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Noise and Fault Diagonois Using Control Theory

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Abstract

The goal of this paper is to describe an advanced method of the fault diagnois using Control Theory with reference to a crack detection, a new way to localize the crack position under infulence of the plant disturbance and white measurement noise on a rotating shaft. As a first step, the shaft is physically modelled with a finite element method as usual and the dynamic mathematical model is derived from it using the Hamilton - principle and in this way the system is modelled by various subsystems. The equations of motion with crack is established by adaption of the local stiffness change through breathing and gaping from the crack to the equation of motion with undamaged shaft. This is supposed to be regarded as reference for the given system.
Based on the fictitious model of the time behaviour induced from vibration phenomena measured at the bearings, a nonlinear State Observer is designed in order to detect the crack on the shaft. This is elementary NL- observer(EOB). Using the elementary observer, an Estimator(Observer) Bank is established and arranged at the certain position on the shaft. In case a crack is found and its position is known, the procedure for the estimation of the depth is going to begin.

1 Introduction

In this paper, as an indicator for the existence of a crack, the nonlinear dynamic effects, given rise by the change of the stiffness coefficients due to the rotation of the cracked shaft, is going to be investigated. These effects related to the measurements on the bearings, are one of the important clue to determinate the existence of the crack on the rotating shaft. But it is very difficult to set up the clear relation between crack and caused phenomena in the time domain operation. This is the main task in the area of the crack problem too. As a classical method, there are some simple ways to find the split on the shaft. For example to analyse the vibration peak, acoustics and to measure the oil temperature by the costdown and by the transition of the resonance. These methods are hardly to offer the solution to the localization clear relationships between phemomen and stiffness. Here a new method based on the theory of disturbance rejection control for the detection of the crack, estimating the position with respect to constant crack depth and the depth of a certain crack respectively. First of all, the basic state observer is established in the way to modify the given system into extended system with a linear fictitious model for the nonlinear system behaviour. In this consideration, the effects of the extended system which may be nonlinear, are interpreted as a internal or external disturbance which is unknown at the initial stage.

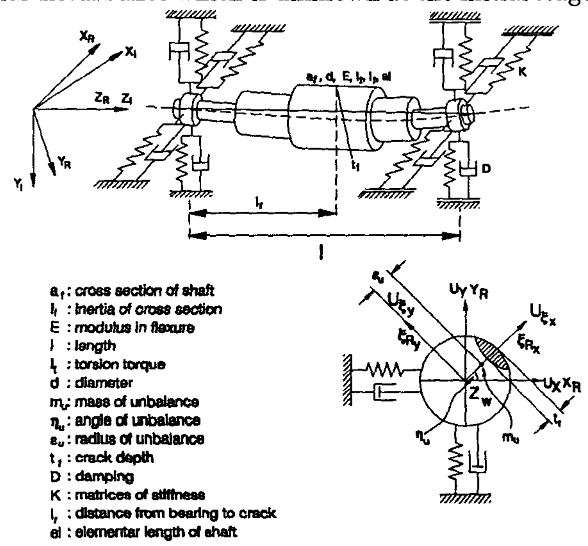

figure 1-1 : physical Model of the Rotor

The unknown nonlinear effects is going to be approximated by the additional time signals yielded by elementary state observer. Because of using FEM model, it is not necessary to calculate the relative compliance of the crack. Normally the elementary stiffness matrix for undamaged rotor is given in the stage of the construction and the stiffness corresponding to the crack is able to be calculated.
As an example the physical model, which is divided into N(=7) finite sub-shafts is modelled. Every one is called a subsystem. At both ends of the shaft there exist dynamic of the bearings. They have a task of system control. On the bearings at the left and right side of the shaft the measurements are taking place. It is assumed that the material properties are homogenous. The geometical data and other detailed informations are given in appendix.

