

The Coupling Effects of Excitatory and Inhibitory Connections Between Chaotic Neurons Having Gaussian-shaped Refractory Function With Hysteresis

Changkyu Choi and Ju-Jang Lee

Department of Electrical Engineering

Korea Advanced Institute of Science and Technology

FAX: +82-42-869-3410, Email: flyers@iliad.kaist.ac.kr, jjlee@ee.kaist.ac.kr

Abstract—Neural Networks, modeled succinctly from the real nervous system of a living body, can be categorized into two folds; artificial neural network(ANN) and biological neural network(BNN). While the former has been developed to solve practical problems using function approximation capability, pattern classification, clustering algorithm, etc, the latter has been focused on verifying the information processing capability to which brain research gives an impetus, by mimicking real biological systems. However, BNN suffers from severe nonlinearities dealt with. A bridge between two neural networks is chaotic neural network(CNN), which simply delineate the real nervous system and comprises almost all the ANN structures by selecting parameters. Main research theme of this area is to develop an explanation tool to clarify the information processing mechanism in biological systems and its extension to engineering applications. The CNN has a Gaussian-shaped refractory function with hysteresis effect and the chaotic responses of it have been observed for a wide range of parameter space. Through the examination of the coupling effects of excitatory and inhibitory connections, the secrets of information processing and memory structure will appear.

I. INTRODUCTION

Currently used artificial neural networks employ mostly simple neuron models that greatly differ from the real biological neurons. To ensure progress in biologically inspired neural processing, more advanced neuron models must be developed that better reflect the biological functionality. Biological neurons reveal rich dynamics and they are connected with each other via synaptic connection to show locked, unlocked and even chaotic responses. The main interest of this paper is to gain an insight of the complex dynamics in biological neural networks and one of the aims of studying them is to understand information processing in the brain.

Modeling of the dynamics of biological neurons for information processing dating from forties include McCulloch-Pitts's [1], Hodgkin-Huxley's [2], Fitzhugh's [3], Caianiello's [4], Nagumo-Sato's [5], Freeman's [6], Aihara's [7], Hansel-Sompolinsky's [8], Szu's [9] and more. These models are categorized into differential flows and

discrete maps and these differ in delay strategy for generating chaos. Most of the models relies on adjustable synaptic weighting of incoming excitatory and inhibitory signals, post synaptic summation, and static thresholding. To make the models be more efficient in information processing, they must also include adjustable synaptic time-delays, short-term memories, dynamic thresholding to obtain variable refractory periods, and so on.

The chaotic neuron model proposed in this paper was originally developed to solve the local minima problem in gradient based search [10] [11]. However, it also has interesting response characteristics, suitable for information processing, which is mainly due to the Gaussian-shaped refractory function with hysteresis. The analysis on the dynamics of the excitatory and inhibitory couplings of chaotic neurons will enable us to make a whole network structure and to learn it.

II. SINGLE CHAOTIC NEURON MODEL AND ITS DYNAMICS

The proposed chaotic neuron reveals the chaotic responses of a biological neuron, which models the refractory effect with Gaussian-shaped hysteresis function [10][11]:

$$p(t+1) = f \left(A(t) - \alpha \sum_{d=0}^t k^d g(p(t-d)) - \Theta \right), \quad (1)$$

where $p(t)$ is the neuron output with an analogue value between -1 and 1 at the discrete time t , f is the output function, $A(t)$ is the external stimulation at the time t , g is the refractory function and α , k , and Θ are the refractory scaling parameter, the refractory decay parameter, and the threshold, respectively.

