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An Eigenvalue Method Used in Impedance Computed Tomography

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Abstract

We have developed an eigenvalue method for impedance computed tomography to improve the ill-conditioning problem. We have compared the performance of this method and the balancing method by computer simulations. As a result, it was proved that this method is better than the balancing method very much. It was found that the initial value condition is not so severe to obtain good images.

1. Introduction

Tissue impedance reflects the individual information related to the constitution and the function of the tissue.

The goal of electrical impedance tomography is to obtain the resistivity distribution at all points in a plane(or volume) of interest. The resistivity distribution can be obtained by injecting current through the cross section and measuring the voltages at the surface of the cross section between different pairs of electrodes. The procedures are repeated in several angular directions. Once all the current and voltage data are collected at the periphery, the resistivity distribution can be reconstructed by the various reconstruction algorithms.

The Newton-Raphson algorithm is one of the most theoretically sound algorithms used in electrical impedance tomography. However, it is limited by a divergence problem of the updating equation. The divergence of the updating equation is caused by the Jacobian matrix of the estimated voltage and its inversive matrix to form a ill-conditioning matrix. It results in large reconstruction errors or sometimes failure to obtain any image.

In recent years, various methods have been suggested to improve the ill-conditioning. In this paper, an eigenvalue method is introduced, which is one of the algorithms to solve this problem in our laboratory.

2. Algorithms

In the impedance computed tomography, the real resistivity distribution is unknown. Thus, we can only take an indirect measurement to comparing a reconstruction error with the reconstructed one. Namely, we compared the objective function, $\phi(\rho)$, in the form of the measured voltage v_0 and estimated voltage $f(\rho)$ as follows.

$$\phi(\rho) = (1/2)[f(\rho) - v_0]^T[f(\rho) - v_0]$$

where ρ is the resistivity distribution and T means transpose.

We used a numerical technique such as the Finite Element Method(FEM) to obtain the estimated voltage $f(\rho)$. In the FEM, we change Poisson's calculus equation ($\nabla\rho^{-1}\nabla V = 0$) into a linear system of equations ($Yv = c$), then we solve the linear system of equations using various numerical techniques. In this paper, we used the eigenvalue method and compared with the balancing method (Mingji Li et. al.).

For the resistivity distribution ρ, the updating equation is obtained by Newton-Raphson method,

$$\Delta\rho = -[f'(\rho)^T f'(\rho)]^{-1}[f'(\rho)]^T[f(\rho) - v_0]$$

where $[f']^T[f']$ is the above ill-conditioning matrix. The ill-conditioning matrix is a complicated function of current, voltage and the unknown resistivity distribution matrices. Therefore, it is so difficult to improve the conditioning of the matrix by choosing a measurement method.

We have developed various algorithms to improve the conditioning of the matrix by changing some parameters, such as the eigenvalue method and the balancing method.

In Hua et al's regularization method, the objective function to be minimized is

$$\phi(\rho) = (1/2)[f(\rho) - v_0]^T[f(\rho) - v_0] + \lambda g(\rho)$$

where λ is smoothing coefficient. The regularization method adopts a fixed smoothing parameter λ. A large λ distorts the real information, a small λ can not improve the conditioning. Thus, the performance of the regularization method is closely related to the smoothing coefficient.

We have developed an automatic algorithm by calculating an eigenvalue of the ill-conditioning matrix. We defined the smoothing parameter λ as the eigenvalue Dd of the ill-conditioning matrix $[f']^T[f']$. Namely, in the automatic eigenvalue regularization method, the objective function to be minimized is

$$\phi(\rho) = (1/2)[f(\rho) - v_0]^T[f(\rho) - v_0] + Ddg(\rho).$$

Therefore, we obtain the updating equation for ρ

$$\Delta\rho = -[f'(\rho)^T f'(\rho) + Dd]^{-1}[f'(\rho)]^T[f(\rho) - v_0].$$

Namely, we revised the ill-conditioning matrix by a penalty term Dd. We improved the regularization method (Hua et. al.) by setting the smoothing parameters to some different constant values instead of one constant smoothing parameter. And the eigenvalues of the ill-conditioning matrix are defined as these smoothing parameters.

This method has some advantages comparing to other algorithms (Balancing method, Regularization method). (1) It still maintains the penalty term, which is the feature of the regularization method. (2) The penalty smoothing parameters reflect the leading information from the resistivity distribution and data collection method.

In the balancing method, we adjusted the updating equation and the resistivity distribution to minimize the crumple error, then changing the pivots into the elements which are the large absolute value of equation coefficients.

