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System Realization by Using Inverse Discrete Fourier Transformation for Structural Dynamic Models

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Abstract : The distributed-parameter structures expressed with the partial differential equations are considered as the infinite-dimensional dynamic system. For implementation of a controller in multivariate systems, it is necessary to derive the state-space reduced order model. By the eigensystem realization algorithm, we can yield the subspace system with the Markov parameters derived from the measured frequency response function by the inverse discrete Fourier transformation. We also review the necessary conditions for the convergence of the approximation system and the error bounds in terms of the singular values of Markov-parameter matrices. To determine the natural frequencies and modal damping ratios, the modal coordinate transformation is applied to the realization system. The vibration test for a smart structure is performed to provide the records of frequency response functions used in the subspace system realization.

Keywords : inverse discrete Fourier transformation, Markov parameter, eigensystem realization, frequency response function, modal parameter.

1 Introduction

It has been the central problems of vibration specialists to determine the accurate control system models from the measured frequency response data. Undoubtedly, the engineering techniques for an experimental determination of the structural vibration modes have been developed for design improvements of passive vibration suppression systems. The accurate in-situ determination of the structural dynamic models far beyond the traditional modal testing and model determination accuracy levels are demanded in the active vibration control and the structural health monitoring. The control system model have to be build prior to the controller synthesis of the vibration suppression system, like as a smart structure attenuating the structural vibration levels actively. The distributed-parameter vibration systems expressed as the partial differential equations can be allowed the infinite-dimensional dynamic order to be truncated for the control system realization.

However, the structural system identification have notable progress in structural dynamics and is facilitated by the key theoretical framework of state-space modeling from FRF measurement data. The state-space-based structural system identification can provide the minimum set of model parameters that participate in the impulse responses induced from one of the finite dimensional model set. The impulse response or Markov parameters can be obtained from the FRFs obtained from the input and output spectral density functions with inverse discrete Fourier transformation(IDFT), or from the input and output time histroies with discrete wavelet transformation. The IDFT technique may be considered to still have overall advantages in the respect of modal testing practices, while it can have some of demerits in the system realization like as the Gibbs phenomenon, leakage, end effects and aliasing, and windowing of frequency response functions. Modern structural identification have been developed recently, whereas the eigensystem realization algorithm(ERA) [7] and the Q-Markov COVER [8] are widely used in the system realization algorithm. These realization methods can provide the balanced minimal order model of the discrete-time state space system.

Little attention has been given to the point how accurate the state space system of the reduced finite order model can be approximated to the infinite-dimensional transfer function obtained by the structural vibration testing. The present paper investigate the approximation error bounds and conditions for the transfer function. It also presents the ERA realization method with IDFT to find the natural frequencies and modal damping ratios of structural vibrations.

2 $\mathcal{H}_\infty$ Approximation by Using IDFT

We begin the approximation of the continuous transfer function by reviewing the mathematical preliminaries. The Hardy space, $\mathcal{H}_p(\mathcal{U})$, $p \geq 1$, and $\mathcal{H}_\infty$ consists of all analytic functions f on $\mathcal{U}$ with the following property, respectively, that

$$\sup_{\sigma>0}\left[\left(\frac{1}{2\pi j}\int_{-j\infty+\sigma}^{j\infty+\sigma}\|f(s)\|^p ds\right)^{\frac{1}{p}}\right]=\|f\|_p<\infty \quad (2.1)$$

$$\sup_{\sigma>0}\sup_{\omega\in[0,\infty]}|f(j\omega+\sigma)|=\|f\|_\infty<\infty \quad (2.2)$$

We also know that if $f\in\mathcal{H}_p(\mathcal{U})$, then a function $f(j\omega)$ on $\partial\mathcal{U}$ exists almost everywhere and belongs to Lebesgue space $\mathcal{L}_p$. The space $\mathcal{H}_p(\mathcal{U})$ is a Banach space and the $\mathcal{H}_2(\mathcal{U})$ is a Hilbert space with the inner product. For the matrix-valued functions analytic on $\mathcal{U}$, the quadratic $\|\mathbf{G}\|_2$ and uniform norms $\|\mathbf{G}\|_\infty$ of the $G_{ij}(s)\in\mathcal{H}_p(\mathcal{U})$ are defined by

$$\|\mathbf{G}\|_2:=\left(\frac{1}{2\pi j}\int_{-j\infty}^{j\infty}\mathrm{tr}\{\mathbf{G}^T(-s)\mathbf{G}(s)\}ds\right)^{\frac{1}{2}} \quad (2.3)$$

$$\|\mathbf{G}\|_\infty:=\operatorname*{ess\,sup}_{\omega\in[0,\infty]}\{\bar\sigma(\mathbf{G}(j\omega))\} \quad (2.4)$$

