Proceedings of the 13th KACC, October 1998

An improvement for system identification by use of M-transform

H. Kashiwagi, M.Liu, H. Harada and T. Yamaguchi
Faculty of Engineering,Kumamoto University
Kurokami 2-39-1,Kumamoto 860-8555,Japan
E-mail:kashiwa@gpo.kumamoto-u.ac.jp

Abstract

In this paper, the authors propose a new method for improving identification method of linear system by using M-transform. The authors has recently proposed a new method for linear system identification by use of M-transform. In this method, the input signal $x(i)$ must have the same period N as that of the M-sequence. When N becomes large, it will take a long time to compute. To overcome this difficulty, we propose a new approach of system identification by using a small size matrix. The results of simulation show a good agreement with the theoretical considerations.

1. Introduction

Any time function can be transformed by use of an orthogonal signal. Since M-sequence has the pseudo-orthogonal property, M-sequence may be used for such an orthogonal transformation. When we define the signal transformation by use of M-sequence, we call this transformation as "M-transform" (9) .

Just like in case of Fourier transform where any time signal is thought as a weighted sum of sinusoidal signals by use of Fourier Transform, "M-transform" is such a transform that any time signal is considered to be a weighted sum of M-sequences. It is shown theoretically that the M-transform has almost the same properties as time signal $x(i)$ with respect to such statistical properties as autocorrelation function, crosscorrelation function, input-output relation of linear systems and so on (9).

One application of M-transform is the identification of linear control system. When an input $x(i)$ and output $y(i)$ of a control system are obtained, the input $x(i)$ is M-transformed and is considered to be a weighted sum of M-sequence. Thus, by using M-sequence correlation method, we obtain the impulse response of the system.

In the method described above, the input signal $x(i)$ must have the same period N as that of the M-sequence. When N becomes large, it will take a long time to compute. To overcome this difficulty, we propose here a new approach of system identification by using a small size matrix. The results of simulation show a good agreement with the theoretical considerations.

2. Definition of M-transform

In this chapter, we shall simply introduce the definition of M-transform. Let us denote an element $a_i(= 0 \text{ or } 1)$ of an M-sequence of a period of $N(= 2^n - 1)$. We make a series of $\{m_i\}$ of $+1$ or -1 by $m_i = 1 - 2a_i$. The autocorrelation function $\phi_{mm}(k)$ of m_i is

$$\phi_{mm}(k) = \frac{1}{N}\sum_{i=0}^{N-1} m_{i-k}m_i = \begin{cases} 1 & (\, k = 0, N, 2N, \cdots) \\ -1/N & (\text{ otherwise}) \end{cases} \tag{1}$$

We call this property as "pseudo-orthogonal property of M-sequence". A matrix $\boldsymbol{M}_i$ of $N \times N$ degree is defined as follows by use of m_i.

$$\boldsymbol{M}_i = \begin{bmatrix} m_i & m_{i-1} & \dots & m_{i-N+1} \\ m_{i+1} & m_i & \dots & m_{i-N+2} \\ \vdots & \vdots & \ddots & \vdots \\ m_{i+N-1} & m_{i+N-2} & \dots & m_i \end{bmatrix} \tag{2}$$

According to the property of M-sequence, we have

$$\boldsymbol{M}_i^T\boldsymbol{M}_i = \begin{bmatrix} N & -1 & \dots & -1 \\ -1 & N & \dots & -1 \\ \vdots & \vdots & \ddots & \vdots \\ -1 & -1 & \dots & N \end{bmatrix} \tag{3}$$

where T denotes transpose of a matrix.

The inverse of the above matrix becomes(8):

$$(\boldsymbol{M}_i^T\boldsymbol{M}_i)^{-1} = \frac{1}{N+1}\begin{bmatrix} 2 & 1 & \dots & 1 \\ 1 & 2 & \dots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & \dots & 2 \end{bmatrix} \tag{4}$$

M-transform $\boldsymbol{A}$ for the sampled signal $x(i\Delta t)$ of a time signal $x(t)$ which has a period of $N\Delta t$ is defined as follows:

$$\boldsymbol{X}_i = \boldsymbol{M}_i\boldsymbol{A} \tag{5}$$

where

$$\boldsymbol{X}_i = \{x(i), x(i+1), \cdots, x(i+N-1)\}^T$$
$$\boldsymbol{A} = (\alpha_0, \alpha_1, \cdots, \alpha_{N-1})^T$$

M-transform $\boldsymbol{A}$ is uniquely determined as:

$$\boldsymbol{A} = (\boldsymbol{M}_i^T\boldsymbol{M}_i)^{-1}\boldsymbol{M}_i^T\boldsymbol{X}_i \tag{6}$$

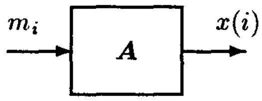

Figure 1: Definition of M-transform

The definition of M-transform $\boldsymbol{A}$ is shown in Fig. 1. Any time signal X_i can be considered as the output of a filter whose input is an M-sequence. We call this filter "M-filter".