3. Experiment

We have compared the performance of this method and the balancing method by computer simulations. As a result, it was proved that this method is better than the balancing method. It was found that the initial value has a wide range (Figure 1) and many good sensitive images can be gained

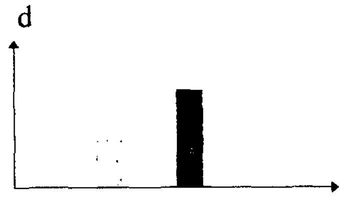
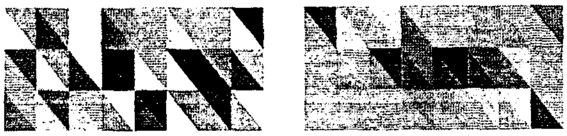


Figure 1. The initial value dynamic range.

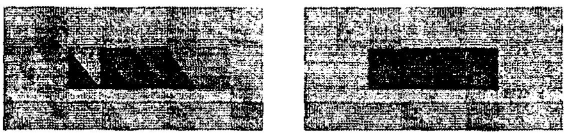
:Balancing method ■: Eigenvalue method

depending upon these several different initial data(Figure 2). Thus, a good image can be obtained after any of the predefined value of iteration times. Meanwhile, this method uses a small amount of memories like the balancing method and it is very



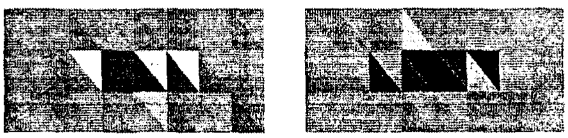
(a)Balancing Method for k=1 (b)Eigenvalue Method for k=3

The initial value is r=90 、 r0=100.



(a)Balancing Method for k=1 (b)Eigenvalue Method for k=1

The initial value is r=100 、 r0=200.

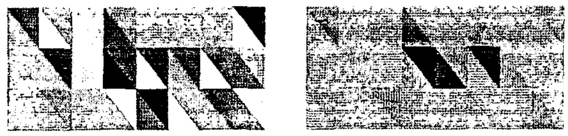


(a)Balancing Method for k=1 (b)Eigenvalue Method for k=1

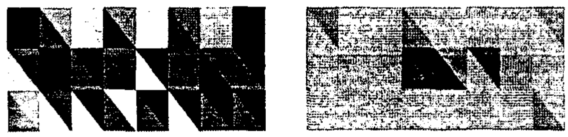
The initial value is r=100 、 r0=300.

Figure 2.The images reconstructed from simulation data

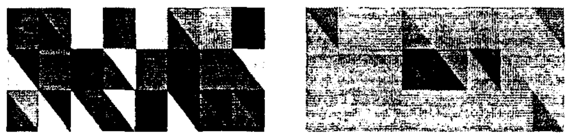
effective to the multi-elements numerical calculating problem. Also it possesses fast convergence, usually only after three times iteration. Even if it can not converge after 25 iterations, but it does not diverge. However, A good image can be



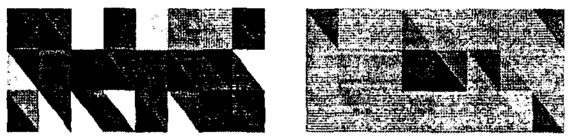
(a)Balancing Method for k=1 (b)Eigenvalue Method for k=1



(a)Balancing Method for k=2 (b)Eigenvalue Method for k=2



(a)Balancing Method for k=3 (b)Eigenvalue Method for k=3



(a)Balancing Method for k=4 (b)Eigenvalue Method for k=4

(a)Balancing Method for k=10 (b)Eigenvalue Method for k=10

Reconstruction of the Simulation data.
The initial value is r=95 、 r0=80.

Figure 3. The image performance progress with the iteration times increase

obtained only after about one time iteration and the iteration times differ with the various initial values in the balancing method. The image performance

Figure 4. The resistivity distribution to be reconstructed is an object of 150 Ω(r0) imbedded in a 100 Ω(r) background.

degrades strikingly progress with the iteration times increased in the balancing method than the eigenvalue method(Figure 3).

Figure 4 is the image of resistivity distribution to be reconstructed simulation phantom in which an object of 150 Ω(r0) imbedded in a 100 Ω(r) background.

4. Conclusions

We have developed an automatic algorithm which determines a good smoothing coefficient depending on the ill-conditioning matrix. We have simulated the two methods by computer, and proved that this method is better than the balancing method. It was found that the initial values have wide ranges d. Also the method helps to stabilize the system and to solve the divergence problem of the updating equation greatly. Therefore, many good images can be obtained after a couple of iterations or by the predefined value of iteration times.

We have proved that the eigenvalue automatic method is effective and can be used in impedance tomography imaging.

5. References

[1] Mingji Li, Akihiko Uchiyama, "Impedance computed tomography by the balancing method and the conventional method", *Proceedings of the 1998 IEICE general conference*, Tokyo, Japan, pp. 132.

[2] Mingji Li, Akihiko Uchiyama, "A fast modified calculation method for the reconstruction of impedance computed tomography", *Proceedings of the 37th conference the Japan society of medical electronics & biological engineering*, Kurashiki, Japan, pp. 424.