume of the oil compressed by the pressure P_4, B_5 is the bulk modulus of the oil in the volume V_5, k_{PAD} is the equivalent spring constant of left and right disk pads, d_1 is the total gap between the disk pads and the disk plate, d_2 is the small gap between the calipers piston and its cap, F_0 is the constant force given by the spring shown in Fig.1, C_T is a coefficient dependent on the opening of throttle valve, and Q^* and P^* are nominal values to make the eq.(14) non-dimensional. In eq.(13), d_2 and F_0 are introduced to detect the constant of disk pads and plate. This detection is required to control the driving voltage(see Fig.11).

3.2 Driving voltage In the experiments of the piezoelectric pump, we used the driving voltage of 700V in order to draw a maximum pumping power from the element of 22mm diameter. However, from the viewpoint of reliability of the brake device, we limit the driving voltage within 600V which is the rated value of the element modeled by eq.(2).

3.3 Diameter of the piezoelectric element If we use the piezoelectric pump described in the previous section as a power source of the brake device, maximum values of force F and speed v of the calipers piston will approximately become as $F_{max} = 4.5\,(MPa) \times A_5$ and $v_{max} = 52\left(cm^3/s\right) \div A_5$. Therefore, their product is $F_{max} \times v_{max} = 234\,W$. However, since the driving voltage E is decreased from 700V to 600V, piezoelectric elongation βE will reduce to 0.81 times of the original one. Applying this reduction ratio to both the maximum pumping pressure and maximum flow rate, $F_{max} \times v_{max}$ will be reduced to 154W where the diameter of the element is 22mm.

On the other hand, an example of the requirement for the brake device is $F_{max} = 20\,kN$, $v_{max} = 15\,mm/\mathrm{sec}$ and $F_{max} \times v_{max} =$ 300W. Therefore, we have to use the element which has the cross-sectional area A_e larger than $(300/154) \times \pi (22)^2/4 = 741\,mm^2$. So, we use the element of 33mm diameter ($A_e = 855\,mm^2$) in the following simulations.

4. Simulations

Simulations of the brake device was carried out by using the following parameters;

pump:

$$\left.\begin{array}{l} A_2 = 15.0\,cm^2\ ,\ A_e = 8.55\,cm^2\ ,\ A_p = 28.3\,mm^2 \\ B_2 = 600\,MPa\ ,\ f = 300\,Hz\ ,\ L_1 = 0.4\,m \\ L_3 = 0.5\,m\ ,\ P_{ACC} = 3.0\,MPa \end{array}\right\} \quad (16)$$

calipers:

$$\left.\begin{array}{l} A_5 = 32.0\,cm^2\ ,\ B_5 = 900\,MPa\ ,\ C_5 = 100\,Ns/m \\ C_T = 0.7\ ,\ d_1 = 1.5\,mm\ ,\ d_2 = 0.12\,mm \\ F_0 = 1\,kN\ ,\ k_{PAD} = 8.0\,kN/\mu m\ ,\ k_5 = 200\,N/mm \\ P^* = 1\,MPa\ ,\ Q^* = 1\,cm^3/s\ ,\ V_5 = 9.0\,cm^3 \end{array}\right\} \quad (17)$$

where L_1 and L_3 are the length of the inlet and outlet resonant pipes. The remainder parameters for the pump are the same as eqs.(9)-(10).

The maximum speed and force of calipers piston for the driving voltage of 600V are 17.0mm/s and 20.1kN, respectively, as shown in Fig.8. Although the maximum speed of the piston can be enlarged by using another value of L_3 which causes stronger pressure resonance in the outlet resonant pipe, the cavitation occurs in the pumping chamber for that case. From this reason, values of $L_1 = 0.4\,m$ and $L_3 = 0.4\,m$ were used here. The corresponding time histories of pressure, flow rate, applied voltage and so on are shown in Fig.9. As for the variation of the driving frequency , the following results were obtained.

$$f = 200\,Hz:\ v_{max} = 14.3\,mm/s\ ,\ F_{max} = 20.5\,kN \quad (L_1 = 0.6\,m\ ,\ L_3 = 0.95\,m)$$

$$f = 250\,Hz:\ v_{max} = 16.0\,mm/s\ ,\ F_{max} = 20.9\,kN \quad (L_1 = 0.5\,m\ ,\ L_3 = 0.7\,m)$$

Fig.10 shows the relation between the driving voltage E and the piston force F. Since the piezoelectric constant $\bar{\beta}$ is not constant for the variation of E, F is not proportional to E. In order to cover this point, a simulation of feedback control of the force were carried out by using PD compensator , and its result is shown in Fig.11. Since the proportional gain was set to a relatively small value to avoid the unstability, complete tracking to the reference force was not achieved. Therefore, an additional compensator such as the disturbance observer seems to be necessary.