2 Equation of Motion

Assuming that there are only small deviation from motion and without redundant coordinate, the system including 3 harmonic unbalances in the 3rd, 4th and 5th subsystems in the middle of the shaft

$$(M_g)\,\ddot{q}(t) + (D_{dg} + G_g)\,\dot{q}(t) + K_g\,q(t) = f_u(t) + f_g(t) + Ls(j)\,n(q(t),t) \quad (1)$$

can be accepted as linear system and it is able to be discretized into N(=7) sub - finite systems and it's equation of motion with crack at some subsystem j is described by,

$$i_e = 1, ..., N \quad (2)$$

$$j_k(i_e) = \left[(i_e - 1)\,\frac{n}{2} + 1\right]_{(i_e:1,...,N)} \quad (3)$$

$$i = j_k, ..., j_k + n - 1 \quad (4)$$

$$j = j_k, ..., j_k + n - 1 \quad (5)$$

With i_e, j_k, i and j the vector in explizit Form and the equation of motion can be given as follows:

$$q_{(i_e+1)(i)_{(i=1,..,\frac{n}{2}+1)}} = q_{(i_e-1)(\frac{n}{2}+i)} \quad (6)$$

$$\sum_{i_e=1}^{N} \sum_{j_k=j_k(i_e)}^{j_k(i_e)+n-1} [M_e\ddot{q}_{j_k(i_e)}(t) + (D_e + G_e)\dot{q}_{j_k(i_e)}(t) + K_e q_{j_k(i_e)}(t)]\} = [f_u(t)](i_e)_{(i_e=3,4,5)} + [f_g(t)](i_e)_{(i_e=1,...,N)} + Ls(n_f, i_e)[n(q_{(t)}(t),t)]_{(i_e=1,...,N)} \quad (7)$$

where the index g denotes the whole system and the index e presents the elementary subsystem and with

- $q(t), \dot{q}(t), \ddot{q}(t)$: displacement vector, velocity vector and acceleration of the system.
- M_g, K_g : mass matrix, stiffness matrix of undamaged section,
- $D_{dg},\ G_g = -G_g^T$ matrix of the damping and gyroscopic matrix.
- $q_e(t), \dot{q}_e(t), \ddot{q}_e(t)$: displacement vector, velocity vector and acceleration of the elementary sub systems. $q_e(t) \in \Re^n$, n(=8) and nn(=32) are degree of freedom of considered elementary sub system and total system. The $q_e(t)$ consists of $q_e(t) = (x_l, y_l, \theta_{xl}, \theta_{yl}; x_r, y_r, \theta_{xr}, \theta_{yr})$, the indices l and r denote the left and right node and $x_r, y_r, \theta_{xr}, \theta_{yr})$ are the coordinates at the sub system.
- $f_u(t), f_g(t)$, n(q(t),t) : vector of unbalance, gravitation input vector and vector of the nonlinearities caused by unexpected influence(crack)
- M_e, K_e : mass matrix, stiffness matrix of undamaged secton,
- $D_{de},\ G_e = -G_e^T,\ Ls_{(n_f,i_e)}$: matrix of the damping , gyroscopic effects and distribution vector with regard to the crack at subshaft number i_e.

All system matrices are constant and the distribute matrix is given as follows

$$Ls(i_e) = \underbrace{\begin{bmatrix} 000 & ,..., & \overbrace{1000}^{i_e.thposition} & ,..., & 000 \\ 000 & ,..., & \underbrace{0100}_{i_e.thposition} & ,..., & 000 \end{bmatrix}^T}_{(2,N)} \quad (8)$$

From now on the index j will be left out with respect to the whole dynamic system. It is normally convienient for further operation to write the above equation via state space notation with $x(t) = [q(t)^T, \dot{q}(t)^T]$ inclusive the nonlinearties of the motion created by a crack and under assumption that it concerns randum disturbance in plant with $s(t)$.

$$\dot{x}(t) = A\,x(t) + B\,u(t) + N_R n_R(x(t)) + W\,s(t) \quad (9)$$

The equation of the measurement into following form

$$y = C\,x(t) + w_m(t) \quad (10)$$

where A is $(N_n \times, N_n)$ dimensional system matrix which is responsible for the system dynamic with $N_n = 2nn$, u(t) denotes r -dimensional vector of the excitation inputs due to gravitation and unbalances and C presents $(m_e \times N_n)$-dimensional measurement matrix. W is the $(N_n \times N_n)$ dimensional matrix and $s(t)$ presents the plant vector of noise. w_m denotes the white measurement noise. x(t)is N_n -dimensional state vector, and y(t) is m_e -dimensional vector of measurements respectively.
Here the vector $n_R(x(t))$ characterize the n_f -dimensional vector of nonlinear functions due to the crack. N_R is the input matrix of the nonlinearties and the order of N_R is of (Nn, n_f). It is presupposed that the matrices A, B, C, N_R and the the vector u(t) and y(t) are already known. On the assumption that the plant noise and measurement noise are uncollated