We can define the Hardy space $\mathcal{H}_p(\mathcal{D})$ on the unit disk by the bilinear transformation, $z=\frac{\beta-s}{\beta+s}$. We use Laguerre model for representation of the transfer function since it have some efficiency for the bilinear transformation, while the other models have been developed in system identification [1]. The classical Laugrre functions obtained by inverse Laplace transformation of the Laguerre model have orthogonality in $\mathcal{L}_2(0,\infty)$ and completeness in $\mathcal{L}_2$ and $\mathcal{L}_1$ [13]. Assume the transfer function $\mathbf{G}(s)$ is strictly proper and continuous in Re s. We can choose a sequence $\{\mathbf{g}_k\}$ for some β such that

$$\mathbf{G}(s)=\sum_{k=0}^{\infty}\mathbf{g}_k\phi_k(s), \quad (2.5)$$

$$\text{where}\quad \phi_k(s)=\frac{\sqrt{2\beta}}{\beta+s}\left(\frac{\beta-s}{\beta+s}\right)^k,\quad \mathbf{g}_k\in\mathcal{R}^{m\times p} \quad (2.6)$$

$\mathbf{G}(s)$ admits the Fourier series expansion with the complete orthogonal basis for $\mathcal{H}_2(\mathcal{U})$, and $\mathbf{g}_k$ is given by

$$\mathbf{g}_k=<\mathbf{G},\,\phi_k>=\frac{1}{2\pi j}\int_{-j\infty}^{j\infty}\mathbf{G}(s)\phi_k(-s)\,ds \quad (2.7)$$

Since $\phi_k(s)$ consists of a first-order low-pass term and a kth-order all-pass factor, we can define a transfer function $\mathbf{G}(s)$ by aparting the first-order term in the basis functions $\phi_k(s)$. Then, we get

$$\mathbf{R}(s):=\frac{\sqrt{2\beta}}{\beta+s}\mathbf{G}(s)=\sum_{k=0}^{\infty}\mathbf{r}_k\phi_k(s) \quad (2.8)$$

$$\mathbf{G}_d(s)=\sum_{k=0}^{\infty}\mathbf{r}_kz^k,\quad\text{where}\quad z^k=\left(\frac{\beta-s}{\beta+s}\right)^k \quad (2.9)$$

The bilinear transformation $s=\beta\frac{1-z}{1+z}$ is a conformal mapping of the unit disk to the right-half plane. Let $\mathbf{G}_d(z)=\mathbf{G}(\beta\frac{1-z}{1+z})$, then we find $\mathbf{G}_d(z)\in\mathcal{H}_2(D)^{m\times p}$ since $\mathbf{G}$ and $\mathbf{R}$ belongs to $\mathcal{H}_2(\mathcal{U})^{m\times p}$. The sequence $\{\mathbf{r}_k\}$ in $\mathbf{G}_d$ is the inverse Fourier series coefficients of $\mathbf{R}(s)$.

We know that if an orthogonal system is complete, the Fourier series of every $f\in\mathcal{H}_2(\mathcal{U})$ converges to f in the H_2 norm. Also it can be established that the partial sum, referred to as Cesaro partial sum, converges to $\mathbf{G}(s)$ in $\mathcal{H}_\infty$ norm if only if $\mathbf{G}(s)\in\mathcal{H}_2(\mathcal{U})^{m\times p}$ is continuous on the imaginary axis including point at ∞ [11]. The fact that partial sum $\mathbf{G}_N:=\sum_{k=0}^{N}\mathbf{r}_kz^k$ converges to $\mathbf{G}(s)$ in $\mathcal{H}_\infty$ norm as N approaches ∞ can be obtained from the result that $\frac{d\mathbf{G}(s)}{ds}\in\mathcal{H}_1(\mathcal{U})^{m\times p}$ implies $\{\|\mathbf{r}_k\|\}\in l_1$, which can be proved by using the Hardy inequality [9].

The approximation theory of Hankel operators have been used to develop the $\mathcal{H}_\infty$ approximation of infinite-dimensional system model [10]. The Hankel operator is compact if $\mathbf{G}$ is continuous on the boundary of $\mathcal{U}$. Let $\sigma(\mathbf{r}_k)$ be the kth singular value of the Hankel operator $\mathbf{G}$. The system transfer function $\mathbf{G}$ is said to be nuclear whenever $\sum_1^\infty\sigma(\mathbf{r}_k)<\infty$. The nuclearity of $\mathbf{G}$ is the necessary condition for convergence in the Hankel norm $\sigma(\mathbf{r}_k)$. It can implies the $\frac{d\mathbf{G}(s)}{ds}\in H_1(U)^{m\times p}$ and $\|\frac{d\mathbf{G}(s)}{ds}\|\le C\sum_1^\infty\sigma_k(\mathbf{r}_k)$ for some constant C, shown by the Rosenblum [15]. Therefore, $\mathbf{G}_N$ converges to $\mathbf{G}$ as N approaches ∞ under condition $\frac{d\mathbf{G}(s)}{ds}\in\mathcal{H}_1(\mathcal{U})^{m\times p}$.