Defining an internal state $q(t+1)$ of the neuron by

$$q(t+1) = A(t) - \alpha \sum_{d=0}^t k^d g(p(t-d)) - \Theta, \quad (2)$$

we can obtain the following reduced difference equation

on the internal state:

$$q(t+1) = kq(t) - \alpha g(f(q(t))) + a(t), \quad (3)$$

where $a(t) \equiv A(t) - kA(t-1) - \Theta(1-k)$. The output $p(t+1)$ of the neuron is calculated from the internal state $q(t+1)$ through the output function f as follows:

$$p(t+1) = f(q(t+1)). \quad (4)$$

Based upon Caianiello's neuron equation and Nagumo-Sato model the chaotic neuron model includes conventional models of neural networks. On the contrary to the Hodgkin-Huxley model, whereas the chaotic neuron model is far from the realistic biological neuron mechanism, it can qualitatively illustrate the phenomenon of the real neuron. Hence, the use of chaotic neuron model will be one challenging way to compromise the realistic nature of biological neuron model and the applicability of various practically interesting problems, because of their mathematical clearness and hidden potential.

Without loss of generality, the proposed chaotic neuron can be extended to the m D vector form:

$\mathbf{p}(t) = [p_1(t) \dots p_m(t)]^T$, $\mathbf{q}(t) = [q_1(t) \dots q_m(t)]^T$ and $\mathbf{a}(t) = [a_1(t) \dots a_m(t)]^T$, where

$$p_i(t+1) = f(q_i(t+1)), \quad (5)$$

$$q_i(t+1) = kq_i(t) - \alpha g_i(\mathbf{p}(t)) + a_i(t), \quad (6)$$

and $i = 1, 2, \dots, m$. $p_i(t+1)$ is the i th component of the neuron output at the discrete time $t+1$, which takes an analog value between -1 and 1; f is a continuous output function, *e.g.*, the sigmoid function; and g_i is a function describing the relationship between the analog output and the magnitude of the refractoriness to the following stimulation. $a(t)$ is the strength of the input at t , α is a positive parameter and k is the damping factor of the refractoriness, which takes a value between 0 and 1.

The output function, f , and the refractory function, g_i , are proposed to have the following forms:

$$f(q) = \frac{2}{1 + e^{-q/\Phi}} - 1, \quad (7)$$

$$g_i(\mathbf{p}(t)) = \begin{cases} \frac{1}{2\pi\sigma^2} e^{-\|\mathbf{p}(t)\|^2/2\sigma^2} & \text{if } p_i(t) - p_i(t-1) \geq 0 \\ -\frac{1}{2\pi\sigma^2} e^{-\|\mathbf{p}(t)\|^2/2\sigma^2} & \text{otherwise,} \end{cases} \quad (8)$$

where Φ and σ control the shape of the functions f and g_i 's, respectively.

Throughout this paper the parameter set consisting of $k = 0.2$, $\alpha = 0.5$, $\Phi = 0.4$, $\sigma = 0.125$ and $q(0) = 0.2$

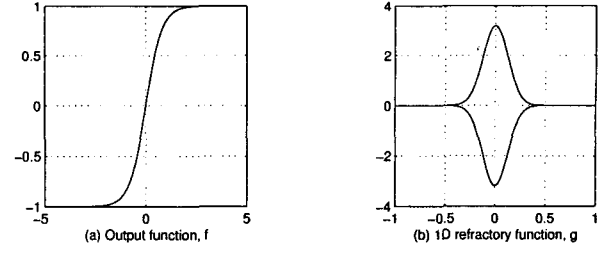


Fig. 1. Functions used in 1D chaotic neuron; (a) output function, f and (b) Gaussian-shaped refractory function with hysteresis – upper Gaussian is traced when the argument is increasing, while lower is traced when it is decreasing.

