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The Short Time Spectra Analysis System Using The Complex LMS Algorithm and It's Applications

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Abstract

B.Widrow established fundamental relations between the least-mean-square (LMS) algorithm and the digital Fourier transform [1]. By extending these relations, we proposed the short time spectra analysis system using the LMS algorithm [2]. In that paper, we used the normal LMS algorithm on the thought of dealing with only real analytical signal. This algorithm minimizes the real mean-square by recursively altering the complex weight vector at each sampling instant. But, the short time spectra analysis sometimes deals with the complex signal that is outputted from complex analog filter. So, in order to optimize and develop this methods, furthermore it is necessary to derive an algorithm for the complex analytical signal. In this paper, we first discuss the new adaptive system for the spectra analysis using the complex LMS algorithm and then derive convergence condition, time constant of coefficient adjustment and frequency resolution by extending the discussion. Finally, the effectiveness of the proposed method is experimentally demonstrated by applying it to the measurement of transfer performance on complex analog filter.

1. Introduction

Frequency response method is a useful and simple method to measure the transfer performance on a system. In this method, the peak-to-peak value of input and output signals is read and then transfer performance is analyzed from the ratio of them. This method also has some problems as follows: First, the lower the ratio between signal and noise becomes, the more difficult the accurate measurement becomes. Second, measuring times are generally said to be long. Further, if input signals consist of real signal and output signals consist of complex signal, then transfer performances such as gains and phases can'tn be gotten. In the reference [3], we proposed the real time measurement of transfer performance using SIMULINK, which could resolve the first and second problems. But, we used the normal LMS algorithm on the thought of dealing with only real analytical signal. So, we could't solve the final problem. In order to optimize and develop the short time spectra analysis system using the LMS algorithm, further it is necessary to derive an algorithm for the complex analytical signal. In this paper, we first discuss the new adaptive system for the spectra analysis using the complex LMS algorithm and, then we derive convergence condition, time constant of coefficient adjustment and frequency resolution by extending the discussion. Finally, the effectiveness of the proposed method is experimentally demonstrated by applying it to the measurement of transfer performance on complex analog filter.

2. Derivation of the optimum Fourier coefficient vector

The complex LMS algorithm to analyze the complex signals is similar to the LMS algorithm to analyze the real signals [2], except that the rules of complex algebra must be taken into account. **Fig. 1** shows the short time spectra analysis system using the complex LMS algorithm. In this figure, $G(k)$ and $W(k)$ are Fourier coefficient vector and input signal vector of the adaptive system, and are given by

$$G = [G_1, G_2, \cdots G_n]^T \tag{1}$$

$$X(k) = [\exp(j2\pi f_1 kT), \exp(j2\pi f_2 kT), \cdots, \exp(j2\pi f_n kT)]^T \tag{2}$$

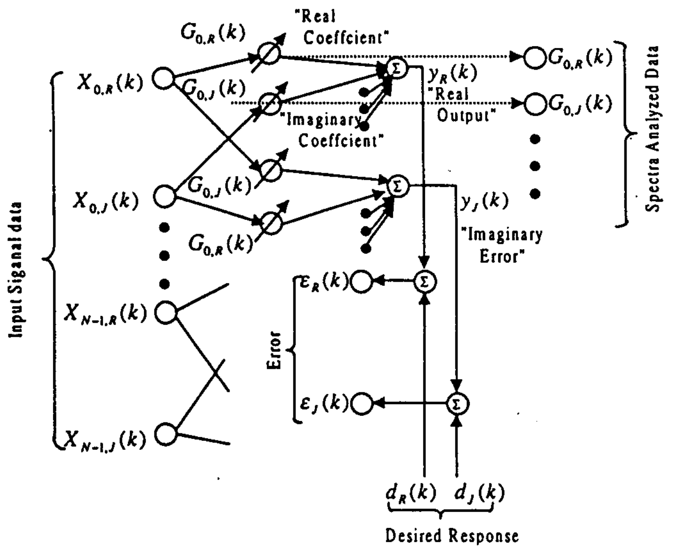

Fig. 1 Spectra analysis system with the complex LMS algorithm.

