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Stochastic Design Approach for the Guidance and Control System of an Automatic Landing Vehicle

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Abstract

In this paper, a stochastic approach based on a Monte Carlo simulation method for the design of a guidance and control (G & C) system of an automatic landing flight experiment (ALFLEX) vehicle is presented. The aim of this study is to design a G & C system robust against uncertainties in the vehicular dynamics. In this study, uncertain parameters and disturbances are treated as random variables in the Monte Carlo simulation. Then, some controller gains in the G & C system are tuned to satisfy conditions concerning the states at touchdown. The proposed method was applied to the ALFLEX vehicle. The simulation results showed the effectiveness of the present approach.

1. Introduction

The ALFLEX project is an automatic landing flight experiment for an unmanned reusable space transportation system called HOPE-X which has been developed by the collaboration of the National Aerospace Laboratory (NAL) and National Space Development Agency (NASDA) of Japan. This project has led to the successful obtaining of much data concerning all 13 automatic landings in 1996. In this experiment, the vehicle landed automatically on a target runway after its release from a helicopter at an altitude of 1,500m. However, a robustness-related problem occurred associated with the parameters of the sensors in longitudinal flight i.e., the vehicle had a tendency to float prior to touchdown in all 13 flights. A primary factor causing this problem was a measurement bias error in V_{EAS} from Air Data Sensor (ADS). In order to solve this problem, a stochastic design approach for tuning some gains in the G & C system has been proposed. In this paper the presented approach has been introduced and its availability is discussed

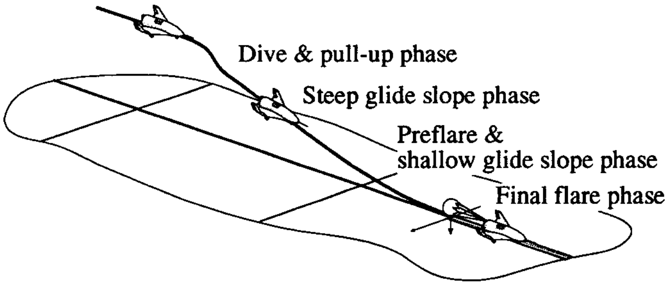

Figure 1 ALFLEX project

2. A problem in the longitudinal G & C system for automatic landing

Generally, in the design of an aircraft G & C system, the design of a latitudinal G & C system is more difficult than that of a longitudinal one. However, the longitudinal G & C system of a reentry space transportation vehicle has a problem with respect to automatic landing, that is, the vehicle has to flare for soft-landing. For the landing of a conventional aircraft, the glide path angle is about 1.5 degrees, however for the reentry vehicle, the glide path angle needs to be larger due to a low lift-drag ratio (L/D). In the case of an ALFLEX vehicle, the glide path angle in the approach phase is 30 degrees. Therefore , the flare phase for curving the flight path is necessary. Thus, the latitudinal G & C system can be considered as a regulator problem while, in contrast, the longitudinal one has to be considered as a problem of tracking to a reference flight path.

The longitudinal G & C system of the pre-flare phase of the ALFLEX vehicle is shown in Figure 3. The guidance law is composed of a PID feedback control system and an open-loop control system. The output is a nominal acceleration command (A_{Zcom}). The advantage of this system is that it can improve the response of the vehicle using the open-loop control system. Since the input to the attitude controller is a pitch-rate command (q_{com}), a second order filter is used to convert A_{Zcom} into q_{com}. This second order filter requires an equivalent air speed (V_{EAS}) input. If the V_{EAS} has a bias error, the pitch-rate command becomes unsuitable and a path error occurs.

As a result of an automatic landing experiment, the vehicle passed 3m to 5m higher than the nominal altitude, due to the bias error in V_{EAS} measured by ADS. These flight paths from pre-flare to touchdown are plotted in Figure 2.

