Proceedings of the 13th
KACC, October 1998

A force-Guided Control with Adaptive Accommodation for Complex Assembly

Sungchul Kang Munsang Kim Chong W. Lee Kyo-Il Lee†
Korea Institute of Science and Technology †Seoul National University
P.O.BOX 131, Cheongryang, Seoul, Korea
Tel: +82-2-958-6815
Fax: +82-2-958-5629
Email: Kasch@kist.re.kr

ABSTRACT

In this paper, a target approachable force-guided control with adaptive accommodation for the complex assembly is presented. The complex assembly (CA) is defined as a task which deals with complex shaped parts including concavity or whose environment is so complex that unexpected contacts occur frequently during insertion. CA tasks are encountered frequently in the field of the manufacturing automation and various robot applications. To make CA successful, both the bounded wrench condition and the target approachability condition should be satisfied simultaneously during insertion. By applying the convex optimization technique, an optimum target approaching twist can be determined at each instantaneous contact state as a global minimum solution. Incorporated with an admissible perturbation method, a new CA algorithm using only the sensed resultant wrench and the target twist is developed without motion planning nor contact analysis which requires the geometry of the part and the environment. Finally, a VME-bus based real-time control system is built to experiment various CA tasks. T-insertion task as a planar CA and double-peg assembly task as a spacial assembly were successfully executed by implementing the new force-guided control with adaptive accommodation.

1. Introduction

Among manufacturing processes, assembly process is one of the most labor-intensive sectors in which the share of the cost of the assembly can amount from 25 to 75 per cent of the total production costs[1]. Recently, various robot applications, e.g., assembly (or fixturing) lines in manufacturing, construction, component repairing in space or in hazardous environment and robotic surgery, require an insertion of a complicated object into an positionally uncertain environment. In these applications, the shape of the part inserted is not simple or the environment geometry is not exactly known.

In this paper, a complex assembly (CA) is defined as a task which deals with complex shaped parts including concavity or whose environment is so complex that unexpected contacts occur frequently during insertion. Different from simple peg-in-hole tasks, CA has features that the dimension of the nominal insertion path can usually be more than 1 and the contact states occurring during the insertion for CA are various and complicated. Therefore the conventional compliant motion planning approaches based on various existing compliant control schemes have limitation to be applied to CA due to their computational complexity in generating a multi-dimensional motion plan. Moreover, they cannot represent the nonlinear compliance effect needed to represent multi-contact cases occurring more frequently and severely in the CA process.

Several results related to CA have been presented by extending the active compliance approaches. Shimmels and Peshkin[2-4] proposed an admittance control law for force-guided assembly applicable to a typical fixture assembly task with or without friction. The method requires the contact analysis for every possible contact occurring in fixture assembly. Due to the limitation of linear compliance mapping[5], although all possible contact states can be generated from the geometry of the part and environment[6], the method is hard to be applied to a CA tasks which deals with complex parts with concavities and whose contact states are complicated and change severely. McCarragher[7,8] presented a discrete event controller by imitating human decision making mechanism for assembly task. Also, the control system includes a qualitative matching process[9] to monitor the current contact state during assembly. However, similar to the work of Shimmels and Peshkin's, since it requires the geometry of all possible contact states to build a petri net as a discrete event system model, the more complex the geometry of the parts becomes, the more complicated and time-consuming to build the petri net becomes. Lee and Asada[10] presented an perturbation/correlation method using vibratory end effector. The correlation between the lateral perturbation generated from the piezo-actuator and the longitudinal reaction force sensed from the F/T sensor gives the information of the insertion direction minimizing the reaction force.

As a beginning work for CA, Kang et al[11] presented a cartesian stiffness controller which uses the rotational degree of freedom effectively during insertion by dynamically updating the location of the compliance frame to the current contact point. The proposed stiffness controller was successfully implemented to a real time controller and tested on T-insertion task involving a tight tolerance. However, the controller can only handle single-contact cases to simplify the localization procedure, since it is difficult to uniquely localize the instant center of rotation (or compliance center) in multi-contact states.

As described above, most existing compliant control and planning approaches have limitation to be easily applicable to CA due to their dependency on the part geometry or on the dimension of the motion since they requires contact kinematic analysis procedures for every possible contact or planning procedures to produce a compliant path. To solve these complexity problems inherent in CA, a geometry independent force-guided control having target approachable property is designed and verified in this paper. Using the target approachability condition as a significant goal for CA, a differential twist minimizing the deviation between the current and the target twist admitted by repelling or reciprocal conditions[12] can be determined. To make CA successful, as well as the target approachability condition, the bounded wrench condition[2] should also be satisfied at every contact state during insertion not to exert a larger wrench than the tolerable one. The bounded wrench condition can be satisfied by properly designing the accommodation parameters not to exceed the prescribed contact wrench. To solve the optimum twist at a contact state, an optimization problem statement is formulated. To build the optimization problem, the square of norm of the twist deviation from the target twist is defined as an objective function and repelling and reciprocal conditions as a inequality constraint function. Using the Kuhn-Tucker conditions from convex optimization theory[13], an optimum target approaching twist can be determined as a global minimum solution at each instantaneous contact state. Therefore, by the convex optimization technique incorporated with an admissible perturbation method, a new CA algorithm using only a sensed resultant wrench and an updated target twist can be developed without motion planning nor contact kinematic analysis procedure requiring the geometry of the part and the environment. As a result of experiments of a variety of CA tasks including T-insertion and double-peg-assembly, its feasibility and applicability are verified.