3. Application to identification of linear system

3.1 Principle

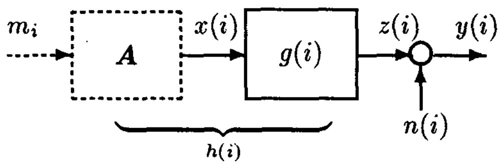

Figure 2: Linear system identification

In this section, we will use M-transform to identify a linear system which is shown in Fig. 2. We consider here the problem of identifying the impulse response of a control system under the condition that we can measure the input $x(i)$ and the signal $y(i)$ which is the sum of output $z(i)$ of the system and noise $n(i)$. Now the input $x(i)$ is M-transformed to be a weighted sum of M-sequence as shown in Fig. 2 as a dotted line. Let $h(i)$ be the impulse response of the virtual system from m_i to $y(i)$, that is, the cascade system of A and $g(i)$. $n(i)$ is an independent noise signal. First, we calculate M-transform A of $x(i)$, then

$$y(i) = \sum_{j=0}^{N-1} h(j)m_{i-j} + n(i) \tag{7}$$

that is

$$\boldsymbol{Y}_i = \boldsymbol{M}_i\boldsymbol{H} + \boldsymbol{N}_i \tag{8}$$

where

$$\boldsymbol{Y}_i = (y(i), y(i+1), \cdots, y(i+N-1))^T \tag{9}$$

$$\boldsymbol{H} = (h(0), h(1), \cdots, h(N-1))^T \tag{10}$$

$$\boldsymbol{N}_i = (n(i), n(i+1), \cdots, n(i+N-1))^T \tag{11}$$

So we get,

$$\boldsymbol{H} = (\boldsymbol{M}_i^T\boldsymbol{M}_i)^{-1}\boldsymbol{M}_i^T\boldsymbol{Y}_i \tag{12}$$

where, we suppose that the crosscorrelation of $n(i)$ and m_i is zero, since $n(i)$ and m_i is statistically independent.

Since $h(i)$ is the impulse response of the cascaded systems of A and $g(i)$, we have

$$h(i) = \sum_{j=0}^{N-1} \alpha(i)g(i-j) \tag{13}$$

That is,

$$\boldsymbol{H} = \boldsymbol{\alpha g} \tag{14}$$

where

$$\boldsymbol{g}_i = (g(0), g(1), \cdots, g(N-1))^T \tag{15}$$

$$\boldsymbol{\alpha} = \begin{bmatrix} \alpha_0 & \alpha_{-1} & \cdots & \alpha_{-N+1} \\ \alpha_1 & \alpha_0 & \cdots & \alpha_{-N+2} \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_{N-1} & \alpha_{N-2} & \cdots & \alpha_0 \end{bmatrix} \tag{16}$$

So the impulse response $g(i)$ can be obtained as follows:

$$\boldsymbol{g} = \boldsymbol{\alpha}^{-1}\boldsymbol{H} \tag{17}$$

Or we can use Fourier transform in Eq.(14) and we have

$$\boldsymbol{H}(j\omega) = \boldsymbol{A}(j\omega)\boldsymbol{G}(j\omega) \tag{18}$$

where $\boldsymbol{H}(j\omega)$, $\boldsymbol{A}(j\omega)$, $\boldsymbol{G}(j\omega)$ are Fourier transform of $h(i)$, $\alpha(i)$, and $g(i)$, respectively.

Thus we have

$$\boldsymbol{G}(j\omega) = \boldsymbol{H}(j\omega)/\boldsymbol{A}(j\omega) \tag{19}$$

3.2 Improvement in case of non-periodic signal

In the case of non-periodic signal, we have to overcome the difference between the start point and end point at a time length of N. In order to do this, we use a window as follows:

$$w(i) = \begin{cases} i/T_w & 0 \le i < T_w \\ (N-i)/T_w & N - T_w < i \le N \\ 1 & (\text{ otherwise}) \end{cases}$$

where T_w is the time interval for window.

Before we go into the process of identification described in 3.1, we multiply this window to both $x(i)$ and $y(i)$, then by use of the mothod described in section 3.1, we can identify the control system when the input is non-periodic.