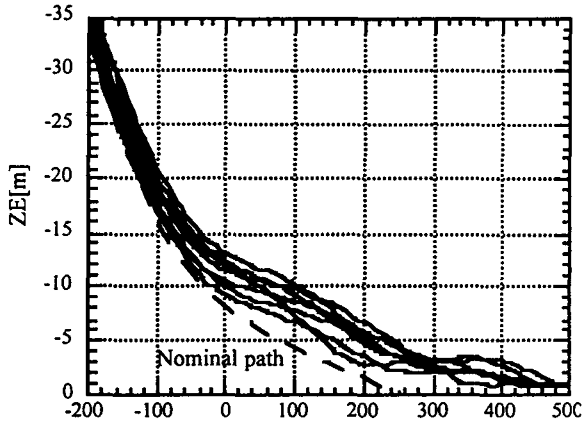

Figure 2 Flight path from pre-flare to touchdown in the experiment

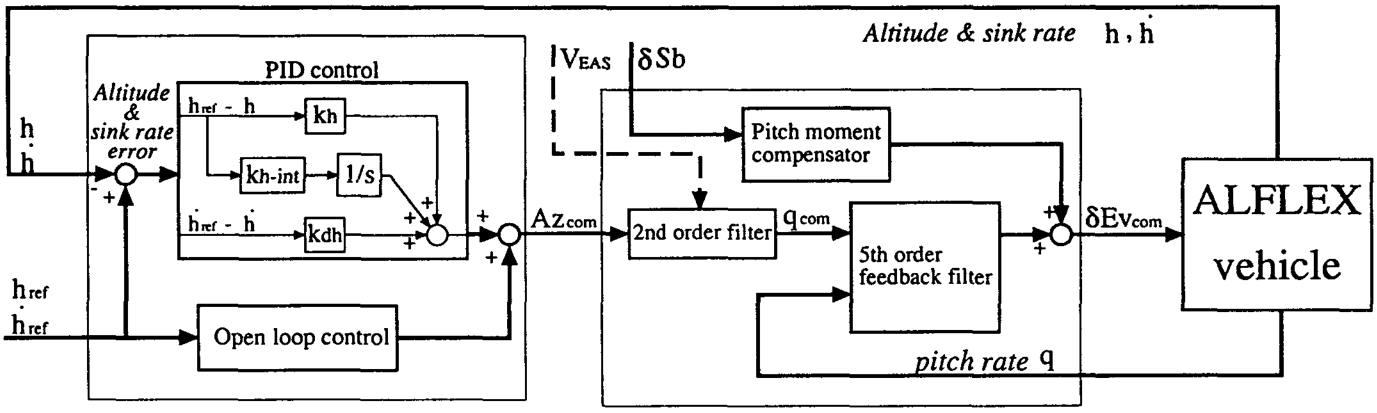

Figure 3 ALFLEX G&C system in Pre-flare phase

3. Stochastic evaluation using Monte Carlo simulation

In this section, a stochastic simulation analysis using Monte Carlo simulation is described.

The nonlinear dynamics of an aircraft are usually expressed in the form

$$\dot{\mathbf{x}} = f(\mathbf{x}, \mathbf{u}), \tag{3.1}$$

where $\mathbf{x}$ is a state vector and $\mathbf{u}$ is an input vector.

However, a system necessarily includes some uncertain parameters due to modeling errors in the vehicle, actuators and sensors. Atmospheric disturbance ($\mathbf{u}_d$) is also included in the system. Therefore, Eq.(3.1) can be rewritten as

$$\begin{cases} \dot{\mathbf{x}} = f(\theta, \mathbf{x}, \mathbf{u}, \mathbf{u}_d) \\ \mathbf{u} = g(\mathbf{x}, \mathbf{k}) \end{cases}, \tag{3.2}$$

where vector $\boldsymbol{\theta}$ represents uncertain parameters and the vector $\mathbf{k}$ is a controller gain which will be explained in detail later.

In this study, in order to apply the Monte Carlo method, such uncertain parameters are treated as random variables defined by a probability density function with normal distribution. Figure 4 shows this scheme.

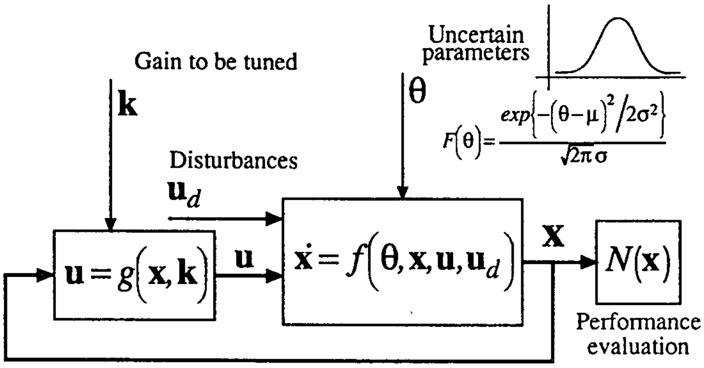

Figure 4 Monte Carlo Simulation
Uncertain Parameters $\theta_i(i=1,2,\cdots,100)$ are tuned as gaussian random variables. $k_i(i=1,2,\cdots,q)$ are gains to be tuned.