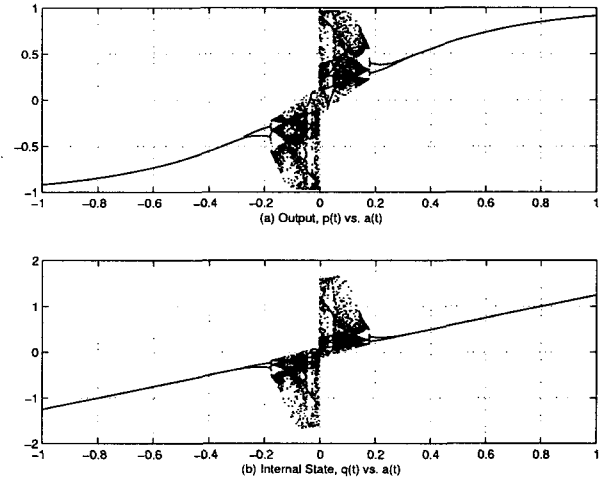


Fig. 2. Output and internal state patterns of 1D chaotic neuron. This pattern shows that the input space is partitioned two regions in fuzzy way. That is, for positive inputs the responses are restricted to positive values except for the inputs near the origin.

is used consistently. Note that the sigmoid function is bipolar. The refractory function g_i is a kind of hysteresis function. As the input increases, the upper Gaussian function is traced. Thus, it gives a directionality to the internal state q and it is analogous to the momentum. The $q(t+1)$ is drastically changed from $a(t)$ when $q(t)$ is near 0. Therefore, the output undergoes unstable motions near the origin.

The patterns of output $p(t)$ with respect to $a(t)$ for the 1D case are shown in Fig. 2 by utilizing the functions in Fig. 1. Also in this figure, 20 steady state evolutions are plotted for the fixed input stimulation after the transient responses vanish. When the magnitude of the input is larger than 0.2, the output is almost the same as the input. If the input is near 0, the output oscillates chaotically. It has the tendency to wander the positive region when the input is positive while it wanders the negative region when the input is negative, due to the refractoriness having hysteresis.

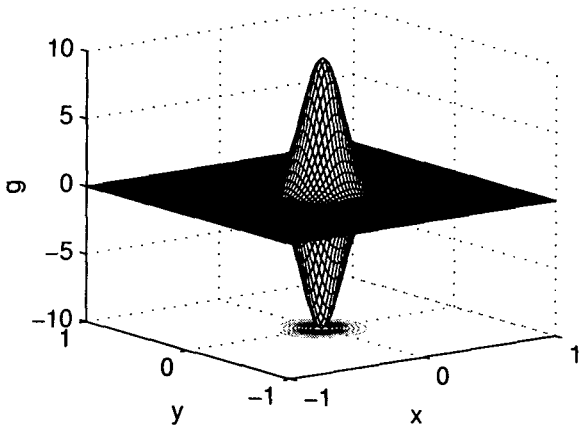


Fig. 3. The 2D Gaussian-shaped refractory function with hysteresis. Similar to the 1D case, upper Gaussian is traced when the argument is increasing, while lower is traced when it is decreasing.

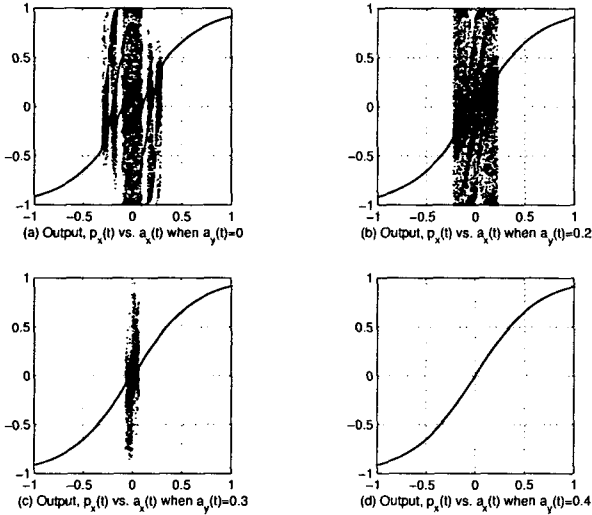


Fig. 4. Output patterns of 2D chaotic neuron, $p_x(t)$ versus $a_x(t)$ for various values of $a_y(t)$. For the inputs far from the origin chaotic response regions lose their sizes drastically and finally disappear. This is another way to express the coupling effect of two neurons at the next section.