Recalling $\mathbf{G}_d(z)$ from (2.9), we can get the M-point inverse DFT of $\mathbf{G}_d(z)$ given as

$$\mathbf{r}_M(n):=\frac{1}{M}\sum_{k=0}^{M-1}\mathbf{G}_d(e^{\frac{2\pi jn}{M}})e^{-\frac{2\pi jnk}{M}}, \quad (2.10)$$

$$\mathbf{G}_N^M(s)=\sum_{k=0}^{N}\mathbf{r}_M(k)z^k,\quad N<M \quad (2.11)$$

$$\text{where}\quad M=2^L,\quad L\in\mathcal{N}_+$$

The key idea is that the sequence $\{\mathbf{r}_M(k)\}$ can be used as an approximation of $\{\mathbf{r}(k)\}$. We can establish the following theorem which is similar to Gu et al [12] to find Markov parameters for simple transfer functions with a delay term. Our motivation is originated from how the modal parameters of the structural dynamic system can be determined from the realized system model using the frequency response functions.

Theorem 2.1. *Let* $\mathbf{G}$ *be in* $\mathcal{H}_2(\mathcal{U})^{m\times p}$. *If* $\frac{d\mathbf{G}}{ds}$ *belongs to* $\mathcal{H}_2(\mathcal{U})^{m\times p}$, *then*

$$\|\mathbf{G}(s)-\mathbf{G}_N^M(s)\|_\infty\to 0$$
$$\text{as}\ (N,M)\to(\infty,\infty)\quad\text{with}\ N<M. \quad (2.12)$$

Proof. We begin the proof by the triangle inequality,

$$\|\mathbf{G}(s)-\mathbf{G}_N^M(s)\|_\infty\le\|\mathbf{G}(s)-\mathbf{G}_N(s)\|_\infty + \|\mathbf{G}_N(s)-\mathbf{G}_N^M(s)\|_\infty \quad (2.13)$$

We know that the first term on the right goes to zero by the nuclearity of $\mathbf{G}(s)$, given by $\frac{d\mathbf{G}(s)}{ds}\in\mathcal{H}_1(\mathcal{U})^{m\times p}$.

To show the convergence of the second on the right, we consider that if $\mathbf{G}(s)$ is stable and $\frac{d\mathbf{G}}{ds} \in \mathcal{H}_1(\mathcal{U})^{m\times p}$, then

$$\lim_{M\to\infty} \sum_{n=0}^{M} \|\mathbf{r}_M(n) - \mathbf{r}(n)\| = 0, \quad \text{with } \|z_k\| = 1 \quad (2.14)$$

We get $\mathbf{r}_M(n)$ from (2.10) as follows

$$\mathbf{r}_M(n) := \frac{1}{M}\sum_{k=0}^{M-1}\left(\sum_{l=0}^{\infty}\mathbf{r}(l)e^{\frac{2\pi jlk}{M}}\right)e^{-\frac{2\pi jkn}{M}} \quad (2.15)$$

$$:= \sum_{l=0}^{\infty}\mathbf{r}(l)\left(\frac{1}{M}\sum_{k=0}^{M-1}e^{\frac{2\pi j(l-n)k}{M}}\right) \quad (2.16)$$

we consider the two cases for the indices l on the summation. For $l = PM + n$ where P is a positive integer,

$$\frac{1}{M}\sum_{k=0}^{M-1} e^{\frac{2\pi j(l-n)k}{M}} = 1 \quad (2.17)$$

When $\quad l \neq PM + n, \quad$ (2.18)

$$\frac{1}{M}\sum_{k=0}^{M-1} e^{\frac{2\pi j(l-n)k}{M}} = \frac{1}{M}\frac{1-e^{2\pi j(l-n)}}{1-e^{\frac{2\pi j(l-n)}{M}}} = 0 \quad (2.19)$$

Substituting (2.17) and (2.18) into (2.16), we get $\mathbf{r}_M(n) = \sum_{P=0}^{\infty}\mathbf{r}_{PM}(n)$. From this, we also obtain the following inequality

$$\sum_{n=0}^{\infty}\|\mathbf{r}_M(n) - \mathbf{r}(n)\| \le \sum_{P=1}^{\infty}\|r_{PM}(n)\| \quad (2.20)$$

we can find the upper bound for $\|\mathbf{G}_N - \mathbf{G}_N^M\|_\infty$ with $\|z^k\|_\infty = 1$ as follows.

$$\|\mathbf{G}_N - \mathbf{G}_N^M\|_\infty = \sum_{n=0}^{N}\|(\mathbf{r}_M(n) - \mathbf{r}(n))z^k\|$$

$$\le \sum_{n=0}^{N}\|\mathbf{r}_M(n) - \mathbf{r}(n)\| \le \sum_{n=0}^{M}\|\mathbf{r}_M(n) - \mathbf{r}(n)\| \quad (2.21)$$

The error estimate in (2.19) is bounded from the fact that $\frac{d\mathbf{G}(s)}{ds}$ belongs to $\mathcal{H}_1(\mathcal{U})^{m\times p}$ implies the $\{\|\mathbf{r}(n)\|\}$ is l_1 sequence. Thus, the last term in (2.21) goes to zero as M approaches to ∞. □

We can notice that the IDFT based approximation has the same convergence properties as the Fourier series based approximation. We can reduce the error in the $\mathcal{H}_\infty$ norm more when we use the real Fourier coefficients. We can find the error bound in the $\mathcal{H}_\infty$ norm as shown by the following theorem.