3.3 Simulation

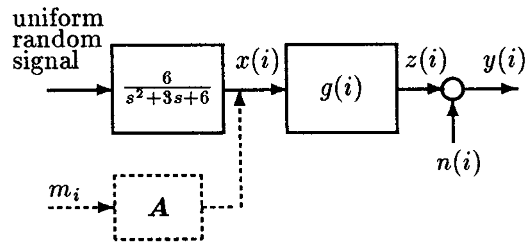

Figure 3: Identification of linear system by use of M-transform

In order to verify the improvement method described above, we carried out simulation on the system shown in Fig. 3. We use a uniform random signal to produce $x(i)$ (Fig. 4) which is then added to the system to be identified ($g(i)$) and $y(i)$ (Fig. 6) is obtained. M-sequence m_i of degree 9 was used as the imaginary input to calculate A and H. Fig. 8 shows M-transform A of $x(i)$. Fig. 9 shows the impulse response H of cascaded A and $g(i)$. Fig. 5 shows the input signal multiplied by window $w(i)$(here $T_w = 40$). Since the period N of the M-sequence is 511 in this case, we see the effect of window $w(i)$ around $i = 511$. Fig. 7 shows the output multiplied by window $w(i)$. The impulse response $g(i)$ can be identified as shown in Fig. 11, where square is calculated value by the improvement method proposed in this paper, and the solid line is theoretical value. The system to be identified, whose impulse response is $g(i)$, is made unknown to the person who carries out the simulation. After the identification is over, the actual system is informed to us, thus enabling us to compare the result of simulation with the theoretical one. The system used for simulation is actually a second-order system with the natural frequency $\omega = \sqrt{2}$ and the damping ratio $\zeta = \sqrt{2}/2$. The results show good agreement with the theoretical considerations even in case of non-periodic input.

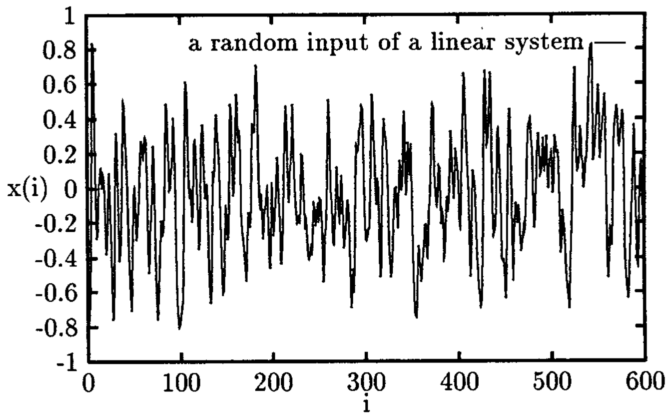

Figure 4: Input signal of the system to be identified

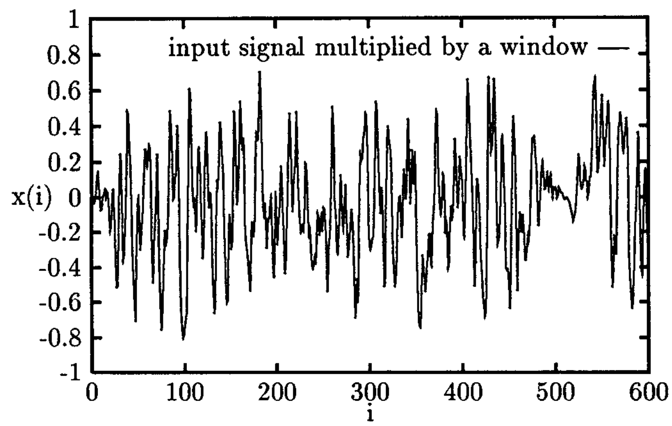

Figure 5: The input signal multiplied by the window $w(i)$

4. Application to signal processing

4.1 Principle

Any time signal can be thought as the output of a filter whose input is white noise. The transfer function of the filter is determined by the power spectrum of the time signal. This way of thinking is called "prewhitening".

M-transform or M-filter plays the role of above prewhitening filter with the use of M-sequence in place of white noise.

M-transform $\boldsymbol{A}$ of a time signal $\boldsymbol{X}_i$ having the peried N is uniquely determined as in Eq.(6) when we use N dimensional $\boldsymbol{A}$. When the signal $\boldsymbol{X}_i$ is non-periodic, we can still make M-transform by use of a window $w(i)$ as shown in 3.2. Here we will show that M-transform $\boldsymbol{A}$ can be also determined in $L(< N)$ dimensional space by minimizing the mean square error.

