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The extremal shift method for the feedback optimal game-control problems

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Abstract: The report presents an approach to constructing of control algorithms for finite dimensional dynamical systems under the deficit of information about dynamical disturbances. The approach is based on the constructions of the extremal shift strategy of the differential game theory.

Keywords: Control, differential games

1. Introduction

One of the main problem of control theory is to construct the feedback controls that would be stable with respect to dynamical disturbances. An approach to solving this problem consists of formalization an optimal control problem for the minimax or maximin of the given quality index as an antagonistic differential game of two person (zero-sum game) and application of the corresponding methods of the differential game theory. Such formalization is used in the present report.

2. Control problem

We consider feedback optimal control problem under deficit or lack of information about the dynamical disturbances. The control object (system) is described by ordinary differential equations

$$\dot{x} = f(t, x, u, v)\,, \qquad (1)$$

where $f(\cdot,\cdot,\cdot,\cdot,) : [t_0, \theta] \times \mathbf{R}^n \times P \times Q \to \mathbf{R}^n$, $P \in \mathbf{R}^r$, $Q \in \mathbf{R}^m$, $u \in P$ is a vector of control parameters, $v \in Q$ is a vector which characterizes unknown disturbances.

In this report we consider the case when the resources of the regulator u and the device v, that introduces the disturbances to the object, are bounded, e.g. the sets P and Q are convex bounded closed sets in the corresponding spaces.

The problem of optimal feedback control is considered during finite time interval $T = [t_0, \theta]$ and the cost of control has the form of some estimation over the whole time interval

$$\gamma = \phi(x[t], u[t], v[t], t_0 \le t \le \theta)\,.$$

We formalize the problem (1) – (2) as the zero-sum differential game and use the so-called "method of extremal shift" [1, 2] for elaborating the corresponding feedback control law $u^0[t, x]$.

To construct the feedback control we choose and fix some partition of the interval T by points τ_i :

$$\tau_0 = t_0 < \tau_1 < ... < \tau_N = \theta,$$

$\Delta = \max\{\tau_i - \tau_{i-1} : i = 1, 2, ..., N\}$.

At every time interval $[\tau_{i-1} - \tau_i)$ we construct the control law according to the extremal shift strategy [1, 2] that consists in solving some special finite dimensional minimax (maximin) problem. The obtained vector $u_i \in P$ is applied to the system (1) during the interval $[\tau_{i-1} - \tau_i)$, i.e. $u^0[t, x] = u_i$ for $t \in [\tau_{i-1} - \tau_i)$.

Thus, though the continuous time nonlinear dynamical process is considered, the construction of the feedback control $u^0[t, x]$ is based on step–by–step discrete time procedure and at every step it is necessary to solve finite dimensional minimizational problem.

In the report we present the simulation results of application of the proposed uniform algorithms to control of the specific dynamical systems.

Reference

[1] A.N. Krasovskii and N.N. Krasovskii, *Control Under Lack of Information.* Birkhauser, Boston, USA, 1995.

[2] N.N. Krasovskii and A.I. Subbotin, *Game-Theoretical Control Problem.* Springer Verlag, Berlin – New York, 1988.