Theorem 2.2. [12] If $\mathbf{G}$ is stable and $\frac{d\mathbf{G}}{ds} \in \mathcal{H}_1(\mathcal{U})^{m\times p}$. Let $\mathbf{r}(n)$ and $\mathbf{r}_M(n)$ be defined as in (2.9) and (2.10) for $n = 0, 1, 2, \cdots, M-1$. Then,

$$\|\mathbf{G}(s) - \mathbf{G}_N^M(s)\| \le \sum_{n=N+1}^{M-1}\bar{\sigma}(\mathbf{r}_M(n)) + 2\sum_{n=M}^{\infty}\bar{\sigma}(\mathbf{r}(n)) \quad (2.22)$$

In general, we use the IDFT with larger point M of the frequency samples of the frequency response function while we take smaller number N of Markov parameters related to the realized system order. Then the approximation error in (2.22) will be dominated by the first term in (2.22). The error bound can estimated as

$$\|\mathbf{G}(s) - \mathbf{G}_N^M(s)\| \approx \sum_{n=N+1}^{M}\bar{\sigma}(\mathbf{r}_M(n)). \quad (2.23)$$

3 Identification of Vibration Models

In this section we will show how the Markov parameters obtained from IDFT can be exploited for the system identification based on the state-space model of system. From the continuous state space system of $[\hat{\mathbf{A}}\ \hat{\mathbf{B}}\ \mathbf{C}]$ with a sampling time Δt, we can obtain the discrete system model for a sampled data system with zero-order holder, which can be expressed as

$$\mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{B}\mathbf{u}(k) \quad (3.1)$$
$$\mathbf{y}(k) = \mathbf{C}\mathbf{x}(k)$$

where

$$\mathbf{A} = e^{\hat{\mathbf{A}}\Delta t}, \quad \mathbf{B} = \int_0^{\Delta t} e^{\hat{\mathbf{A}}(\Delta t-\tau)}\hat{\mathbf{B}}d\tau$$

$$\mathbf{x}(k) = \mathbf{x}(k\Delta t), \quad \mathbf{u}(k) = \mathbf{u}(k\Delta t), \quad \mathbf{y}(k) = \mathbf{y}(k\Delta t)$$

We can also obtain the transfer function approximation in terms of the Markov parameters derived from IDFT, which is expressed as

$$\mathbf{G}(z) = \mathbf{D} + \mathbf{C}(z^{-1}\mathbf{I} - \mathbf{A})^{-1}\mathbf{B} \approx \sum_{k=0}^{N}\mathbf{r}_M(k)z^k \quad (3.2)$$

$$\mathbf{r}_M(k) \approx \mathbf{C}\mathbf{A}^{k-1}\mathbf{B}, \quad \mathbf{r}_M(0) = \mathbf{D} \quad (3.3)$$

We can employ the eigensystem realization algorithm [7] for the system identification. We begin the ERA procedure by constructing the a blocked Hankel matrix $\mathbf{H}_{qd}$ from the IDFT results as follows:

$$\mathbf{H}_{qd}(k) = \begin{bmatrix} \mathbf{r}(k+1) & \mathbf{r}(k+2) & \dots & \mathbf{r}(k+d) \\ \mathbf{r}(k+2) & \mathbf{r}(k+3) & \dots & \mathbf{r}(k+d+1) \\ \vdots & \vdots & \vdots & \vdots \\ \mathbf{r}(k+q) & \mathbf{r}(k+q+1) & \dots & \mathbf{r}(k+q+d-1) \end{bmatrix} \quad (3.4)$$

Then, we use a singular value decomposition of $H_{qd}(0)$ and truncate the singular values following the N largest one as like:

$$\mathbf{H}_{qd}(0) \approx \sum_{i=1}^{N}\sigma_i\mathbf{v}_i\mathbf{w}_i^T = \mathbf{V}_N\mathbf{Q}_N\mathbf{W}_N^T \quad (3.5)$$

where

$$\mathbf{V}_N = \mathbf{V}^{q\times N}, \quad \mathbf{Q}_N = \mathbf{Q}^{N\times N}, \quad \mathbf{W}_N = \mathbf{W}^{d\times N} \quad (3.6)$$