Figure 3 shows the 2D refractory function with the same parameter set as in the 1D case. Concentric circles in $x - y$ plane represent contours. Output patterns of $p_x(t)$ with respect to $a_x(t)$ of the 2D chaotic neuron for various fixed values of $a_y(t)$ are depicted in Fig. 4. As $a_y(t)$ is away from zero, the region that oscillate chaotically lose its size. In other words, the chaotic motions are observed when $\mathbf{a}(t)$ is near $\mathbf{0}$. When only one component of $\mathbf{a}(t)$ is near 0, it does not reveal chaotic motion as seen in Fig. 4(d). Similar characteristics of steady state responses can be obtained in higher dimensional chaotic neurons.

III. COUPLING EFFECTS OF TWO CHAOTIC NEURONS

In neural networks literature hysteresis is mentioned as a phenomenon to avoid [12] and to make the system be able to associate proper memory retrieval [13]. To endow the system with the hysteresis function explicitly, we could make the systems to be synchronized [14][15] and to escape from the local minima [10][11].

The neural action is determined by three components, which are refractoriness (ζ_i), feedback connections from other neurons (η_i), which also involve time-delays, and external inputs (ξ_i):

$$\zeta_i(t+1) = k_r \zeta_i(t) - \alpha g_i(x_i(t)) + \theta_i, \quad (9)$$

$$\eta_i(t+1) = k_f \eta_i(t) + \sum_{j=1}^N W_{ij} p_j(t), \quad (10)$$

$$\xi_i(t+1) = k_e \xi_i(t) + \sum_{j=1}^M V_{ij} a_j(t), \quad (11)$$

where N and M are the numbers of neurons and inputs, respectively; V_{ij} 's represent input connection strength; W_{ij} 's are synaptic weightings; k_r , k_f and k_e are refractory, feedback, external input decay parameters between 0 and 1, respectively; and θ_i is a threshold. Together with the above expressions the output is

$$p_i(t+1) = f(\zeta_i(t+1) + \eta_i(t+1) + \xi_i(t+1)). \quad (12)$$

In this paper, we consider the case with the unit input connection strength, no time delays in external feedback connections, and null thresholds. Therefore, the coupling equations of two chaotic neurons are expressed as

$$q_1(t+1) = W_{11} q(t) - \alpha_1 g(p_1(t)) + W_{12} p_2(t) + a_1(t), \quad (13)$$

$$q_2(t+1) = W_{22} q(t) - \alpha_2 g(p_2(t)) + W_{21} p_1(t) + a_2(t), \quad (14)$$

$$p_1(t+1) = f(q_1(t+1)), \quad (15)$$

$$p_2(t+1) = f(q_2(t+1)), \quad (16)$$

where $W_{ii} = k_i$, $i = 1, 2$ are the self-feedback taking place inside of the neuron and are usually set to be in $[0, 1]$; W_{12} and W_{21} are the external feedback gains between -1 and 1 .

Simulation results are shown in figures 5 through 11. Each figure of figures 6 through 11 shows the change of dynamic response regimes as W_{12} varies for fixed W_{21} . Figures 6 through 8 and figures 9 through 11 have some symmetry because the response of the 1D chaotic neuron has symmetry as in fig. 2. However, taking a closer look at the figures 8 and 9, this similarity seems to