$$E[w_s(t)] = 0, E[w_m(t)] = 0, \quad (11)$$
$$E[w_s(t) \quad w_m^T(t)] = 0, \quad (12)$$
$$E[w_s(t) \quad w_s^T(\tau)] = Q_s(t)\delta(t-\tau), \quad (13)$$
$$Q_s(t) = Q_s^T(t) \quad (14)$$
$$E[w_m(t)w_m^T(\tau)] = R_m(t)\delta(t-\tau), \quad (15)$$
$$R_m(t) = R_m^T(t) \quad (16)$$
$$E[w_s(t)x_0^T] = 0, E[w_m(t)x_0^T] = 0 \quad (17)$$

where the weighting matrix Q corresponding to the plant and R regarding to the measurement should be suitably chosen.

Now it remains to reconstruct the unknown nonlinear vector $n_R(x(t), t)$ which mentions the disturbance force caused by crack. The basic idea is to get the signals from $n_R(x(t))$ approximated by a linear fictitious model

$$n_R(x(t), t) \approx H\, v(t) \quad (18)$$
$$\dot{v}(t) = V\, v(t) \quad (19)$$
$$dim\, v(t) = s \quad (20)$$

that describes the time behaviour of the nonlinearities due to the apperance of the crack approximately as follows:

$$n_R(x(t), t) \approx \hat{n}_R(\hat{x}(t)) = H\hat{v}(t) \quad (21)$$

where $\hat{v}(t)$ follows from(25, see below The matrices H and V have to be chosen according to the technical background considered. In this way the additional forces created by crack are going to be reconstruced through the estimation of disturbance vector v(t).
To make the signals $\hat{n}(\hat{x}(t))$ available, it needs the elementary observer(EOB) to be designed.

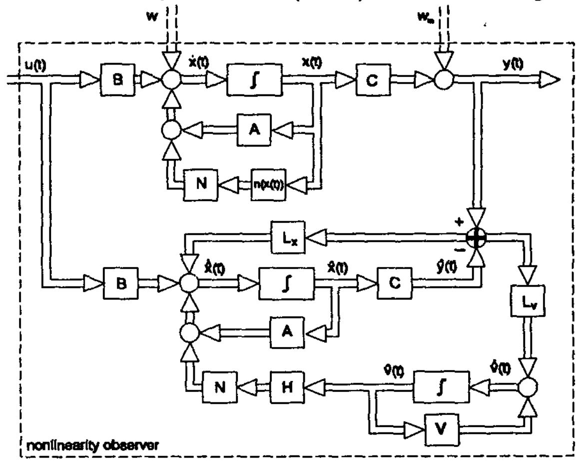

figure 1-2 : elementary observer: EOB

At first the given system(9) has to be extended with the fictitious model(18) into extended model

$$\underbrace{\begin{bmatrix} \dot{x}(t) \\ \dot{v}(t) \end{bmatrix}}_{\dot{x}_e} = \underbrace{\begin{bmatrix} A & N_R H \\ 0 & V \end{bmatrix}}_{A_e} \underbrace{\begin{bmatrix} x(t) \\ v(t) \end{bmatrix}}_{x_e(t)} + \underbrace{\begin{bmatrix} I \\ 0 \end{bmatrix}}_{B_e} \tilde{u}(t) \quad (22)$$

$$y(t) = \underbrace{\begin{bmatrix} C & \vdots & 0 \end{bmatrix}}_{C_e} \begin{bmatrix} x(t) \\ \cdots \\ v(t) \end{bmatrix} \quad (23)$$

