Proceedings of the 13th
KACC, October 1998

MODEL PREDICTIVE CONTROL OF NONLINEAR PROCESSES BY USE OF 2ND AND 3RD VOLTERRA KERNEL MODEL

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Abstract

This paper proposes a new method of Model Predictive Control (MPC) of nonlinear process by using the measured Volterra kernels as the nonlinear model. A nonlinear dynamical process is usually described as Volterra kernel representation. In the authors' method, a pseudo-random M-sequence is applied to the nonlinear process, and its output is measured. Taking the crosscorrelation between the input and output, we obtain the Volterra kernels up to 3rd order which represent the nonlinear characteristics of the process. By using the measured Volterra kernels, we can construct the nonlinear model for MPC. In applying Model Predictive Control to a nonlinear process, the most important thing is, in general, what kind of nonlinear model should be used. The authors used the measured Volterra kernels of up to 3rd order as the process model.

The authors have carried out computer simulations and compared the simulation results for the linear model, the nonlinear model up to 2nd Volterra kernel, and the nonlinear model up to 3rd order Volterra kernel. The results of computer simulation show that the use of Volterra kernels of up to 3rd order is most effective for Model Predictive Control of nonlinear dynamical processes.

1. Introduction

To control a system the dynamic response of the system must be identification, many kinds of methods of identification have appeared on linear system in the world, but in the almost all actual systems are nonlinear.

In this paper, We use Volterra series with M-sequences to identificate nonlinear system. M-sequences are applied to a non-linear system and the crosscorrelation function between the input and the output displays not only the linear impulse response of the linear part of the system, but also crosssections of the high-order Volterra kernel up to 3rd order of non-linear system.

MPC has enjoyed widespread acceptance and success as an effective technique for dealing with difficult control problems of chemical nonlinear processes. The result of computer simulation shows better improvement on the control performance of MPC when we use nonlinear model than the case of linear model, and use 3rd Volterra kernel model is better than 2nd.We applied this method to identification of a chemical nonlinear process and model predictive control is carried out for the process.

2. Principle of the identification

For linear systems, the linear convolution integral

$$y(t) = \int_0^\infty g_1(\tau) \times u(t-\tau)d\tau \tag{1}$$

For nonlinear systems, the systems can be described by Volterra Kernel as follows.

$$y(t) = \sum_{i=1}^{\infty} \int_0^\infty \int_0^\infty \cdots \int_0^\infty g_i(\tau_1, \tau_2, \cdots \tau_i) \times u(t-\tau_1)u(t-\tau_2)\cdots u(t-\tau_i)d\tau_1 d\tau_2 \cdots d\tau_i \tag{2}$$

where $u(t)$ is the input, and $y(t)$ is the output of the nonlinear system, and $g_i(\tau_1, \tau_2, ...)$ is called Volterra kernel of i-th order. When we take the crosscorrelation function between the input $u(t)$ and the output $y(t)$, we have,

$$\begin{aligned}\phi_{uy}(\tau) &= \overline{u(t-\tau)y(t)} \\ &= \sum_{i=1}^{\infty} \int_0^\infty \int_0^\infty \cdots \int_0^\infty g_i(\tau_1, \tau_2, \cdots \tau_i) \\ &\quad \times \overline{u(t-\tau)u(t-\tau_1)\cdots u(t-\tau_i)}d\tau_1 d\tau_2 \cdots d\tau_i\end{aligned} \tag{3}$$

where $\phi_{uy}(\tau)$ is the crosscorrelation function of $u(t)$ and $y(t)$ and $\overline{\quad}$ denotes time average.

When we use an M-sequence as an input to the system, the n-th moment of $u(t)$ can be easily obtained by use of so-called "shift and add property" of the M-sequence. So we can obtain the Volterra kernels $g_i(\tau_1, \tau_2, \cdots \tau_i)$ from simply measuring the crosscorrelation function between the input and output of the nonlinear system.