Table 1 Model Error

Error	Type	3σ	
Equivalent air speed	ADS Bias (V_{EAS})	4	m/s
Stability derivatives C_{m_α}	Model error	0.00211	1/deg
Stability derivatives C_{L_α}	Model error	0.007	1/deg
Mass	Model error	-6 , +24	kg
Steady wind	Air disturbance	1	(uniform)
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.	.		.
.	.		.

In order to evaluate the landing performance, a special function $N(\cdot)$ is defined as

$$N(\mathbf{x}_{tp}) = \begin{cases} 1 & (\mathbf{x}_{tp} \subset \mathbf{S}) \\ 0 & (\mathbf{x}_{tp} \not\subset \mathbf{S}) \end{cases}, \tag{3.3}$$

where S is a set of requirements in which the state $\mathbf{x}_{tp}$ in Eq.(3.2) at touchdown must be satisfied. This function judges whether or not state $\mathbf{x}_{tp}$ satisfies all the members of S. If all the requirements are satisfied, the value of $N(\mathbf{x}_{tp})$ is unity. If one or more requirements remain unsatisfied, this value becomes zero.

After J repetitions of the Monte Carlo simulation, the jth landing performance $\mathbf{x}_{tpj}$ is obtained from the following dynamics.

$$\begin{cases} \dot{\mathbf{x}}_j = f(\theta_j, \mathbf{x}_j, \mathbf{u}_j, \mathbf{u}_{dj}) \\ \mathbf{u}_j = g(\mathbf{x}_j, \mathbf{k}_j, \mathbf{k}_{dj}) \end{cases} \tag{3.4}$$

Then, the probability such that all the requirements are satisfied can be expressed by

$$P(\mathbf{x}_{tp} | \mathbf{x}_{tp} \subset \mathbf{S}) = \lim_{J \to \infty} \frac{\sum_{j=1}^{J} N(\mathbf{x}_{tpj})}{J}. \tag{3.5}$$

When the value of J increases, the reliability of the probability increases.

The Monte Carlo simulation for the present ALFLEX vehicle includes about 100 kinds of uncertain parameters. Table 1 shows important uncertain parameters with their 3σ, where σ indicates a standard deviation. Table 2 shows the landing performance requirements of the ALFLEX vehicle.

Table 2 Landing performance requirement

Evaluation point	**Requirement**
Touchdown	
Position	X: >0m , Y: ±18m
Velocity	Ground speed: <62m/s
	Airspeed: 51.5±8m/s
	Sink rate: <3m/s
Attitude	Pitch angle: <23 deg
	Bank angle: ±10 deg
	Yaw angle: ±8 deg
Ground roll	Y: ±20m
Stop point	X: <1000m

4. Stochastic gain tuning

The gain values of a class with a higher probability, where all the requirements are satisfied, are chosen as a suitable set of gains for a G & C system by the Monte Carlo simulation.

In Eq.(3.2), the vector **k** is a set of controller gains to be designed.

$$\mathbf{k}=\left[k_1,k_2,k_3,\cdots\cdots,k_q\right]^T, \tag{4.1}$$

where k_i is a uniformly distributed random variable, assumed to have the following boundaries.

$$k_{i\,min}\le k_i\le k_{i\,max},\ i=1,2,3\cdots\cdots,q. \tag{4.2}$$

When the Monte Carlo simulation is repeated J times, a set **H** is defined as

$$\mathbf{H}=\left\{\mathbf{k}\middle|N\left(\mathbf{x}_{tp}\right)=1\right\}, \tag{4.3}$$

where all **k**s in the set **H** satisfy all the requirements, i.e. $N(\mathbf{x}_{tp})=1$.