Here,$N_R H$ couples the fictitiuous model(18,7) to the whole system. To enable the successful estimates, it's obligatory to pay attention to the condition $m_e \geq n_f$. i.e the number of the measurements must be at least equal or greater than the modelled nonlinearities. In the case the above requirements are satisfied, then the elementary observer in terms of an identity observer can be designed as follows;

$$\underbrace{\begin{bmatrix} \dot{\hat{x}}(t) \\ \dot{\hat{v}}(t) \end{bmatrix}}_{\dot{x}_o} = \underbrace{\begin{bmatrix} A - L_x C & N_R H \\ -L_v C & V \end{bmatrix}}_{A_o} \underbrace{\begin{bmatrix} \hat{x}(t) \\ \hat{v}(t) \end{bmatrix}}_{x(t)_o} + \underbrace{\begin{bmatrix} I \\ 0 \end{bmatrix}}_{Be} \tilde{u}(t) + \underbrace{\begin{bmatrix} L_x \\ L_v \end{bmatrix}}_{L_o} y(t) \quad (24)$$

$$\hat{y}(t) = \underbrace{\begin{bmatrix} C & \vdots & 0 \end{bmatrix}}_{C_e} \begin{bmatrix} \hat{x}(t) \\ \cdots \\ \hat{v}(t) \end{bmatrix}, \quad (25)$$

where matrices L_x and L_v are the gain matrix of the observer and white noise vector related to the state measurement respectively. The above equation(24) means that the observer consists of a simulated model with a correction feedback of the estimation error between real and simulated measurements. The matrix A_o has $(N_n + n_f, N_n + n_f)$ -dimension and represents the dynamic behaviour of the elementary observer. The asymptotic stability of the elementary observer can be guaranteed by a suitable design of the gain matrices L_x, L_v which is possible under the conditions of detectability or observabilty of the extended system(22, 23). The fictitious model of the crack behaviours is able to be designed as follows,

$$H = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (26)$$

$$V = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \quad (27)$$

$$n_{(R;1,x(t)_1)} \approx v_1(t) \quad (28)$$

$$n_{(R;2,x(t)_2)} \approx v_2(t) \quad (29)$$

The observer gain matrices L_x, L_v can be calculated by pole assignment or by the Riccati equation

$$0 = A{+}P{+}P\ A^T{-}P\ C^T R_m^{-1} C\ P{+} \begin{bmatrix} W \\ \dots \\ 0 \end{bmatrix} Q\left[\ W^T, 0\ \right] (30)$$

3 Estimator(Observer) Bank for the Localization of the Crack

In above section it has been studied how to design the elementary observer(EOB) for the detection at a given local position. It means that a certain place on the shaft is initially given as the position of a crack. In the real running operation there is not any information about the position of the crack, so the elementary observer has to survey not only the assigned local position but also any other place on the shaft and give the signals whether a crack exist or not. As it has been known, it is possible to detect the crack assigned certain place on the shaft. In case a crack appears at any subsystem out of the assigned or located position in running time, it must be detected as well. But in many cases it has been shown that it is impossible or very difficult to estimate the position of the crack at all subsystem on the shaft with one EOB. Generally it depend on the number of the subsystem, the number of observer(arrangement) and depth of the carck. For the estimation of a crack position a method based on Estimator Bank is presented. The main idea is to feel the related crack forces from a certain local position to the arranged elementary observer. This is the main task in this section.