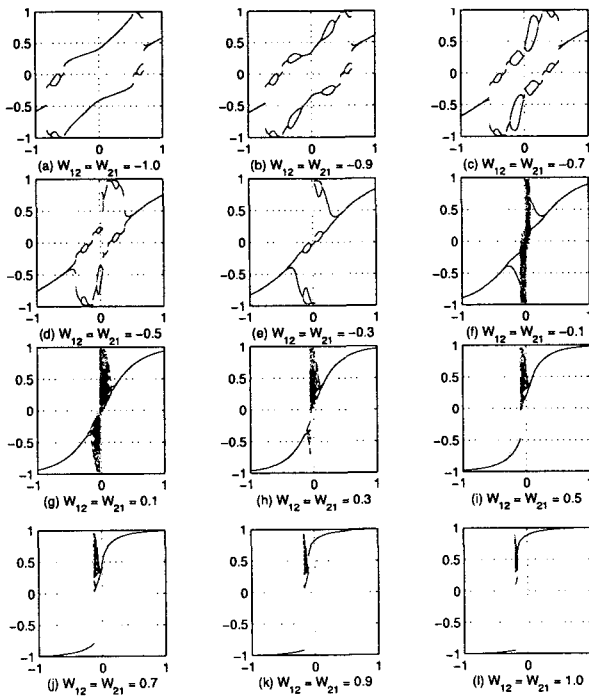


Fig. 5. Output $p_1(t)$ vs. $a(t)$. All the data are obtained from the condition of $W_{12} = W_{21}$. Thus, $p_2(t)$ vs. $a(t)$ has exactly the same response as these. It is an interesting feature that when two identical chaotic neurons are connected each other with the same weights, the external feedback term from the other seems to suppress chaos.

fail because the response characteristics of positive inputs and negative inputs in fig. 2 are not exactly the same. The variations of dynamic regimes are estimated and predicted by 1-D return map analysis and the details of them are shown at presentation.

IV. CONCLUSIONS

We considers the chaotic neuron model which was originally proposed to solve the local minima problem. The chaotic neuron involves hysteresis function and it's not the hard-limit type but the Gaussian function, which makes it more difficult to understand the response characteristics of the neuron.

The output pattern of an 1-dimensional chaotic neuron shows that the input space is approximately partitioned to two regions. For positive inputs the responses are restricted to positive values whereas for negative inputs the responses are restricted to negative values. However, for the inputs near the origin there's fuzziness in partitioning. That is, when a small positive input is applied to an 1D neuron, the output wanders chaotically over the open interval $(-0.2, 1)$. The output pattern of a 2-dimensional chaotic neuron shows that chaos appears for both the in-

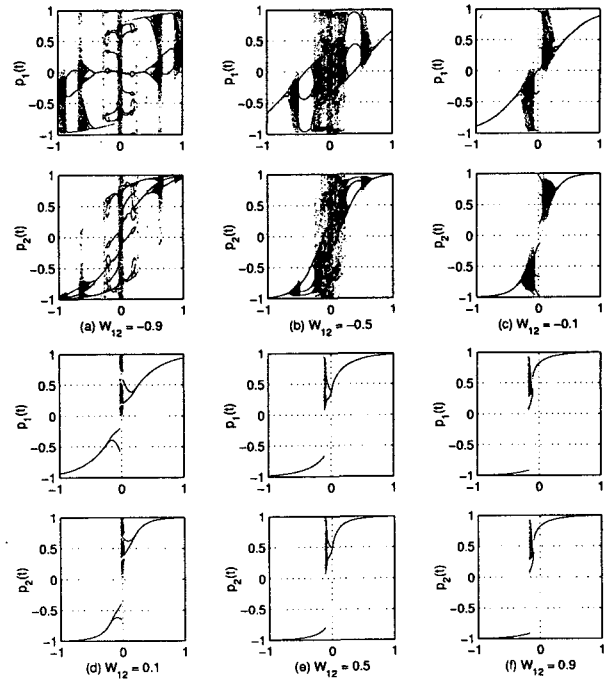


Fig. 6. Output $p_1(t)$ vs. $a(t)$ and $p_2(t)$ vs. $a(t)$. All the data are obtained with the fixed $W_{21} = 0.9$. The stronger the inhibitory connection is, the wider the region of chaotic response is.