We can also get a discrete system realization of (3.1) with the order N:

$$\mathbf{A} = \mathbf{Q}_N^{-\frac{1}{2}} \mathbf{V}_N^T \mathbf{H}_{qd}(1) \mathbf{W}_N \mathbf{Q}_N^{-\frac{1}{2}} \quad (3.7)$$
$$\mathbf{B} = \mathbf{Q}_N^{\frac{1}{2}} \mathbf{W}_N \begin{bmatrix} \mathbf{I}_{p\times p} \\ 0_{(d-p)\times p} \end{bmatrix}$$
$$\mathbf{C} = \begin{bmatrix} \mathbf{I}_{m\times m} & \mathbf{0}_{m\times(q-m)} \end{bmatrix} \mathbf{V}_N \mathbf{Q}_N^{\frac{1}{2}}$$

From the eigenproblem of $\mathbf{A}$, we have the continuous modal model from (3.7) by a change of basis, $\mathbf{x} = \Psi[\mathbf{z}_1 \ \mathbf{z}_2]^T$ with $\Psi = [\psi \ \bar{\psi}]$:

$$\mathbf{z}(k+1) = \Lambda \mathbf{z}(k) + \mathbf{B}_m \mathbf{u}(k) \quad (3.8)$$
$$\mathbf{y}(k) = \mathbf{C}_m \mathbf{z}(k)$$
$$\text{where,} \quad \Lambda = \Psi^{-1} \mathbf{A} \Psi \quad (3.9)$$
$$\text{with } \frac{\ln \Lambda}{\Delta t} = diag\{\sigma_i \pm j\omega_i, \ i = 1, \ldots, N\}$$
$$\mathbf{B}_m = \Psi^{-1}\mathbf{B} = \lfloor \ldots [b_i \ \bar{b}_i] \ldots \rfloor^T$$
$$\mathbf{C}_m = \mathbf{C}\Psi = \lfloor \ldots [c_i \ \bar{c}_i] \ldots \rfloor$$

The state-space structural dynamic model will be derived from the realization model. We begin by considering the linear vibration system as:

$$\dot{\mathbf{x}} = \bar{\mathbf{A}}\mathbf{x}(t) + \bar{\mathbf{B}}\mathbf{u}(t), \quad \mathbf{x}^T(t) = (\dot{\mathbf{q}} \ \mathbf{q}) \quad (3.10)$$
$$\mathbf{y}(t) = \bar{\mathbf{C}}\mathbf{x}(t)$$

where

$$\bar{\mathbf{A}} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}D \end{bmatrix} \quad (3.11)$$
$$\bar{\mathbf{B}} = \begin{bmatrix} \mathbf{0} \\ \mathbf{M}^{-1}\mathbf{B}_q \end{bmatrix} \quad \bar{\mathbf{C}} = [\mathbf{C}_q \ \ \mathbf{0}]$$

where $\mathbf{M}$, and D and $\mathbf{K}$ are the mass, damping and stiffness matrices, respectively; $\mathbf{q}$ is the $\frac{N}{2}$-displacement state vector; $\mathbf{u}$ is the p-input displacement vector; $\mathbf{y}$ is the m-sensor displacement output vector; $\mathbf{B}_q$ and $\mathbf{C}_q$ are the input- and output-state influence matrix, respectively. We can obtain the modal model for the linear vibration system with a proportional damping

$$\begin{pmatrix} \dot{\xi}(t) \\ \ddot{\xi}(t) \end{pmatrix} = \begin{bmatrix} 0 & 1 \\ -\omega_{ni}^2 & -2\zeta_{ni}\omega_{ni} \end{bmatrix} \begin{pmatrix} \xi(t) \\ \dot{\xi}(t) \end{pmatrix} + \begin{bmatrix} 0 \\ \phi_i^T \mathbf{B}_q \end{bmatrix} \mathbf{u}(t)$$
$$\mathbf{y}(t) = \sum_{i=1}^{N} [\mathbf{C}_q \phi_i, \ \ 0] \begin{pmatrix} \xi(t) \\ \dot{\xi}(t) \end{pmatrix} \quad (3.12)$$

by use of the modal basis coordination transformation defined by $\mathbf{q} = \Phi\xi(\mathbf{t})$:

$$\Phi^T \mathbf{M} \Phi = \mathbf{I}_{N\times N}, \quad \Phi^T \mathbf{K} \Phi = diag\{\, \omega_{ni}^2, \ i = 1, \ldots, \frac{N}{2} \,\}$$
$$\Phi^T D \Phi = diag\{\, 2\zeta_{ni}\omega_{ni}, \ i = 1, \ldots, \frac{N}{2} \,\} \quad (3.13)$$