The $(i+1)$th moment of the input M-sequence $u(t)$ can be written as

$$\overline{u(t-\tau)u(t-\tau_1)u(t-\tau_2)\cdots u(t-\tau_i)} = \begin{cases} 1 & \text{(for certain } \tau) \\ -1/N & \text{(otherwise)} \end{cases} \tag{4}$$

where N is the period of the M-sequence. When we use the M-sequence with the degree greater than 16, $1/N$ is in the order of 10^{-5}. So Eqn.(4) can be

approximated as a set of impulses which appear at certain τ's.

Eqn.(4) is due to the so-called shift and add property of the M-sequence, that is, for any integer $k_{i1}^{(j)}, k_{i2}^{(j)}, \cdots k_{i,i-1}^{(j)}$ (suppose $k_{i1}^{(j)} < k_{i2}^{(j)} < \cdots, k_{ii-1}^{(j)}$), there exists a unique $k_{ii}^{(j)}(mod\ N)$ such that

$$u(t)u(t+k_{i1}^{(j)})u(t+k_{i2}^{(j)})\cdots u(t+k_{i,i-1}^{(j)}) = u(t+k_{ii}^{(j)}) \tag{5}$$

where j is the number of a group $(k_{i1}, k_{i2}, \cdots, k_{i,i-1})$ for which Eqn.(5) holds. We assume that total number of those groups is m_i(that is, $j = 1, 2, \cdots, m_i$). Note that when $k_{ir}^{(j)}(r = 1, 2, ...i)$ satisfy Eqn.(5), then $2^p k_{ir}^{(j)}$ also satisfy Eqn.(5) for any integer p. Therefore Eqn.(4) becomes unity when

$$\tau_1 = \tau - k_{i1}^{(j)}, \tau_2 = \tau - k_{i2}^{(j)}, \cdots \tau_i = \tau - k_{ii}^{(j)} \tag{6}$$

Therefore Eqn.(3) becomes

$$\phi_{uy}(\tau) \simeq \sum_{i=1}^{\infty}\sum_{j=1}^{m_i} g_i(\tau - k_{i1}^{(j)}, \tau - k_{i2}^{(j)}, \cdots \tau - k_{ii}^{(j)}) \tag{7}$$

Since $g_i(\tau_1, \tau_2, \cdots \tau_i)$ is zero when any of τ_i is smaller than zero, each $g_i(\tau - k_{i1}^{(j)}, \tau - k_{i2}^{(j)}, \cdots \tau - k_{ii}^{(j)})$ in Eqn.(7) appear in the crosscorrelation function $\phi_{uy}(\tau)$ when $\tau > k_{ii}^{(j)}$.

Here the problem is how to obtain $g_i(\tau_1, \tau_2, \cdots \tau_i)$ from Eqn.(7), in which $\phi_{uy}(\tau)$ is the measured crosscorrelation function and those $g_i(\tau_1, \tau_2, \cdots \tau_i)$ are unknown. Generally speaking, the precise determination of $g_i(\tau_1, \tau_2, \cdots \tau_i)$ from Eqn.(7) is impossible since there are many variables with only one equation. But when we consider the general property of $g_i(\tau_1, \tau_2, \cdots \tau_i)$ which usually exists locally, that is, $g_i(\tau_1, \tau_2, \cdots \tau_i)$ appears only in a limited range of τ_i's (say $0 < \tau_1, \tau_2 \cdots \tau_i < 50\Delta t$, where Δt is the time increment of the measurement time), we can separate those g_i's from $\phi_{uy}(\tau)$ by suitable choice of the M-sequence. That is, if the $k_{ii}^{(j)}$ of i-th Volterra kernel g_i are sufficiently apart from each other (say, more than $50\Delta t$), we can separate each Volterra kernel $g_i(\tau - k_{i1}^{(j)}, \tau - k_{i2}^{(j)}, \cdots, \tau - k_{ii}^{(j)})$ from Eqn.(7). Volterra kernels $g_i(\tau_1, \tau_2, \cdots \tau_i)$ are obtained as a set of cross-sections along 45 degree lines in $(\tau_1, \tau_2, \cdots, \tau_i)$ space. In order for this to be realized, we have to select suitable M-sequences, which make the appearance of each cross-section of Volterra kernels sufficiently apart each other. Since an M-sequence is generated by a characteristic polynomial which is primitive polynomial over GF(2), we have searched all primitive polynomials over GF(2) of the degree up to 34 (total number of the polynomials is 6187) to find those suitable M-sequences. That is, we searched by computer those $k_{ir}^{(j)}(r = 1, 2, \cdots i)$which satisfy Eqn.(5) and selected those set of $k_{ir}^{(j)}$ where $k_{ii}^{(j)}$ is apart from any other $k_{pp}^{(j')}(p \neq i)$ by more than $30\Delta t$. We obtained some of the usable M-sequences with which we can obtain Volterra kernels of up to 3rd order with maximum argument of the Volterra kernels being less than $30\Delta t$. (See reference 3).