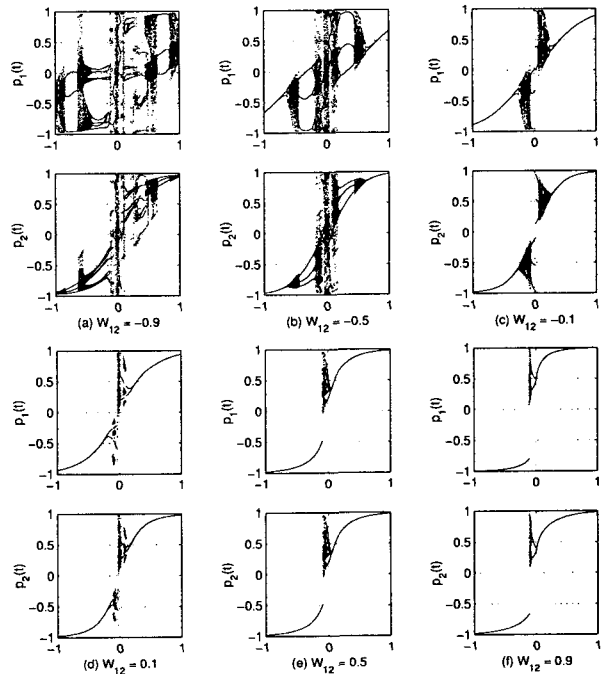


Fig. 7. Output $p_1(t)$ vs. $a(t)$ and $p_2(t)$ vs. $a(t)$. All the data are obtained with the same $W_{21} = 0.5$. Excitatory connections between neurons have a tendency to reduce chaos.

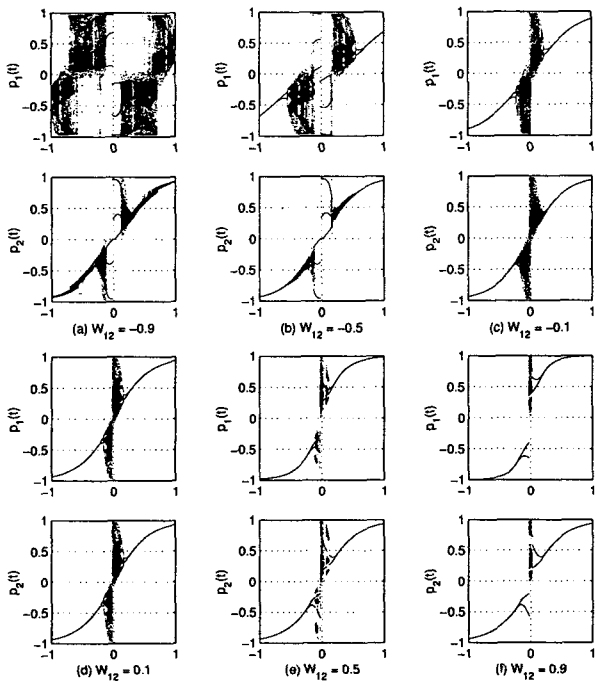


Fig. 8. Output $p_1(t)$ vs. $a(t)$ and $p_2(t)$ vs. $a(t)$. All the data are obtained with the fixed $W_{21} = 0.1$.