Since each band of k_i is classified into T_i ($i=1,2,\cdots,q$), the class interval a_i is

$$a_i=\frac{k_{i\,max}-k_{i\,min}}{T_i}. \tag{4.4}$$

The frequency n of a class $\mathbf{t}(t_1,t_2,\cdots,t_q)$ can be written as

$$n(t)=\sum_{t_1,t_2,\cdots,t_q}\left\{N\left(\mathbf{x}_{tp}\right)\middle|\dot{\mathbf{x}}=f\left(\theta,\mathbf{x},\mathbf{u},\mathbf{u}_d\right),\mathbf{u}=g\left(\mathbf{x},\mathbf{k}\right)\right\} \tag{4.5}$$

Since the elements of **k** have been defined as uniform between the boundaries in Eq.(4.2), if the number of repetitions J is large enough, the number of repetitions $J|_{\mathbf{t}}$ in each class can be expressed as

$$J|_{\mathbf{t}}=\frac{J}{\sum_{i=1}^{q}T_i}. \tag{4.6}$$

Thus, the probability that all the requirements are satisfied in each class is

$$P\left(\mathbf{x}_{tp}\middle|\mathbf{x}_{tp}\subset\mathbf{S},\dot{\mathbf{x}}=f\left(\theta,\mathbf{x},\mathbf{u},\mathbf{u}_d\right),\mathbf{u}=g\left(\mathbf{x},\mathbf{k}|_{\mathbf{t}}\right)\right) =n(t)/J\ . \tag{4.7}$$

A class **t**, which has a higher probability yields a more suitable set of gains. Thus, the highest-probability class in the obtained histogram is the optimal one for the controller gains to be adopted.

This approach is not so difficult. For example, if the number of the design gains is 2, a 2-dimensional histogram is obtained.

5. Gains tuning of the ALFLEXvehicle

In section 4, higher probability gains to satisfy all requirements were discussed. However, in contrast in this section, lower probability gains, which do not satisfy one or more requirements, are dealt with. In the G & C system for the ALFLEX vehicle, two controller gains were tuned; these gains are shown in the PID control block in Figure 3. One of them is a feedback gain k_h for the difference between the altitude and the reference one. Another gain is a feedback gain k_{dh} for the difference between the sink-rate and the reference one.

In Figure 5, sets of gains which do not satisfy one or more requirements in S are plotted. The number of repetitions of the Monte Carlo simulation is 10,000. A open circle in Figure 5 shows an original set of gains. A glance at Figure 5 reveals that the original set of gains

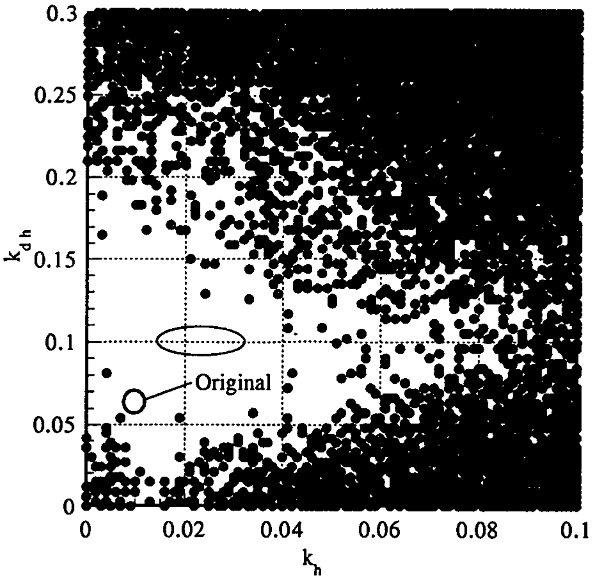


Figure 5 Unsatisfictory combinations of gains

A dot shows a set of gains that do not satisfy the requirements. The ellipse shows the highest probability set of gains that satisfy all the requirements.