3. Measurement of Volterra kernels up to 3rd order

Let us consider the case where the nonlinear system can be expressed by up to 3rd order Volterra kernels. That is,

$$\begin{aligned} y(t) &= \int_0^\infty g_1(\tau_1)u(t-\tau_1)d\tau_1 \\ &+ \int_0^\infty\int_0^\infty g_2(\tau_1,\tau_2)u(t-\tau_1)u(t-\tau_2)d\tau_1 d\tau_2 \\ &+ \int_0^\infty\int_0^\infty\int_0^\infty g_3(\tau_1,\tau_2,\tau_3) \\ &\times u(t-\tau_1)u(t-\tau_2)u(t-\tau_3)d\tau_1 d\tau_2 d\tau_3 \end{aligned} \tag{8}$$

When we take the crosscorrelation function between $u(t)$ and $y(t)$, we have

$$\begin{aligned} \phi_{uy}(\tau) &= \int_0^\infty g_1(\tau_1)\overline{u(t-\tau)u(t-\tau_1)}d\tau_1 \\ &+ \int_0^\infty\int_0^\infty g_2(\tau_1,\tau_2)\overline{u(t-\tau)u(t-\tau_1)u(t-\tau_2)}d\tau_1 d\tau_2 \\ &+ \int_0^\infty\int_0^\infty\int_0^\infty g_3(\tau_1,\tau_2,\tau_3) \\ &\times\overline{u(t-\tau)u(t-\tau_1)u(t-\tau_2)u(t-\tau_3)}d\tau_1 d\tau_2 d\tau_3 \\ &\simeq \Delta t g_1(\tau) + 2(\Delta t)^2\sum_{j=1}^{m_2} g_2(\tau - k_{21}^{(j)}, \tau - k_{22}^{(j)}) \\ &-2(\Delta t)^3 g_3(\tau,\tau,\tau) + 3(\Delta t)^3\sum_{g=1}^{N} g_3(\tau,q,q) \\ &+6(\Delta t)^3\sum_{j=1}^{m_3} g_3(\tau - k_{31}^{(j)}, \tau - k_{32}^{(j)}, \tau - k_{33}^{(j)}) \end{aligned} \tag{9}$$

Therefore Eqn.(8) becomes

$$\begin{aligned} \phi_{uy}(\tau) &= \Delta t g_1(\tau) + F(\tau) \\ &+2(\Delta t)^2\sum_{j=1}^{m_2} g_2(\tau - k_{21}^{(j)}, \tau - k_{22}^{(j)}) \\ &+6(\Delta t)^3\sum_{j=1}^{m_3} g_3(\tau - k_{31}^{(j)}, \tau - k_{32}^{(j)}, \tau - k_{33}^{(j)}) \end{aligned} \tag{10}$$

where

$$F(\tau) = (\Delta t)^3 g_3(\tau,\tau,\tau) + 3(\Delta t)^3\sum_{q=1}^{m_1} g_3(\tau,q,q) \tag{11}$$

In general case, we have,

$$\begin{aligned} \phi_{uy}(\tau) &= \Delta t g_1(\tau) + F(\tau) \\ &+ \sum_{i=2}^{\infty} i!(\Delta t)^i\sum_{j=1}^{m_i} g_i(\tau - k_{i1}^{(j)}, \tau - k_{i2}^{(j)}, \cdots, \tau - k_{ii}^{(j)}) \end{aligned} \tag{12}$$

Here the function $F(\tau)$ is the function of τ and sum of the odd order Volterra kernels when some of its argument are equal. Since $F(\tau)$ appears together with $g_1(\tau)$ in a overlapped manner, $F(\tau)$ must be calculated from the odd order Volterra kernels and be subtracted from the measured $g_1(\tau)$ in order to obtain the accurate $g_1(\tau)$.