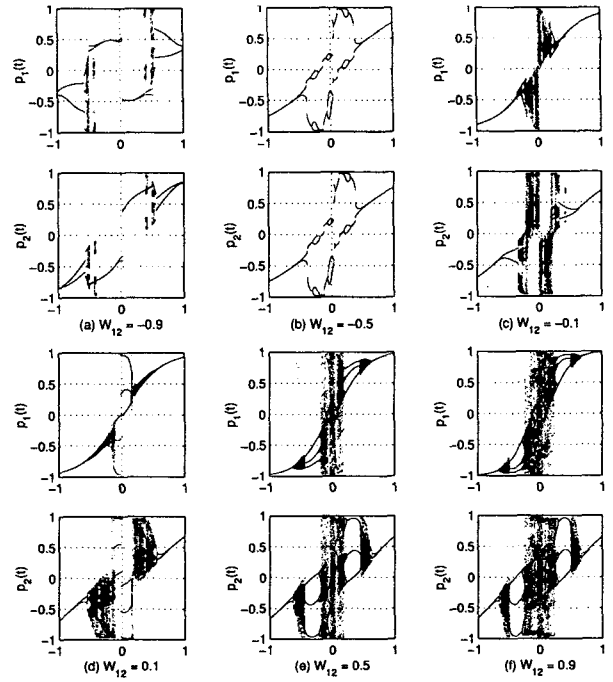


Fig. 10. Output $p_1(t)$ vs. $a(t)$ and $p_2(t)$ vs. $a(t)$. All the data are obtained with the fixed $W_{21} = -0.5$. Inhibitory connections between neurons have a tendency to reduce chaos.

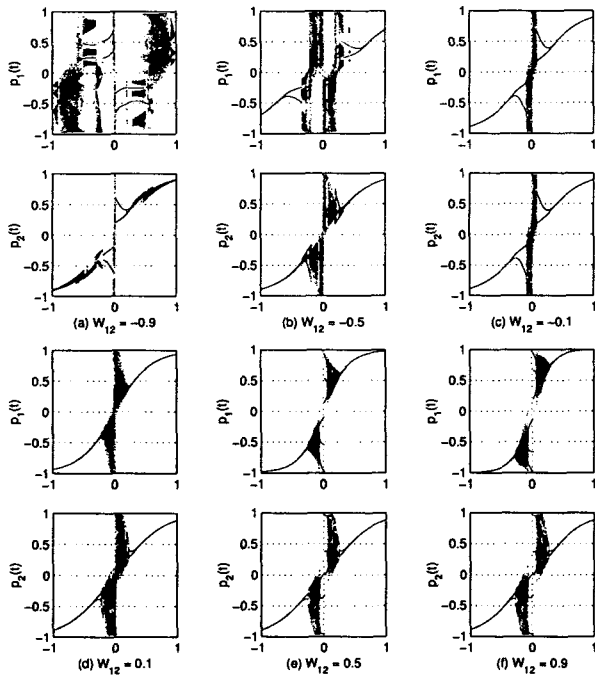


Fig. 9. Output $p_1(t)$ vs. $a(t)$ and $p_2(t)$ vs. $a(t)$. All the data are obtained with the fixed $W_{21} = -0.1$.

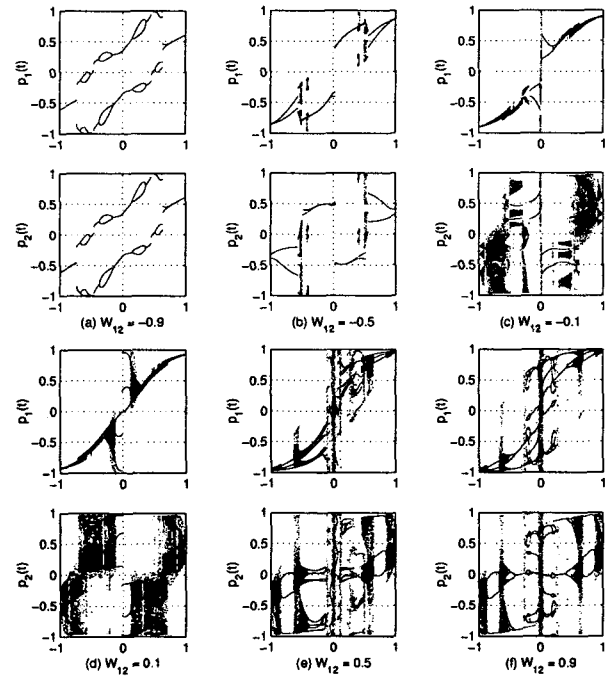


Fig. 11. Output $p_1(t)$ vs. $a(t)$ and $p_2(t)$ vs. $a(t)$. All the data are obtained with the fixed $W_{21} = -0.9$. The stronger the excitatory connection is, the wider the region of chaotic response is.

put arguments are near zero. When one input argument is far from zero, chaos disappears even if the other input argument is exactly on zero.