We can find the relationship between the natural frequencies and damping ratios of the vibration system and the complex paired eigenvalues of the realized state-space system, viz.,

$$\omega_{ni}^2 = \omega_i^2 + \sigma_i^2, \quad -\zeta_i\omega_{ni} = \sigma_i \quad (3.14)$$

and we can notice that there exist the unique similarity transformation $\mathbf{z}_i = \mathbf{E}_i \mathbf{F}_i \xi_i$ such that

$$\bar{\Lambda}_i \mathbf{E}_i = \mathbf{E}_i \Omega_i, \quad \mathbf{F}_i \Omega_i = \Omega_i \mathbf{F}_i \quad (3.15)$$

where

$$\bar{\Lambda}_i := \begin{bmatrix} \sigma_i + j\omega_i & 0 \\ 0 & \sigma_i - j\omega_i \end{bmatrix}, \quad \Omega_i := \begin{bmatrix} 0 & 1 \\ -\omega_{ni}^2 & -2\zeta_{ni}\omega_{ni} \end{bmatrix}$$

The transformation $\mathbf{E}_i$ and $\mathbf{F}_i$ are two rotation operators of the modal coordination vectors, the first for rotating the diagonal state transition matrix to the canonical second-order form and the second for preserving the transformation while zeroing out the the first term of $\mathbf{B}_m$. We want to provide the result of [18] for the similarity transformation, which is obtained by the solution of the simultaneous linear equations.

$$\mathbf{E}_i = \frac{-j}{2\omega} \begin{bmatrix} j\omega_i - \sigma_i & 1 \\ j\omega_i + \sigma_i & -1 \end{bmatrix}, \mathbf{F}_i = -d_i \begin{bmatrix} r_i\omega_i + \sigma_i & -1 \\ \omega_i^2 + \sigma_i^2 & r_i\omega_i - \sigma_i \end{bmatrix} \quad (3.16)$$

where r_i is given by $r_i = \Im \mathbf{b}_i / \Re \mathbf{b}_i$ and d_i is the scaling factor for normalizing the mode shape. Finally, we can obtain the input and output participation vector from (3.12) as shown in (3.17) .

$$\phi_i^T \mathbf{B}_q = \frac{2\Re \mathbf{b}_i^T}{d_i}, \quad \mathbf{C}_q \phi_i = d_i r_i^2 \Im \mathbf{c}_i \omega_i \quad (3.17)$$

We can recorver the the physical information of the real controlled system from the realization model derived from the frequency response function measurements, while it can be lost in the state space system realization model.

4 Modeling Results and Discussion

The flexural structure designed for implementing this realization model, as a smart structure, is comprised with the carbon-fibre-reinforced-plastic rectangular composite plates with aspect ratio of about 1.5, which is stacked by the lamina with fiber orientation $[\,0\backslash 45\backslash -45\backslash 90\,]_{2s}$, and a piezoelectric actuator patched with finite dimensions. We use the composite material in a structural dynamic system since it has higher strength and modulus on its specified weight than metal materials. We perform the experimental modal testing to get the frequency response functions of the composite plate by energizing the ceramic actuator. We measure the vibrational displacement of the free end of the cantilevered plate as the output voltage of a gap sensor, which is then feed through the signal analyzer to compute the sampled frequency response data. We fulfill the simulation of the realization model and find the natural frequency and the modal damping ratios of the structural vibration of the composite plate. We also investigate the estimation error bound according to the variation of the number of frequency response sampled data points applied in the IDFT. The Fig. 1 represents the frequency response function of the cantilevered composite plate. We can

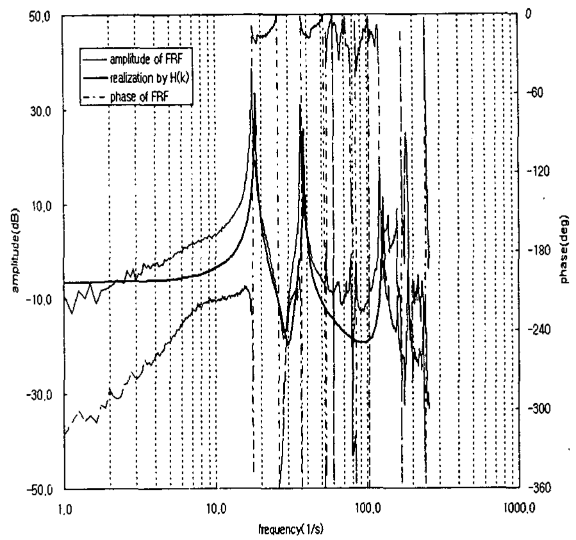

Figure 1: The frequency response function of the composite plate vibration and the sixth-order realization model for the $\mathbf{r}^{1024}$ Markov parameters.

obtain the Markov-parameter sequence by using IDFT of the frequency response function, which is filtered out the FFT weighted window and the first order filter. The Fig. 2 shows the history of the Markov-parameter sequence of $\{\mathbf{r}^{1024}\}$. An discrete state-space model of the

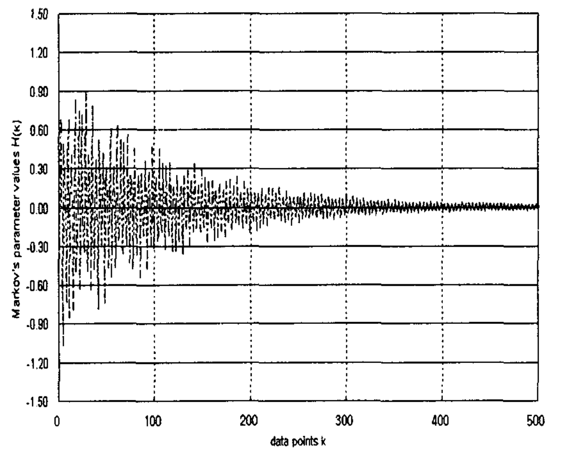

Figure 2: The Markov-parameter sequence $\{\mathbf{r}^{1024}\}$

realization results is given as an example, which is derived from the Markov parameter sequence of $\{\mathbf{r}_4^{512}\}$.