where f_i is frequency. The output signal at time k is

$$y(k) = G^T X(k) = X^T(k)G \tag{3}$$

The complex error signal $\varepsilon(k)$ required for adaptive is defined as the difference between the desired response $d(k)$ and output signal $y(k)$:

$$\begin{aligned}\varepsilon(k) &= d(k) - g(k)\\ &= d(k) - G^T X(k)\end{aligned} \tag{4}$$

The conjugate of the complex error is

$$\bar{\varepsilon(k)} = \bar{d(k)} - \bar{G}^T \bar{X}(k) \tag{5}$$

where the bar above $\bar{\varepsilon(k)}$ designates the complex conjugate. The complex LMS algorithm for the short time spectra analysis system must be able to be adapted to the real and imaginary parts of $G(k)$ simultaneously, minimizing in some sense both $\varepsilon_R(k)$ and $\varepsilon_J(k)$. A resonable objective is to minimize the average total error power $\mathrm{E}[\varepsilon(k)\bar{\varepsilon}(k)]$,

$$\mathrm{E}[\varepsilon(k)\bar{\varepsilon}(k)] = \mathrm{E}[\varepsilon_{R,k}^2 + \varepsilon_{J,k}^2] = \mathrm{E}[\varepsilon_{R,k}^2] + \mathrm{E}[\varepsilon_{J,k}^2] \tag{6}$$

where E designates expected value. Since the two components of the error are quadrate relatives to each other, they can't be minimized independently. Substituting equ.(4) and(5) into this equation, we can obtain the following equation.

$$\begin{aligned}\mathrm{E}[\varepsilon(k)\bar{\varepsilon}(k)] &= \mathrm{E}[\{d(k) - G^T X(k)\}\\ &\quad \cdot\{\bar{d}(k) - \bar{G}^T \bar{X}(k)\}]\\ &= \mathrm{E}[d(k)\bar{d}(k)] - \mathrm{E}[d(k)\bar{X}^T(k)]\bar{G}\\ &\quad -\mathrm{E}[\bar{d}(k)X^T(k)]G\\ &\quad +G^T\mathrm{E}[X(k)\bar{X}^T(k)]\bar{G}\end{aligned} \tag{7}$$

The gradient of $\mathrm{E}[\varepsilon(k)\bar{\varepsilon}(k)]$ is

$$\begin{aligned}\nabla &= \frac{\partial \mathrm{E}[\varepsilon(k)\bar{\varepsilon}(k)]}{\partial \bar{G}}\\ &= -\mathrm{E}[d(k)\bar{X}^T(k)] + \mathrm{E}[X(k)\bar{X}^T(k)]G\end{aligned} \tag{8}$$

where $\mathrm{E}[X(k)\bar{X}^T(k)]$ is

$$\begin{aligned}&\mathrm{E}[X(k)\bar{X}(k)]\\ &= \mathrm{E}\left[\begin{pmatrix}\exp(j2\pi f_1 kT)\\ \exp(j2\pi f_2 kT)\\ \vdots\\ \exp(j2\pi f_n kT)\end{pmatrix}(\exp(-j2\pi f_1 kT),\cdots\cdots)\right]\\ &= \mathrm{E}\begin{bmatrix}1 & \exp(j2\pi(f_1 - f_2)kT) & \cdots & \exp(j2\pi(f_1 - f_N)kT)\\ \vdots & 1 & \cdots & \exp(j2\pi(f_2 - f_N)kT)\\ \vdots & \vdots & \ddots & \vdots\\ \vdots & \vdots & \cdots & 1\end{bmatrix}\\ &= I\end{aligned} \tag{9}$$

From this equation, we can get the optimum coefficient vector solution as follow, where the average total error power $\mathrm{E}[\varepsilon(k)\bar{\varepsilon}(k)]$ is minimized.