To investigate the coupling effects of two interconnected chaotic neurons, we connect two neurons with excitatory and inhibitory synaptic weights. When the external weights are equal, *i.e.* $W_{12} = W_{21}$, the region of chaos is reduced for the weakly connected case and chaos is suppressed for the strongly connected case. For the unequal external weights, (i) if the weights have the same sign – *i.e.* one neuron excites another and vice versa, or one neuron inhibits another and vice versa, the region of chaos tends to shrink. and (ii) if the weights have opposite signs, the region of chaos tends to widen.

So far, we didn't examine the case of existence of time delay in external synaptic weightings. A strict comprehension of the coupling effects and incorporating time delays into the chaotic neuron remain as further studies.

REFERENCES

- [1] W. S. McCulloch and W. H. Pitts, "A logical calculus of the ideas immanent in neural nets," *Bull. Math. Biophys.*, Vol. 5, pp. 115–133, 1943.
- [2] A. L. Hodgkin and A. F. Huxley, "A quantitative description of membrane current and its application to conduction and excitation in nerve," *Journal of Physiology-London*, Vol. 117, pp. 500–544, 1952.
- [3] R. Fitzhugh, "Impulses and Physiological states in theoretical models of nerve membrane," *Biophysics*, Vol. 1, pp. 445–466, 1961.
- [4] E. R. Caianiello, A. Deluca, "Decision equation for binary systems: Application to neural behavior," *Kybernetik*, Vol. 3, pp. 33–40, 1966.
- [5] J. Nagumo and S. Sato, "On a response characteristic of a mathematical neuron model," *Kybernetik*, Vol. 10, pp. 155–164, 1972.
- [6] W. J. Freeman, "Simulation of chaotic EEG patterns with a dynamic model of the olfactory system," *Biological Cybernetics*, Vol. 56, pp. 139–150, 1987.
- [7] K. Aihara, T. Takabe, and M. Toyoda, "Chaotic Neural Networks," *Physics Letters A*, Vol. 144, pp. 333–340, 1990.
- [8] D. Hansel, H. Sompolinsky, "Synchronization and computation in a chaotic neural network," *Physical Review Letters*, Vol. 68, pp. 718–721, 1992.
- [9] H. Szu, G. Rogers, "Single neuron chaos," in proceedings of IJCNN, pp. III:103–108, Baltimore, 1992.
- [10] C. Choi and J. J. Lee, "Dynamical path-planning algorithm of a mobile robot: local minima problem and nonstationary environments," *Mechatronics*, Vol. 6, No. 1, pp. 81–100, 1996.
- [11] C. Choi and J. J. Lee, "Finding multiple local minima using chaotic jump," *International Journal of Cooperative Information Systems*, Vol. 7, No. 1, pp. 105–115, 1998.
- [12] Z. Tang, O. Ishizuka, M. Sakai and H. Matsumoto, "Analysis of hysteresis in Hopfield and T-Model neural networks," in Proceedings of IJCNN, pp. 1477–1480, 1993, Nagoya, Japan.
- [13] Y. Hayashi, "Ambiguously fluctuating associative memory model with hysteresis dependency," in Proceedings of IJCNN, pp. 2323–2326, 1993, Nagoya, Japan.
- [14] Y. Matsumoto and T. Saito, "Chaos, torus and related phenomena from a hysteresis neural network," in Proceedings of IJCNN, pp. 2343–2346, 1993, Nagoya, Japan.
- [15] H. Torikai and T. Saito, "Occasional linear connection for synchronization of chaos," *IEEE Trans. on Circuits and Systems – I: Fundamental theory and applications*, Vol. 43, No. 5, pp. 374–385, 1996.