Table 1: Natural Frequencies of the Realization Models

Markov Sampled Data	ω_{n1}(Hz)	ω_{n2}(Hz)	ω_{n3}(Hz)
$\mathbf{r}_8^{2048}$	17.4	36.4	121.0
$\mathbf{r}_6^{1024}$	13.9	34.8	72.9
$\mathbf{r}_4^{512}$	69.2	110.2	

Table 2: Damping Ratios of the Realization Models

Markov Sampled Data	ζ_{n1} (%)	ζ_{n2} (%)	ζ_{n3} (%)
$\mathbf{r}_8^{2048}$	17.4	36.4	121.0
$\mathbf{r}_6^{1024}$	13.9	34.8	72.9
$\mathbf{r}_4^{512}$	69.2	110.2	

The system of $[\mathbf{A}_4\ \mathbf{B}_4\ \mathbf{C}_4\ \mathbf{D}_4]$ represent the fourth-order-system approximation of the flexible vibration system of the composite plate. The corresponding system model (3.8) with the diagonaliszed transition matrix for $\mathbf{A}_4$ is given by $[\bar{\Lambda}_4\ \mathbf{A}_{m4}\ \mathbf{B}_{m4}\ \mathbf{C}_{m4}]$.

$$\mathbf{A}_4 = \begin{bmatrix} -0.13 & -0.98 & 0.012 & 0.01 \\ 0.99 & -0.14 & -0.03 & 0.04 \\ 0.02 & -0.03 & -0.90 & -0.41 \\ 0.04 & -0.04 & 0.43 & -0.89 \end{bmatrix}$$

$$\begin{aligned} \mathbf{B}_4 &= [\ 0.17, -0.06, -0.12, -0.15\ ]^T \\ \mathbf{C}_4 &= \ [\ 4.20, -0.22, 1.90, -0.74\ ],\ \ \mathbf{D}_4 = -0.29 \\ \bar{\Lambda}_4 &= [\ -0.90 - 0.42j, -0.90 + 042j, \\ &\quad -0.14 - 0.98j, -0.14 + 0.98j\ ], \\ \mathbf{B}_{m4} &= [\ -0.11 - 0.09j, -0.11 + 0.09j, \\ &\quad -0.06 + 0.11j, -0.06 - 0.11j\ ]^T, \\ \mathbf{C}_{m4} &= [\ -0.67 + 1.31j, -0.67 - 1.31j, \\ &\quad -0.59 + 2.89j, -0.59 - 2.89j\ ] \end{aligned}$$

We also find the natural frequencies and damping ratios, shown in table 1 and 2, of the realization models from (3.14) according to the system model orders for three cases of the Markov parameter numbers.

The Fig. 3 represent the error bound of singular value for the Markov parameter $\{\mathbf{r}_8^{2048}\}$ given by (2.10), as expressed in (2.22), in sense of the $\mathcal{H}_\infty$ approximation. The singular values for the Markov parameter $\{\mathbf{r}_4^{512}\}$ are also plotted in Fig. 3. The error bounds of the squared singular values for the three sequences $\{\mathbf{r}_4^{512}\}$, $\{\mathbf{r}_6^{1024}\}$ and $\{\mathbf{r}_8^{2048}\}$. are also plotted in the Fig 4.

References

[1] S. C. Heuberger, et.al, *A Geralized orthonomal basis for linear dynamical system*, IEEE, A.C. Vol. 40(3), pp. 451-165, 1995.

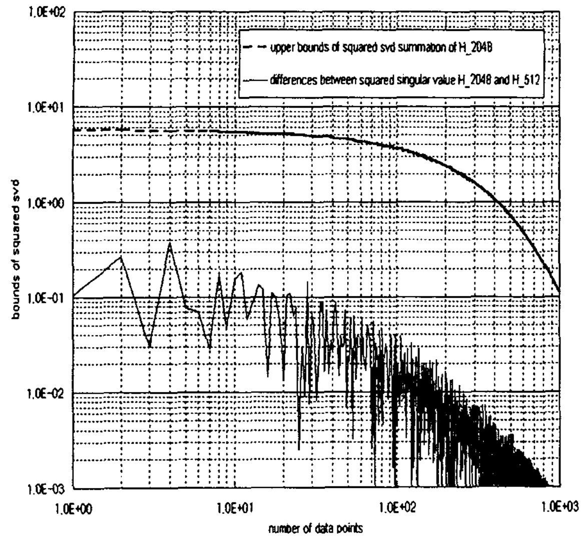

Figure 3: The singular value error bound for $\{\mathbf{r}_8^{2048}\}$ and the singular values for $\{\mathbf{r}_4^{512}\}$