3.1 Design of a Observer Bank

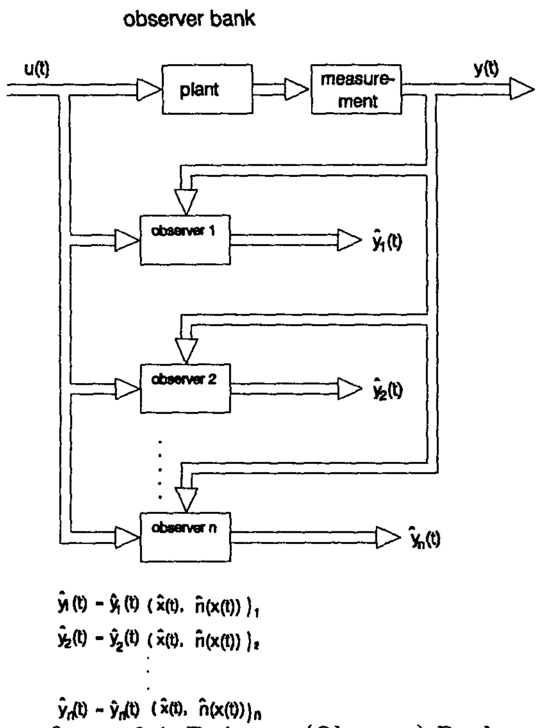

figure: 2-1, Estimator(Observer) Bank

Figure 2-1 shows the structure of the Observer Bank considered. It consists of a few of elementary observers. The number of elementary observer depends on the number of the subsystems modelled. Every elementary observer which is distinguished from the distribution matrix $Ls_{(i_e)}$ gets the same input(excitation) u(t) and the feed back of the measurements and is going to be set up at suitable place on the given system. For the appreciate arrangement of the EOB, the distribution matrix on the analogy of (8) has been applied. In this way the observer bank is established with the EOB. To estimate the local place of the crack, there are two steps. First of all, the EOB must be observable to certain local in the meaning of the asymptotical stability in the system. The requirement has been satisfied by the criteria from Hautus

$$Rank \begin{bmatrix} \lambda I_{N_n} - A & -N_R(Ls_{(i)})H \\ 0 & \lambda I_{n_f} - V \\ C_e & 0 \end{bmatrix}$$
$$= dim(x_e(t)) + dim(v(t)) = N_n + n_f (= s) \quad (31)$$

This means that the EOB has to be cappable of estimating the crack at it's location, where EOB is si-

tuated on the given system.
Secondly the unknown crack position is to be found by the EOB arranged in a certain local place with the related crack forces resulting from the a crack. To guarantee this the condition(31) is supposed to be fulfiled. In this work two EOB are arranged at the 2nd subsystem and 6th like this
$Ls_{(2)}(i=2)=1$, otherweis $Ls_{(2)}(i)=0$
$Ls_{(6)}(i=30)=1$, otherweis $Ls_{(6)}(i)=0$. The equation of the observer bank with EOB A at the 2nd subsystem

$$\underbrace{\begin{bmatrix}\dot{\hat{x}}(t)_{(2)}\\ \dot{\hat{v}}(t)_{(2)}\end{bmatrix}}_{\dot{x}_o} = \underbrace{\begin{bmatrix} A-L_xC & N_R(Ls_{(2)})H \\ -L_vC & V\end{bmatrix}}_{A_o} \underbrace{\begin{bmatrix}\hat{x}(t)_{(2)}\\ \hat{v}(t)_{(2)}\end{bmatrix}}_{x(t)_o} + \underbrace{\begin{bmatrix} I\\ 0\end{bmatrix}}_{b_e=b_o}\tilde{u}(t) + \underbrace{\begin{bmatrix}L_x\\ L_v\end{bmatrix}}_{L_o}(y(t)+w_m) \quad (32)$$

and EOB B at the 6th is descrived by

$$\underbrace{\begin{bmatrix}\dot{\hat{x}}(t)_{(6)}\\ \dot{\hat{v}}(t)_{(6)}\end{bmatrix}}_{\dot{x}_o} = \underbrace{\begin{bmatrix} A-L_xC & N_R(Ls_{(6)})H \\ -L_vC & V\end{bmatrix}}_{A_o} \underbrace{\begin{bmatrix}\hat{x}(t)_{(6)}\\ \hat{v}(t)_{(6)}\end{bmatrix}}_{x(t)_o} + \underbrace{\begin{bmatrix} I\\ 0\end{bmatrix}}_{b_e=b_o}\tilde{u}(t) + \underbrace{\begin{bmatrix}L_x\\ L_v\end{bmatrix}}_{L_o}(y(t)+w_m) \quad (33)$$

The weighting matrix Q and R have to be chosen like eq. in [3].

4 Example

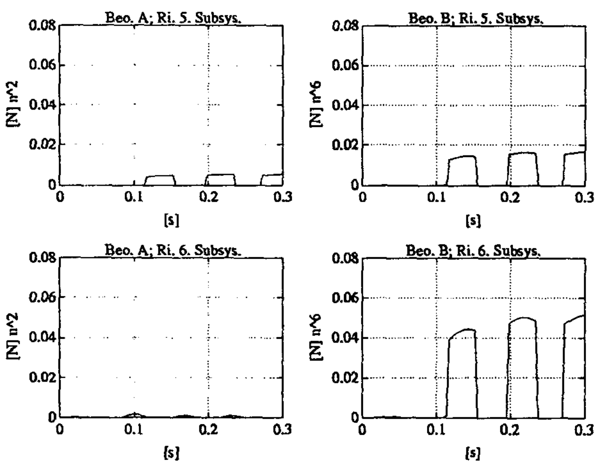

figure: 3-1, EOB A, B: crack in 1st and 2nd Subsystem, $t_{(ri;1)}=0.135$, $t_{(ri;2)}=0.15, t_{(s)}=0.03[s]$ Y coordinate: crack forcee in N , X coordinate: time in [sec],i = 1,7; j= 1, 2