$$G^* = \mathrm{E}[d(k)\bar{X}^T(k)] \tag{10}$$

3. Derivation of the complex LMS algorithm

To develop the complex LMS algorithm from an adaptive algorithm using the previous section's method, we would have to estimate the gradient of $\mathrm{E}[\varepsilon(k)\bar{\varepsilon}(k)]$ by taking instantaneous error $\varepsilon(k)\bar{\varepsilon}(k)$. The instantaneous gradient of $\varepsilon(k)\bar{\varepsilon}(k)$ with respect to the real component of the coefficient vector is

$$\begin{aligned}\nabla_R(\varepsilon(k)\bar{\varepsilon}(k)) &= \frac{\partial \varepsilon(k)\bar{\varepsilon}(k)}{\partial G_R}\\ &= \varepsilon(k)\nabla_R(\bar{\varepsilon}(k)) + \bar{\varepsilon}_k\nabla_R(\varepsilon(k))\\ &= \varepsilon(k)(-\bar{X}(k)) + \bar{\varepsilon}(k)(-X(k))\end{aligned} \tag{11}$$

The instantaneous gradient of $\varepsilon(k)\bar{\varepsilon}(k)$ with respect to the imaginary component of the coefficient vector is

$$\nabla_J(\varepsilon(k)\bar{\varepsilon}(k)) = \varepsilon(k)(j\bar{X}(k)) + \bar{\varepsilon}(k)(-jX(k)) \tag{12}$$

Applying the method of steepest descent to the real and imaginary parts of the coefficient vector by chaining them along their respective negative gradient estimations, we obtain

$$G_R(k+1) = G_R(k) - \mu\nabla_R(\varepsilon(k)\bar{\varepsilon}(k)) \qquad (13)$$

$$G_J(k+1) = G_J(k) - \mu\nabla_J(\varepsilon(k)\bar{\varepsilon}(k)) \qquad (14)$$

Since the complex coefficient vector is $G(k) = G_R(k) + G_J(k)$, the complex coefficient vector iteration rule can be expressed as

$$G(k+1) = G(k) - \mu[\nabla_R(\varepsilon(k)\bar{\varepsilon}(k)) + j\nabla_J(\varepsilon(k)\bar{\varepsilon}(k))] \qquad (15)$$

If the gradients (11) and (12) are now substituted in (15), the complex form of the LMS algorithm for short time spectra analysis system results:

$$G(k+1) = G(k) + 2\mu\varepsilon(k)\bar{X}(k) \qquad (16)$$

where μ is a convergence factor controlling stability and adaptation.

4. Convergence Condition

4.1 Coefficient vector

In this paper, we assume coefficient vector $G(k)$ to be independent of input vector $X(k)$. Taking the expected value of equ.(16) yields the difference equation as follows:

$$\begin{aligned} \mathrm{E}[G(k+1)] &= \mathrm{E}[G(k)] + 2\mu\mathrm{E}[\varepsilon(k)\bar{X}(k)] \\ &= \mathrm{E}[G(k)] + 2\mu\mathrm{E}[\bar{X}(k)\{d(k) \\ &\quad -X^T(k)G(k)\}] \\ &= (1-2\mu)\mathrm{E}[G(k)] + 2\mu G^* \end{aligned} \qquad (17)$$

Now we can define the coefficient error vector $C(k)$ as $C(k) = G(k) - G^*$. So equ.(17) becomes

$$\begin{aligned} \mathrm{E}[C(k)] &= (1-2\mu)\mathrm{E}[C(k-1)] \\ &= (1-2\mu)^k\mathrm{E}[C(0)] \end{aligned} \qquad (18)$$