4. Basic strategy of Model Predictive Control

Model predictive control consists of calculating the control action that will make the predicted output equal to a conveniently selected desired output.

Here we explain the method of Model Predictive Control using Fig.1 briefly.

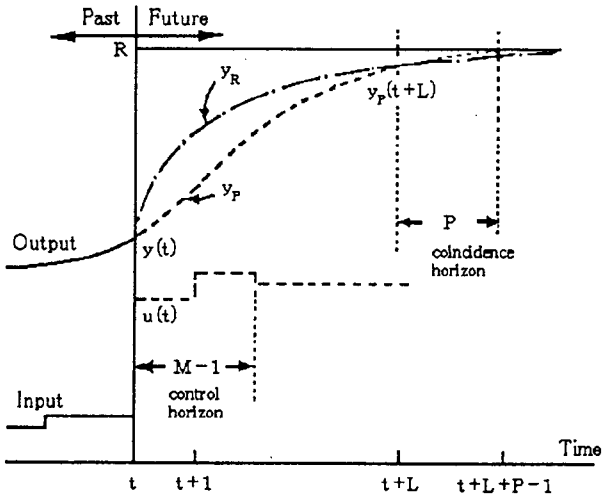

Fig.1 : The basic cocept of Model Predictive Control

At the present time t the process output $y(t)$ is observed. The desired output $y_R(t+j)(L \leq j \leq L+P-1)$ is calculated so as to approach to a target value R and the predicted output $y_P(t+j)(L \leq j \leq L+P-1)$ is calculated. Then the behavior of the process over a coincidence horizon P is checked and an evaluation function J is calculated .

Input $u(t)$ over a control horizon M is calculated so as for the evaluation function J to become minimum; that is $y_P(t+j)$ coincides $y_R(t+j)$ for $(L \leq j \leq L+P-1)$. Only the first value of the calculated input $u(t)$ is implemented at the next time $t+1$.

At the time $k+1$ the calculation is repeated with the time moved to the right by one time interval.

5. Nonlinear MPC

While we can deal with mild nonlinearities just by detuning linear controllers, it is likely that in the presence of strong nonlinearities, nonlinear controllers offer distinct advantages.

Therefore, we made third-order Volterra kernel model and used it for MPC rather than the traditional ways of using linear model. The third order Volterra kernel model is given by the following expression:

$$\begin{aligned} y_M(t+j) &= \sum_{k=1}^{q} g_1(k)u(t+j-k) \\ &+ \sum_{l=1}^{r}\sum_{m=1}^{r} g_2(l,m)u(t+j-l)u(t+j-m) \\ &+ \sum_{f=1}^{n}\sum_{i=1}^{n}\sum_{p=1}^{n} g_3(f,i,p) \\ &\times u(t+j-f)u(t+j-i)u(t+j-p) \end{aligned} \quad (13)$$

where g_1 , g_2 and g_3 are first , second and third order Volterra kernels,respectively.

Other expressions are given by the following expression:

The predicted output $y_P(t+j)$:

$$y_P(t+j) = y(t) + y_M(t+j) - y_M(t) \quad (14)$$

The desired output $y_R(t+j)$:

$$y_R(t+j) = a^j y(t) + (1-a^j)R \quad (15)$$

where $\alpha(0 \leq \alpha \leq 1)$ is a tuning parameter.

From there expressions, the evaluation function is calculated as follows:

$$J := \min_{u(t),\ldots,u(t+M-1)} \sum_{j=L}^{L+P-1} \{y_P(t+j) - y_R(t+j)\}^2 \quad (16)$$

The manipulating input that minimizes the evaluation function J, is searched to obtain the optimum input by use of the hill-climbing method.

6. Simulation of nonlinear MPC

This method of nonlinear MPC is applied to two nonlinear systems:

1. Wiener type nonlinear system
2. chemical process

6.1 Wiener type nonlinear system

Wiener type nonlinear system having linear and nonlinear part is shown in Fig.2.