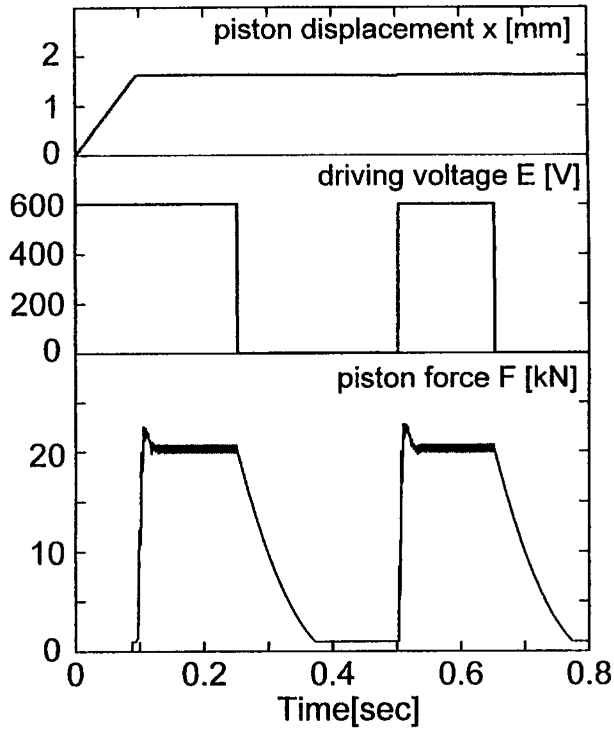


Fig.8 Maximum force and speed of calipers piston

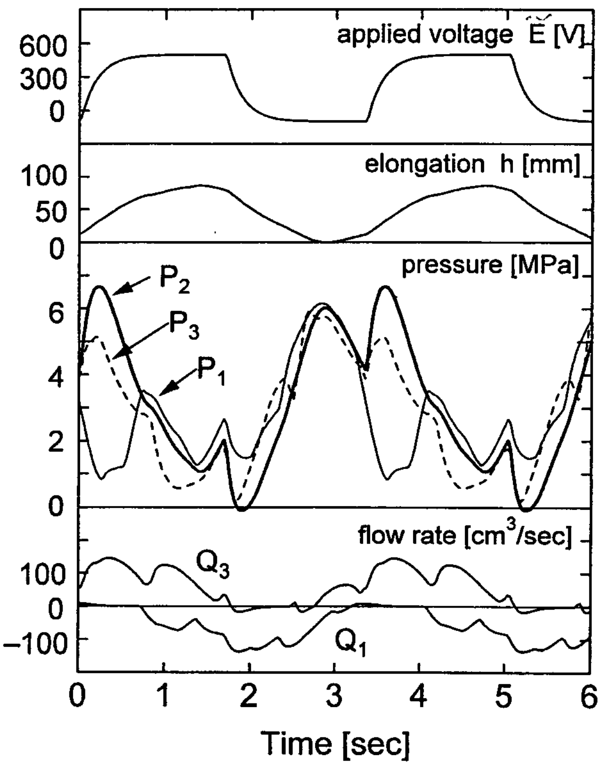


Fig.9 Time histories of variables concerned with the pump

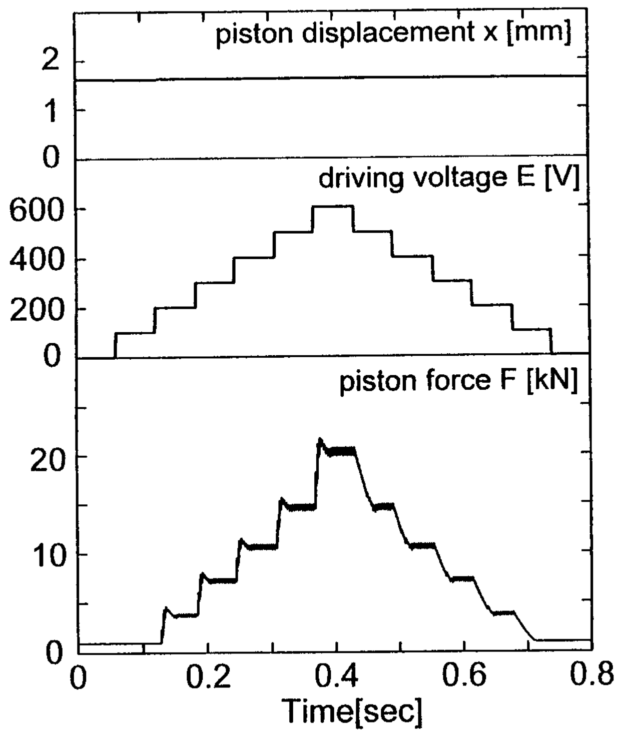


Fig.10 Relation between the driving voltage and the piston force

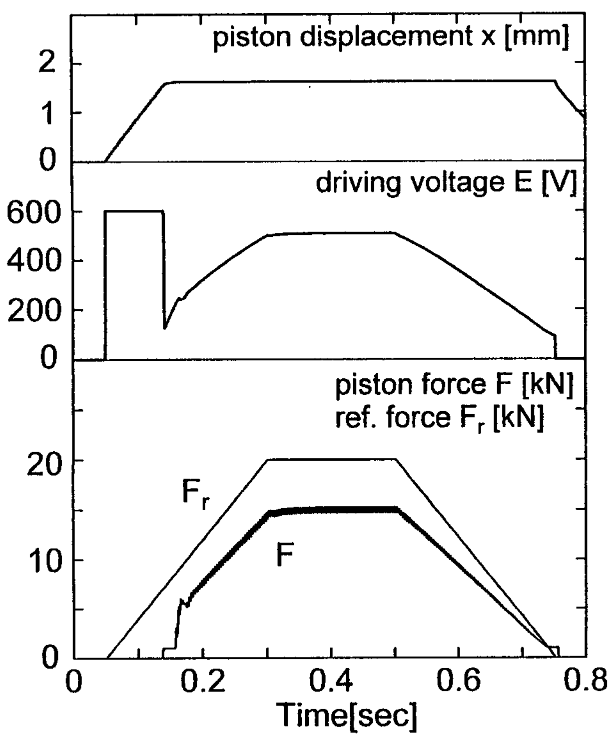


Fig.11 Result of the force control by PD contensator

5. Conclusion

In this paper, we present a mathematical model of the hydraulic pump driven by a multilayered piezoelectric element and its application to a brake device. It is shown by the simulations that the requirement of 20kN and 15mm/s for the brake device can be satisfied by using a piezoelectric element of 33mm diameter and 55.5mm length.

References

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