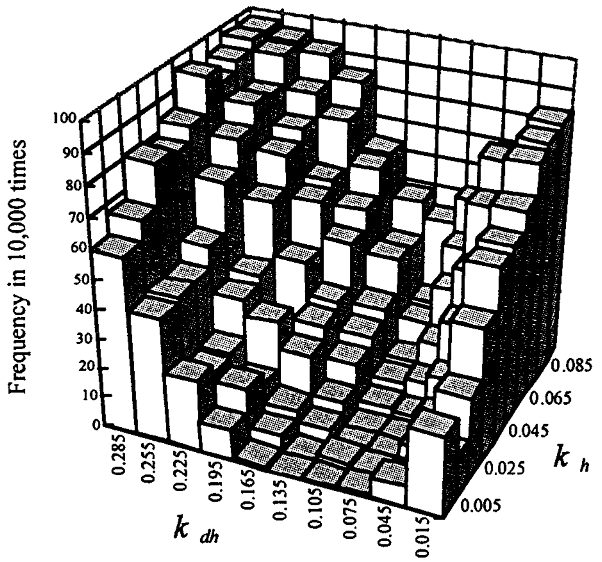
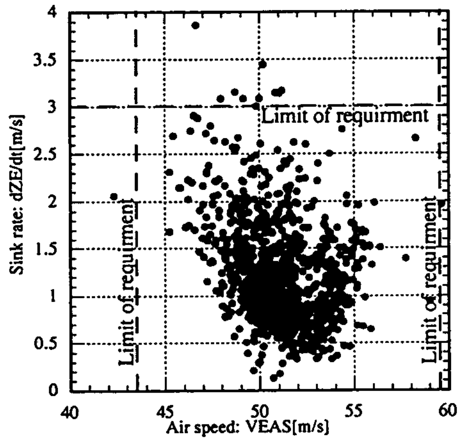
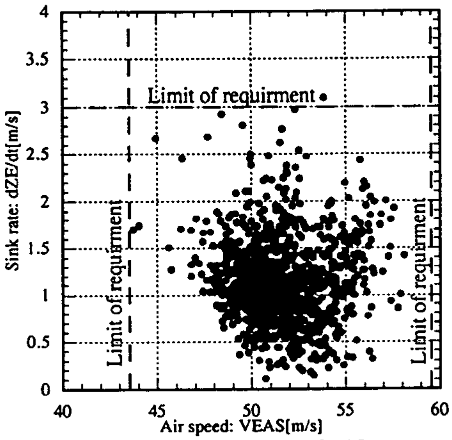


Figure 6 Histogram of spec. out of the requirements Right axis is gain k_h . Left axis is gain k_{dh} . Vertical axis is frequency of spec. out in a class.

has a margin, to make the gains higher. Figure 6 shows a 2-dimension al histogram obtained from Figure5. The lowest probability set of gains which do not satisfy one or more requirements is determined to lie in the ranges $0.02<k_h<0.03$, $0.09<k_{dh}<0.12$ from Figure 6. The averages of this class are $k_h = 0.025$, $k_{dh} = 0.115$. In Figure 7(b), the equivalent air speed and sink-rate at touchdown in 1,000 repetitions of the Monte Carlo simulation are plotted. Comparing the performance using a tuned set of gains with the performance of the original (Figure 7(a)), the sink-rate of spec. out in Figure7(b) is less than that in Figure7(a). Table 3 shows the original set of gains, a set of tuned gains, and other sets around the tuned gain, as well as sink rate, air speed and other factors. It was revealed from Table 3 that the performance using a tuned set of gains is better than that in the original set of gains.



(a) Original G&C



(b) k_h=0.025,k_dh=0.115

Figure 7 Landing Performance

Table 3 Landing performance after 1,000 repetitions of the Monte Carlo simulation

k_h	k_{dh}	Spec. out	Sink rate	Air speed	Other
0.01059 (original)	0.06607	19	10	2	7
0.0250 (after tuning)	0.1150	7	1	1	5
0.0100	0.1150	12	3	3	6
0.0200	0.1150	9	2	1	6
0.0300	0.1150	8	2	1	5

6. Conclusions

As a result of the presented stochastic gain tuning, a set of realistic gains which satisfies the requirements has been obtained. The advantages of this approach are as follows.

- A large number of uncertain parameters in the system can be treated by the present approach.
- The treatment of the G & C system design with respect to the above point is extremely realistic.
- A suitable set of gains can be obtained without the need for a complex theoretical approach.

However, this approach has the following disadvantages.

- This approach cannot be used alone to design a G & C system. Before applying the present approach, the G & C system must be predesigned by a method such as PID, LQR, Kalman Filter, etc.
- This approach requires high computational performance. However, recent dramatic increases in computing power have shortened the required computing time. The computing time is expected to become even shorter in the future.
- The number of gains to be tuned can cause a problem. If there exist q gains to be tuned, 100^q repetitions of the Monte Carlo simulation are required. Thus, in the case of larger **k**, further modifications to this approach are required.

7. References

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