When we take L-dimensional vector $\boldsymbol{A}'$ such as

$$\boldsymbol{A}' = (\alpha_0, \alpha_1, \cdots, \alpha_{L-1})^T \tag{20}$$

then the reconstructed time signal $\boldsymbol{X}'$ is written as

$$\boldsymbol{X}' = \boldsymbol{M}_i'\boldsymbol{A}' \tag{21}$$

where

$$\boldsymbol{X}' = \{x'(i), x'(i+1), \cdots, x'(i+L-1)\}^T$$

$$\boldsymbol{M}_i' = \begin{bmatrix} m_i & m_{i-1} & \dots & m_{i-L+1} \\ m_i+1 & m_i & \dots & m_{i-L+2} \\ \vdots & \vdots & \ddots & \vdots \\ m_{i+N-1} & m_{i+N-2} & \dots & m_{i+N-L} \end{bmatrix} \tag{22}$$

We would like to make $\boldsymbol{X}_i'$ as close as $\boldsymbol{X}_i$, so by using

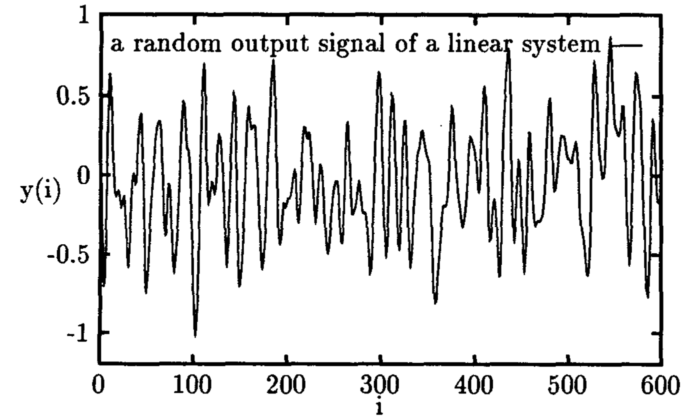

Figure 6: Output signal of the system

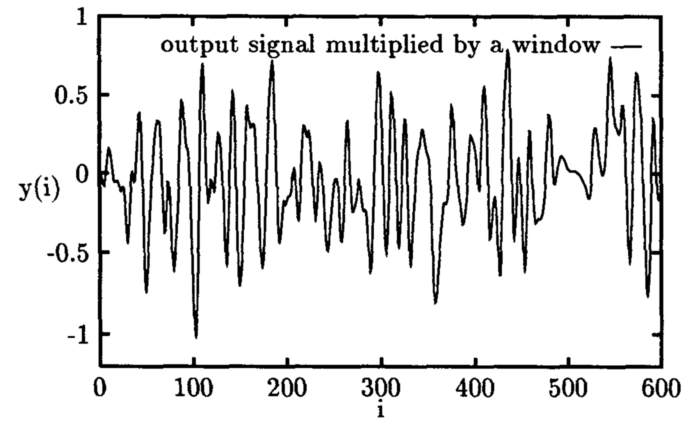

Figure 7: The output signal multiplied by the window $w(i)$

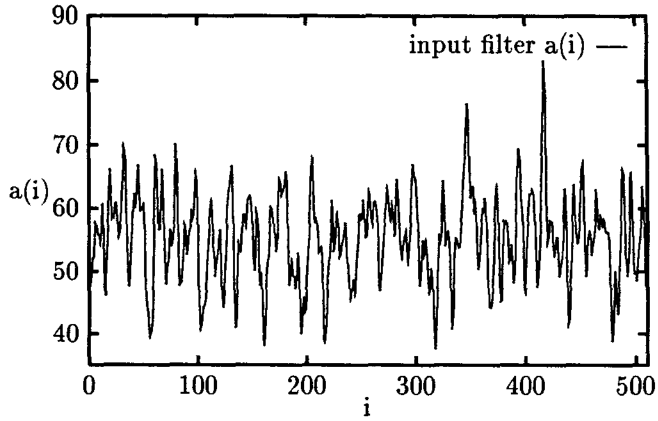

Figure 8: M-transform α_i of $x(i)$

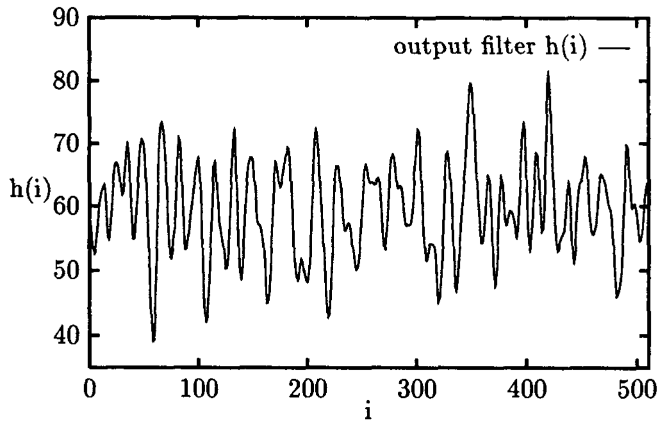

Figure 9: Impulse response $h(i)$ of cascaded A and $g(i)$

least square method, we have

$$A' = (M_i'^T M_i')^{-1} M_i'^T X_i \tag{23}$$

That is, we can represent the time signal $x(i)$ by use of L values of α_i, where $L < N$, in the least square sense.