Thus as k increases without bound, the expected coefficient vector reaches the optimum solution when the right side of this equation converges to zero as follows:

$$\lim_{k\to\infty}(1-2\mu)^k = 0 \qquad (19)$$

where μ is a positive value. Such convergence is guaranteed only when

$$0 < \mu < \frac{1}{2} \qquad (20)$$

4.2 Learning curve

In equ.(19), we have bounds on μ that the convergence of the coefficient vector means to the optimum coefficient vector. But we don't have bounds on μ that the convergence of the mean-square-error $\xi_k = \mathrm{E}[\varepsilon(k)\bar{\varepsilon}(k)]$ means to the minimum mean-square-error ξ_{min}. From the reference [2], the mean-square-error ξ_k and minimum mean-square-error ξ_{min} become

$$\xi_k = \xi_{min} + \mathrm{E}[C^T(k)\bar{C}(k)] \qquad (21)$$

$$\xi_{min} = \mathrm{E}[d^2(k)] - G^{*T}\bar{G}^* \qquad (22)$$

Thus as k increases without any bound, the mean-square-error ξ_k reaches the minimum mean-square-error ξ_{min} only when $\mathrm{E}[C^T(k)\bar{C}(k)]$ converges to zero. So, such convergence is guaranteed only when

$$\mathrm{E}[C^T(k+1)\bar{C}(k+1)] - \mathrm{E}[C^T(k)\bar{C}(k)] < 0 \qquad (23)$$

Substituting equ.(16) and (18) into this equation, we can obtain the following equation.

$$\begin{aligned} &\mathrm{E}[C^T(k+1)\bar{C}(k+1)] - \mathrm{E}[C^T(k)\bar{C}(k)] \\ &= \mathrm{E}[\{G(k) + 2\mu\varepsilon_k\bar{X}(k)\} - G^*\}^T \\ &\quad \cdot\{\bar{G}(k) + 2\mu\bar{\varepsilon}(k)X(k)\} - \bar{G}^*\}] \\ &\quad -\mathrm{E}[\{G(k) - G^*\}^T\{\bar{G}(k) - \bar{G}^*\}] \\ &= \mathrm{E}[-4\mu\varepsilon(k)\bar{\varepsilon}(k) + 4\mu^2\varepsilon(k)\bar{\varepsilon}(k)X^T(k)\bar{X}(k)] \\ &= -4\mu(1-\mu N)\mathrm{E}[\varepsilon(k)\bar{\varepsilon}(k)] < 0 \end{aligned} \qquad (24)$$

where N is the number of coefficients. In this equation, $\mu > 0$ and $\mathrm{E}[\varepsilon(k)\bar{\varepsilon}(k)] > 0$. So, such convergence is guaranteed only when

$$0 < \mu < \frac{1}{N} \qquad (25)$$

4.3 Convergence condition

From equ.(20) and (25), convergence condition becomes

$$\begin{aligned} 0 < \mu_i < \tfrac{1}{2} \quad &\text{for} \quad N = 1 \\ 0 < \mu_i < \tfrac{1}{N} \quad &\text{for} \quad N > 2 \end{aligned} \qquad (26)$$

5. Time Constant of Coefficient Adjustment

The spectra analysis system proposed in this paper is used to the LMS algorithm, so the number of sampling data (desired responses), which are reguired to analyze the spectral component of signals, can be determined by the time constant of coefficient adjustment. From equ.(20), the coefficient vector becomes geometric sequence of the geometric rate $1-2\mu$. Now, we can construct an exponential envelope through the geometric sequence of the samples. Let the envelope be described by $\exp(\frac{-T}{\tau})$, then we can write

$$\exp(\frac{-T}{\tau}) = 1 - 2\mu \qquad (27)$$

where T represents the sampling time and τ represents the time constant. When τ is large (100 or greater) and $1-2\mu$ is small (less than but near 1), the following equation is a useful approximation:

$$1 - 2\mu \cong 1 - \frac{T}{\tau} \qquad (28)$$

So we have the time constant of coefficient adjustment as follows:

$$\tau = \frac{1}{2\mu} \qquad (29)$$

From this equation, we can understand that the time constant of coefficient adjustment is inverse proportional to the parameter of convergence factor μ.