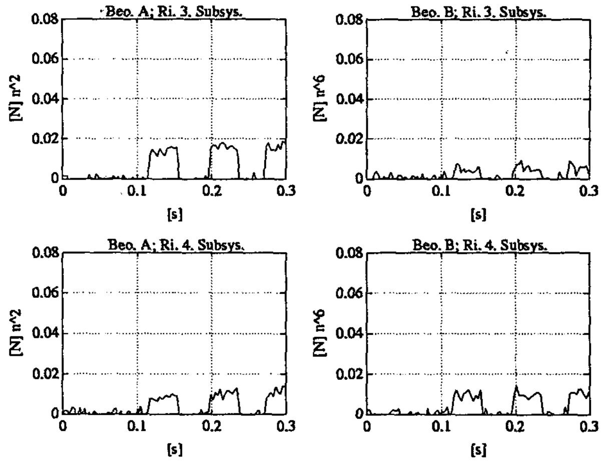

figure: 3-2, EOB, A, B: crack in 3rd and 4th Subsystem, $t_{(ri)}=0.15$, $t_{(s)}=0.03[s]$ Y coordinate: crack forcee in N , X coordinate : time in [sec] i = 2,6; j= 3, 4

The figures from 3-1 and 3-2 present the reults of the theoretical investigation and shows the crack forces at 1st, 2nd of node in vertical direction respectively and their corrupted signals due to randum disturbance in plant. It means also to estimate the local crack position under constant depth with respect to crack forces. These forces related from certain position of crack to EOB A and EOB B are supposed to be interpreted as mechanical forces due to the brathing and gaping from the Gash model[1]. The nummerical value of the ρ_q concerned with the weighting matrix Q are of $Q_{(i,j;i=j=1,\dots,32)}=10, Q_{(i,j;i=j=33,\dots,62)}=15$, $Q_{(i,j;i=j=63)}=2*10^4$, $Q_{(i,j;i=j=64)}=10^{4.25}$ respectively. The factor ρ_r of the weighting matrix R is of 0.975 and $diagR_{(i,j)}$ is of 1. It has been noticed that the observer estimates the signals very well. The external signal exists in case of the oppend crack. On analogy of the system model, the minimal and maximal values depend on the depth if only the crack is situated at the position where the EOB are located. Otherweise the position of the crack plays a part in the values of the forces regarding to the excited inputs as well. However, the crack forces are clear indicator for the apperance of a crack in operating time. The other figures which have been left out, shows that EOB B which is arranged at the right bearing, is not able to estimate the crack in 1st subsystem. In the simulation the given depth is of 2 mm and the time of appearance of the crack makes 0.2 sec.

5 Summary and Conclusions

Using FEM the mathematical model of the rotating shaft including a crack has been presented. Based on the mathematical model including plant randum disturbance, the elementary observer and an observer bank have been developed. With this observer bank the task of the crack detection and localization have been done. The above method give a clear relation between the damaged shaft by a crack and the caused phenomena in vibration by means of the measurement at both bearings. Successful theoretical results have been given. The forces in the results are the internal forces, which have been reconstructed as disturbance forces created by the crack. It has been theoretically shown, that it is possible to localize a crack with the very small depth of 1 mm under the randum plant noise. The measurement regarded as clean. By estimating the depth, the simulation has been succeeded in ascertaning the depth of 12 mm. The suggested methods are very significant not only for the further theoretical research and development but also for the transfer in experiments.
crack depth has been mentioned.