This means that the M-transform α_i 's are a kind of extracted features of signal $x(i)$ representing some characteristics of the original signal $x(i)$.

The M-sequence correlation method (MSEC method) for functional fault detection of logical circuit, which was developed by the authors in 1987, can be interpreted as one application of M-transform. In MSEC method, an M-sequence is applied to a logical circuit under test, and the crosscorrelation function between the input and output is calculated, from which we judge whether or not the circuit has any faults.

In this case, the crosscorrelation function between the input M-sequence and output is actually M-transform of the output of the logical circuit.

4.2 Simulation on system identification when $L < N$

In order to verify the case where $L < N$, we carried out simulation on the system shown in Fig. 3. We use $L = 400$ and $N = 511$. We use a uniform random signal to produce $x(i)$ (Fig. 4) which is then added to the system to be identified ($(g(i))$) and the $y(i)$ (Fig. 5) is obtained. M-sequence m_i of degree 9 was used as the imaginary input to calculate A and H. The impulse response $g(i)$ can be identified as shown in Fig. 11, where square is calculated value by the method proposed in this paper, and the solid line is theoretical value. The results show a good agreement with the theoretical considerations. We also carried out simulation on the system above but the noise $n(i)$ exist

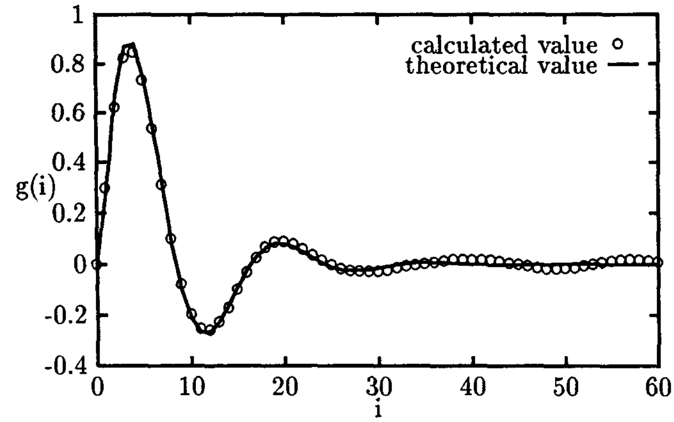

Figure 10: Comparison of the obtained g(i) with theoretical one($L = N$)

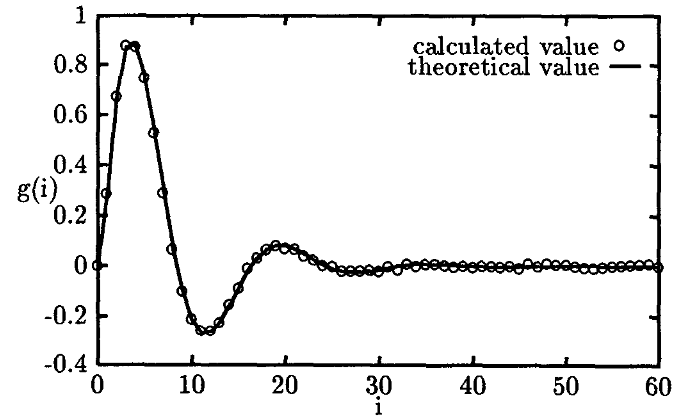

Figure 11: Comparison of the obtained g(i) with theoretical one ($L < N$)

(S/N ratio is 30dB). The results also show a good agreement with the theoretical considerations although there are small dispersions.

5. Conclusion

A new method of signal transformation, called M-transform, is proposed in this paper with application to linear system identification. M-transform is originally defined for periodic signal whose period is the same as that of M-sequence. In this paper, we propose an improvement method in which M-transform can be used for nonperiodic signals by use of a window function. The application of the improvement method to linear system identification is shown with satisfactory simulation results. The result of simulation shows that M-transform is useful for general linear system identification. The application of M-transform to signal processing is also considered. The other applications of M-transform are now under development.

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