6. Resolution in Frequency

Substituting equ.(16) into (4), and taking the expected value of this equation, we can obtain the following equation.

$$\mathrm{E}[G(k+1)] = (1-2\mu)\mathrm{E}[G(k)] + \mathrm{E}[X(k)d(k)] \quad (30)$$

Now, the i'th element $G_i(k)$ of the coefficient vector $G(k)$ is considered as follows:

$$\mathrm{E}[G_i(k+1)] = (1-2\mu)\mathrm{E}[G_i(k)] + \mathrm{E}[X_i(k)d(k)] \quad (31)$$

The transfer performance of this equation becomes

$$H_i(s) = \frac{G_i(s)}{D(s)} = \frac{2\mu}{\{z-(1-2\mu)\}\{z-\exp(j2\pi f_i T)\}}\Big|_{z=\exp(sT)} \quad (32)$$

Assuming T to be small, the following equation is a useful approximation:

$$z = \exp(sT) = 1 + sT + \frac{1}{2!}(sT)^2 \cdots = 1 + sT \quad (33)$$

Therefor equ.(32) becomes

$$H_i(s) = \frac{2\mu f_s}{(sT+2\mu)(s-j2\pi f_i)} \quad (34)$$

where f_s is the sampling frequency ($\frac{1}{T}$). Now Q-value of i'th element is defined as $Q_i = \frac{f_i}{2\chi_i}$, where f_i is the i'th frequency component of input signal vector. $2\chi_i$ means the full width of the frequency where the amplitude is one-half of the maximum value. From reference [4], Q_i becomes

$$Q_i \cong \frac{\pi f_i}{2\mu f_s} \quad (35)$$

From this equation, we get the fact that resolution in the frequency is proportional to the frequency component of input signal vector and is inverse proportional to the parameter of the convergence factor.

7. Complex analog filter and analysis method

Another purpose of this study is to analyze the transfer performance of Complex coefficient analog filter. Hence in this section, we will describe a complex analog filter and analysis method for transfer performance of a complex analog filter.

7.1 Complex analog filter

Complex coefficient digital filters, with applications for processing real sequences, have been investigated. The method allows any real rational transfer function to be expressed in terms of a complex rational transfer function of reduced order. Being implemented in complex hardware form, the reduction of filter order can provide an increase in computational efficiency and speed. From the same reason, complex coefficient analog filters have been investigated. In general, the method converting a real coefficient filter into a complex filter with an unsymmetrical frequency response is used, and the output signal from this complex filter is complex signal. Consider the complex filter $C(s) = G(s) + jH(s)$, where $G(s)$ and $H(s)$ are conjugate-symmetric and conjugate-antisymmetric parts of $C(s)$, respectively. We may express $c(t) = g(t) + jh(t)$ as the corresponding complex impulse response, where the Dirac delta impulse function is real by definition. Letting $x(t) = x_R(t) + jx_J(t)$ denote the complex input to the complex filter, where $x_R(t)$ and $x_J(t)$ are real signals, the complex output $w(t)$ becomes

$$\begin{aligned} w(t) &= [g(t)\otimes x_R(t) - h(t)\otimes x_J(t)] \\ &\quad + j[g(t)\otimes x_J(t) + h(t)\otimes x_R(t)] \\ &= w_R(t) + jw_J(t) \end{aligned} \quad (36)$$

where $w_R(t)$ and $w_J(t)$ are real signals, and $\otimes$ denotes convolution. The Laplace transform of equ.(36) yields a real-equivalent system representation

$$\begin{bmatrix} W_R(s) \\ W_J(s) \end{bmatrix} = \begin{bmatrix} G(s) & -H(s) \\ H(s) & G(s) \end{bmatrix} \begin{bmatrix} X_R(s) \\ X_J(s) \end{bmatrix} \quad (37)$$

for $C(s) = G(s) + jH(s)$. **Fig. 2** shows equivalent circuit for complex analog filter.