6 References

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7 Appendix

Using the abbreviation $ii = i - j_k + 1, jj = j - j_k + 1$ the sum of the matrices with accordance to equations (2) and (3) can be descrived as follows;

$$M^{(g)}_{(j_k,j_k)}(i_e) = \sum_{i_e=1}^{N}\left[\sum_{j_k=1}^{(i_e-1)\frac{n}{2}+1}\left(\sum_{i,j=j_k}^{j_k+n-1} M_e(ii,jj)\right)\right]_{(i_e)} + M^0_{(dime,dime)} \quad (34)$$

$$K^{(g)}_{(j_k,j_k)}(i_e) = \sum_{i_e=1}^{N}\left[\sum_{j_k=1}^{(i_e-1)\frac{n}{2}+1}\left(\sum_{i,j=j_k}^{j_k+n-1} K_e(ii,jj)\right)\right]_{(i_e)} + K^0_{(dime,dime)} \quad (35)$$

$$G^{(g)}_{(j_k,j_k)}(i_e) = \sum_{i_e=1}^{N}\left[\sum_{j_k=1}^{(i_e-1)\frac{n}{2}+1}\left(\sum_{i,j=j_k}^{j_k+n-1} G_e(ii,jj)\right)\right]_{(i_e)} + G^0_{(dime,dime)} \quad (36)$$

$$D^{(g)}_{(j_k,j_k)}(i_e) = \sum_{i_e=1}^{N}\left[\sum_{j_k=1}^{(i_e-1)\frac{n}{2}+1}\left(\sum_{i,j=j_k}^{j_k+n-1} D_e(ii,jj)\right)\right]_{(i_e)} + D^0_{(dime,dime)} \quad (37)$$

The matrices used in equation(9) are follows
$A =$

$$\underbrace{\begin{bmatrix} 0 & \vdots & I_{(nn)} \\ \cdots & \cdots & \cdots \\ -(M_g)^{-1}K_e & \vdots & -(M_g)^{-1}(D_{dg}+G_g) \end{bmatrix}}_{(64,64)} \quad (38)$$

The index i denotes the number of the subsystem. The vector of the order of the excitation and the matrix of nonlinearites,

$$\tilde{u}(t) = \begin{bmatrix} 0 \\ \cdots \\ M_g^{-1}\, f_e \end{bmatrix}, N_R(Ls_{(i)}) = \begin{bmatrix} 0 \\ \cdots \\ -M_g^{-1}\, Ls_{(i)} \end{bmatrix}_{(64,1)} \quad (39)$$

is of (64 x 1). where the vector of the excitation consists of graviation and harmonic unbalnce , is presented by

$$f_e = f_{(g,i_e;i=1,..,N)} + f_{(u,i_e=3,4,5)} \quad (40)$$

$$f_{(g;2)} = f_{(g;30)} = 0, \quad (41)$$

$$f_{(g;6)} = f_{(g;10)} = f_{(g;14)} = f_{(g;18)} = f_{(g;22)} = f_{(g;26)} = -mg, \quad (42)$$

The order of the f_g is of (32 x 1) and f_u is of (32 x 1).

$$f_{(u;17)} = f_{(u;21)} = f_{(u;25)} = -e_m \, \Omega^2 \, m_{(ex)} \, sin(\Omega t + \beta) \quad (43)$$

$$f_{(u;18)} = f_{(u;22)} = f_{(u;26)} = e_m \, \Omega^2 \, m_{(ex)} \, cos(\Omega t + \beta) \quad (44)$$

where angle of the phase:β =0, length of the subsystem of rotor $el = 2m$, Diameter of the subsystem of rotor makes $ed = 0.25m$. The mass of elemetal subsystem: $m = \pi \; el \; \rho \frac{ED^2}{4}$, The density is of $\rho = 7860 \frac{kg}{m^3}$ excentricity: e_m= 0.0001, mass of the excentricity: $m_{(ex)}$= 3 m respectively. The modulus $E \; i_f$ is of $2.1 * 10^5 N/mm^2$. The stiffness of bearing: $K_{beaing} = 15 * 10^5 N/mm^2$. The measurement matrix of order(4 x 64), $C_{(i=1,\ldots 4, j=1,\ldots,64)} = 0$, except $C_{(1,1)} = C_{(2,2)} = C_{(29,29)} = C_{(30,30)} = 1$. The number of the nonlinearities n_f are of 1 and the number of the measurements m_e makes 4. The elementar matrices K_e, M_e and D_g which depends on the geometry, are given in[4, 5, 6].