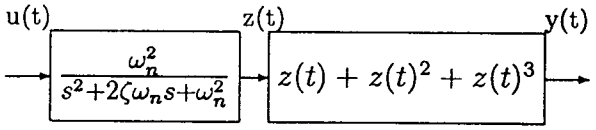

Fig.2 : Wiener type nonlinear system

where ζ is the damping ratio, and ω_n is the natural frequency of the second order system. This system

is identified by use of M-sequence, and first, second and third order Volterra kernels are calculated.

After that, nonlineaer MPC is carried out by use of up to third order Volterra kernel model, and the result is shown in Fig.3.

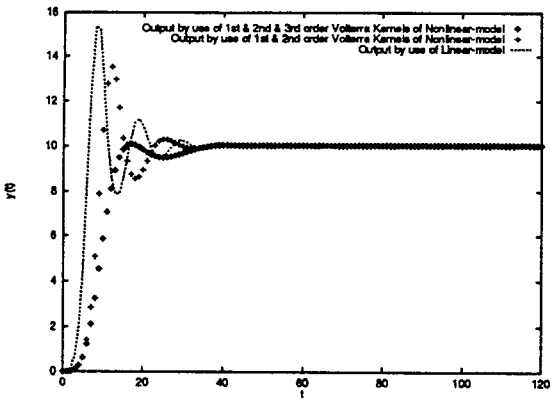

Fig.3 : Output data
Solid: Result of linear MPC
Plus: Result of nonlinear MPC by use of 1st and 2nd Volterra Kernels
Square: Result of nonlinear MPC by use of 1st, 2nd and 3rd Volterra Kernels

6.2 Nonlinear chemical process

The next example of nonlinear system is a chemical reactor having the next differential equation.

$$\begin{aligned} \frac{dx_1}{dt} &= \frac{1}{Tp_1}(-x_1 + Kp_1 u_1) \\ \frac{dx_2}{dt} &= \frac{1}{Tp_2}(Kp_2 x_1 x_2 - x_2 + Kp_3 u_2) \\ y &= x_2 \end{aligned} \tag{17}$$

where x_1 is the consumption velocity of catalyst, x_2 is gas density, u_1 is the supply quantity of catalyst, u_2 is the supply quantity of polyethylene, and Tp_1, Tp_2, Kp_1, Kp_2, Kp_3 are constant. Here the input u(t) to this process is thought to be u_1 and the output y(t) is x_2. Applying suitable input u(t), we would like to make the output approach to a predetermined volue R.

This system is identified by use of M-sequence , and first , second and third order Volterra kernels are calculated. After that, nonlinear MPC is carried out by use of up to third order Volterra kernel model, and the result is shown in Fig.4.

From Fig.4, the simulation results show that the use of up to 3rd order Volterra kernel model provides us the best control performance than the case of linear or up to 2nd nonlinear model.

7. Conclusion

The method of MPC for nonlinear process by use of third order Volterra kernel model is shown that the result of nonlinear MPC by use of up to 3rd order Volterra model is superior to the result of conventional linear MPC where inaccurate linear model is used for nonlinear process and the result of nonlinear MPC by use of up to 2nd order Volterra model. The problems which should be solved is development of more efficient calculation for input $u(t)$.

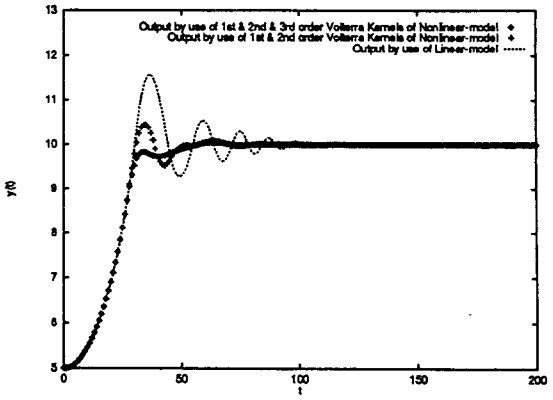

Fig.4 : Output data
Solid: Result of linear MPC
Plus: Result of nonlinear MPC by use of 1st and 2nd Volterra Kernels
Square: Result of nonlinear MPC by use of 1st, 2nd and 3rd Volterra Kernels

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