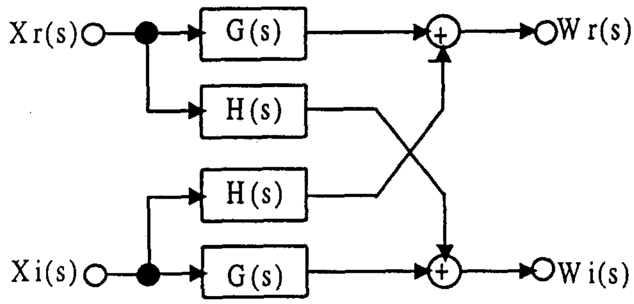

Fig. 2 Equivalent circuit for complex analog filter.

7.2 Analysis method

The purpose of this study is to analyze the transfer performance of Complex coefficient analog filter. Frequency response method is a useful method to analyze transfer performance. It is widely used because it is easy to use. In this method, the peak-to-peak value of input and output signals is measured and then transfer performance is analyzed from the ratio of them. This method also has

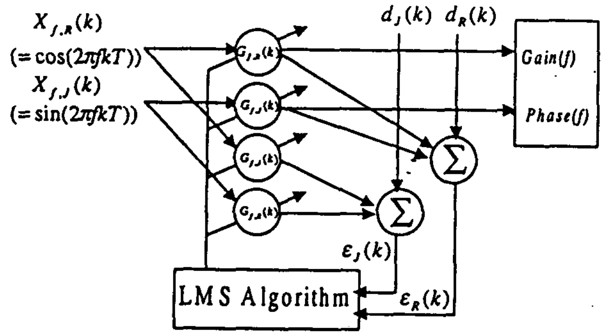

Fig. 3 Spectra analysis system for the purpose of analyzing transfer performance of complex coefficient analog filter.

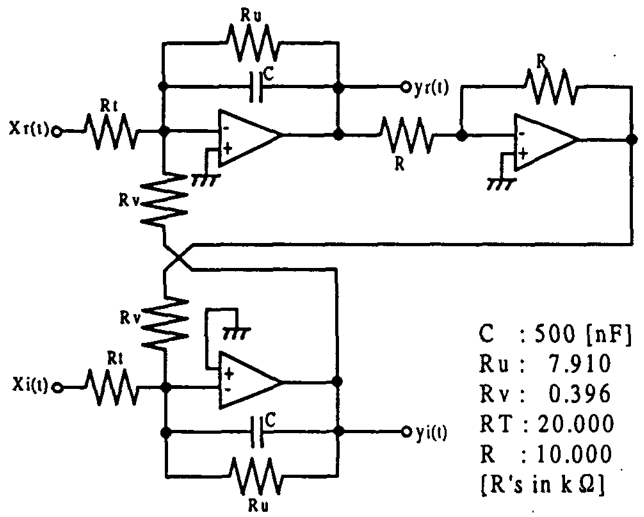

Fig. 4 Circuit realization of the complex coefficient analog filter.

some problems as follows: First, input and output signals are complex signals, and the lower the ratio between signal and noise becomes, the more difficult the accurate measurement becomes. Second, measuring times are generally said to be long. When transfer performance is analyzed using FFT, spectral leakage occurs by virtue of side lobe. Hence accurate analysis of transfer performance is difficult. To overcome these problems, we used the short-term analysis using the LMS algorithm. we assumed that the input signal of Complex coefficient analog filter is complex exponential function, equ.(36) becomes as follows,

$$w(t) = c(t)\exp(2\pi ft) \tag{38}$$

This equation is equal to equ.(3), so we can analyze transfer performance complex coefficient analog filter by using the spectra analysis system as shown in Fig. 3.