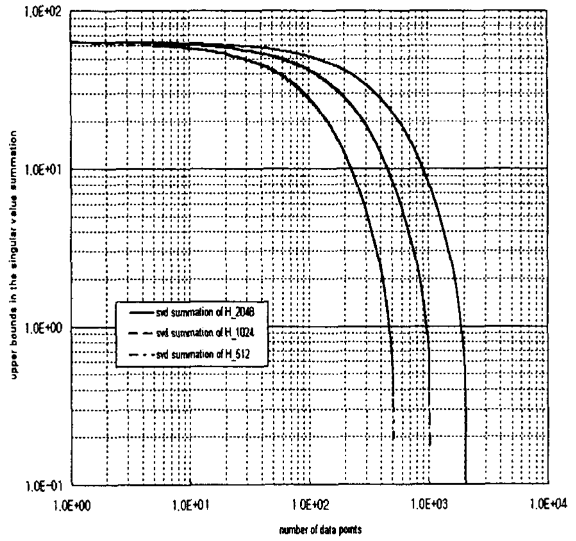

Figure 4: The error bounds of the squared singular values for $\{\mathbf{r}_4^{512}\}$, $\{\mathbf{r}_6^{1024}\}$ and $\{\mathbf{r}_8^{2048}\}$

[2] C.-D. Yang and F.-B. Yeh, *A simple alogrithm on minimal realization for transfer function matrices*, IEEE, A.C. Vol. 34(8), pp. 879-882, 1989.

[3] P. A. Parrilo, M. Sznajer and R. S. Pena, *Mixed time/frequency-domain based robust identification* FA18,Proceeding of the 35th Conf. on Dec. and Contrl, Kobe, Japan. pp. 4196-4201, Dec. 1995.

[4] N. L. C. Chui and J. M. Maciejowski, *Uniqueness of minimal partial realization and markov parameter identification*, FA18,Proceeding of the 35th Conf. on Dec. and Contrl, Kobe, Japan. pp. 3642-3647, Dec. 1995.

[5] X. Li and J. Young, *Optimal control theory for infinte dimensional system*, Birkhauser Boston 1995.

[6] K. Glover, *All optimal Hankel-norm approximationof linear multivariable system and their L^{∞}-error bounds*, Int. J. Contr., Vol. 39, pp. 1115-1193, 1984.

[7] J. N Juang and R. S. Pappa, *An eigensystem realization algorithm for modal parameter identification and model reduction applcation to flexible structural identification*, Jr. Guidance, Control, and Dynamics, Vol. 8(5), pp. 583-592, 1985.

[8] Ketao Liu and R. E. Skleton, *Q-Markov covariance equvalent realization and its applcation to flexible structural identification*, Jr. Guidance, Control, and Dynamics, Vol. 16(2), pp. 308-319, 1993.

[9] Walter Rudin, *Real and complex analysis(3rd edition)*, McGraw-Hill: Marcel-Dekker, 1987.

[10] K. Glover,R. F. Curtain and J. R. Partngton, *Realization and approximation of linear infinite-dimensional Systems*, SIAM J. Contr.Optimiz., Vol. 26, pp. 863-898, 1988.

[11] E. W. Kamen, P. P. Khargoneker and A. Tannenbaum, *Stabilization of time-delay system with finite-dimensional compensator*, IEEE, A.C., Vol. 30, pp. 75-78, 1985.

[12] G. Gu, P. P. Khargoneker and E. B. Lee, *Approximation of infinite-dimenensional Systems*, IEEE, A.C., Vol. 34(6), pp. 161-168, 1989.

[13] Bo Wahlberg, *System Identification using Lagurre models*, IEEE, A.C., Vol. 36(5), pp. 551-562, 1991.

[14] J. Helmicki, C. A. Jacobson and C. N. Nett, *Control oriented system identification: A wrost-case/determinstic approach in H_∞*, IEEE, A.C. Vol. 36(10), pp. 1164-1175, 1991.

[15] S. C. Power, *Hankel operators on Hilbert space*, New York: Pittman, 1982.

[16] K.F.Alvin and K.C.Park, *Second-order structural identification procedure via state-spaced system sdentification*, AIAA Jr, Vol. 32(2), pp. 397-406, 1994.

[17] R. L. Wheeden and A. Zygmund, *Measure and integral: An introduction to real analysis*, New York: Marcel-Dekker, 1977.

[18] M. C. Tsai and Y. P. Shih, *Balanced minimal realization via singular value decomposition of Sarason operator*, Automatica, Vol. 24(5), pp. 701-705, 1988.