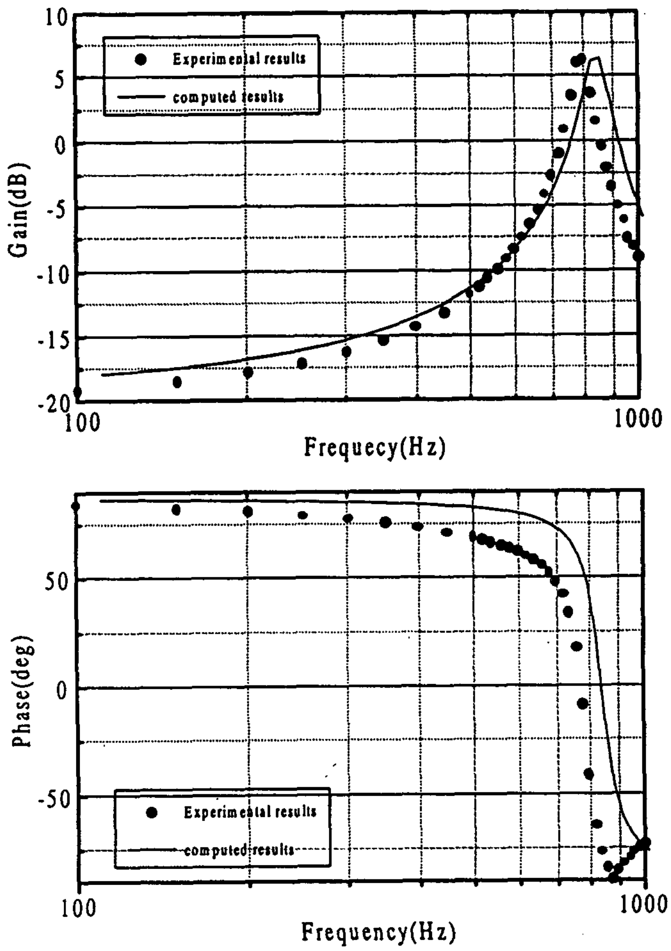

Fig. 5 Comparison between experimental results and computed results of equ.(39))

8. Experiment

Fig. 4 shows the circuit realization of the complex coefficient analog filter which was used in this experiment. The complex transfer performance of this circuit realization becomes

$$C(s) = \frac{1.000E^2S + (2.528E^4 + j5.051E^5)}{S^2 + 5.057E^2S + 2.557E^7} \tag{39}$$

Fig. 5 shows the Comparison between the experimental results and the compued results of equ.(39). From this figure, the experimental results of the proposed system are similar to the results of the computed results of equ.(39).

9. Conclusions

The main points of this study are summarized as follows:

(1) We can constract the spectra analysis using the LMS algorithm by extending the discussion to the fundamen-

tal relations between the LMS algorithm and the digital Fourier transform established by B.Widrow [1].

(2) We assumed the coefficient vector $G(k)$ to be independent of input vector $X(k)$. In addition to it, taking the expected value of equ.(4), we had bounds on μ that the convergence of the coefficient vector means to the optimum coefficient vector and leaning curve.

(3) We derived that both the time constant of coefficient adjustment and the resolution in frequency are inverse proportional to the parameter of convergence factor μ.

(4)Applying of short-term spectrum analysis using the LMS algorithm to the frequency response method, we could measure transfer performances. And experimental results using this method were in good agreement with the results of computed results of equ.(39).

References

1) B.Widrow et al.,"Fundamental relations between the LMS algorithm and the digital Fourier transform",IEEE Trans. Circuits syst., vol.CAS-34,No.7,814-820(1987)

2) T.Umemoto and N.Aoshima,"The adaptive spectrum analysis for trabscription",Trans. of SICE,28-5,619-625 (1992)

3) T.Umemoto and N.Aoshima,"Selection of step size parameter of adaptive filter for spectrum analysis",Trans. of SICE,28-10,1257-1262(1992)

4) T.Umemoto and N.Aoshima,"Construct of Transcription System and Comparison of Peak Extraction Method",Trans. of SICE,29-10,1227-1231(1993)

5) T.Umemoto and N.Aoshima,"Adaptive DFT and its application",Trans. of SICE,30-8,959-965(1994)

6) Regalia P.A. and Mitra S.K.,"Low-sensitivity active filter realization uing a complex all-pass filter",IEEE Trans. Circuits syst., vol.CAS-34,No.4,390-399(1987)

7) C.Muto and N.Kambayashi,"A realization of real filters using complex resonators",IEICE Trans.(A),Vol.J75-A,No.7,1181-1188(1992)