2. Approach to complex assembly

Definition of complex assembly

Fig.1 shows an example of complex assembly which handles complex shaped part and a complex environment. In this study, the complex assembly process is assumed to handle complex-shaped objects. The complex-shaped object (CSO) is defined as Definition 1.

Definition 1 *A complex-shaped object is defined as an object whose geometry is composed of polyhedra including concavities.*

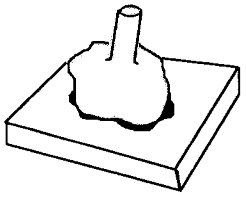

Fig.1 Complex Assembly

Using the Definition 1, complex assembly (CA) is defined as follows.

Definition 2 *A complex assembly is an insertion/outsertion task whose part or environment is a CSO.*

Complex assembly algorithm

As a first step to formulate the CA algorithm in this work, we assume that there is no nominal motion plan nor prescribed contact states determined from the geometry of the part and environment. This assumption corresponds to the situation where a blindfold human inserts a part into a hole without any geometric information of the part and the hole. In this situation contact states occur unexpectedly due to the geometric complexity or the misalignment between the part and the environment. When the situation is implemented to a robotic assembly system, the information usually available to the robot are 1) the end point position/orientation, 2) the sensed *resultant* contact wrench(forces/torques) and 3) target position/orientation with uncertainty.

When there is no planned nominal path for insertion, the error reducible property[2] is no longer meaningful. Rather, the target approachability is valid, which is to make the inserted part move closer to a known target point in the admissible motion space constrained by the constraints due to contacts.

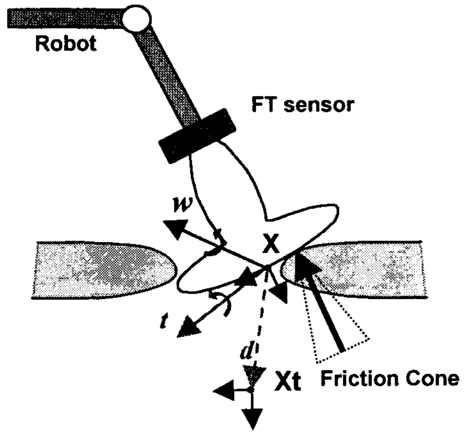

Fig.2 Target approaching twist in contact

At a contact state during a typical CA task shown in Fig.2, a differential twist (translational and angular velocities) which minimizes the deviation between a target twist and an admissible twist satisfying the reciprocal and the repelling conditions is defined as a *target approaching twist.* According to the screw theory[12], the reciprocal and the repelling constraints representing the admissible motion space in contact are described by a contact wrench w and a differential twist t as an inequality form as follows,

$$g(t) = -w^T t \le 0 \tag{1}$$

where $w = (\tau_x, \tau_y, \tau_z, f_x, f_y, f_z)^T$ and

$$t = (\omega_x, \omega_y, \omega_z, v_x, v_y, v_z)^T$$

Note that both wrench and twist are described in TCP (Tool Center Point) frame which is described as X in Fig.2. The resultant wrench measured in a FT sensor can be transformed to that in the TCP frame by using the static equilibrium relation of forces and moments.

The objective function $f(t)$ is defined as a half of the square of the normed deviation between the *target twist d* determined from the current(X) to the target position/orientation(Xt) and the differential twist t. The target twist d is also a vector in the TCP frame to maintaining the consistency with the current and the target pose vectors described in the TCP frame. Therefore, the objective function is described as

$$f(t) = \frac{1}{2}\|d - t\|^2 . \tag{2}$$

Both the quadratic objective function $f(t)$ and constraint function $g(t)$ are convex functions since the Hessian matrix of each function is positive-semi-definite[13]. Therefore, this is a convex optimization problem which guarantees a global minimum at an instantaneous quasi-static contact state. Finally, the CA problem can be formulated as a simple quadratic convex optimization to find a target approachable twist t^* as follows :

Find a twist t^* which minimizes $f(t) = \frac{1}{2}\|d - t\|^2$ ***subject to*** $g(t) = -w^T t \le 0$.

By applying the Kuhn-Tucker conditions for the optimization problem with inequality constraints, the Lagrange function L is described as

$$L = f(t) + u \cdot g(t) \tag{3}$$

Then there exists a Lagrangian multiplier u^* such that the Lagrangian is stationary with respect to each twist component t_i, *i.e.* $v_x, v_y, v_z, \omega_x, \omega_y$ or ω_z and u as follows.

$$\frac{\partial L}{\partial t_i} \equiv \frac{\partial f}{\partial t_i} + u^* \frac{\partial g}{\partial t_i} = 0, \tag{4}$$

$$g(t^*) \le 0, \tag{5}$$

$$u^* \cdot g(t^*) = 0 \text{ and} \tag{6}$$

$$u^* \ge 0. \tag{7}$$

Adaptive accommodation law

It is important to note that the condition (6) can be divided into two cases, which are $u^* = 0$ and $g(t^*) = 0$. When $u^* = 0$, the inequality constraint is inactive. Thus the optimum target approaching twist is simply same as *d.* In this case, the contact wrench does not constrain the part to be inserted to the target pose. This case corresponds to the situation illustrated in Fig.3. Fig.3(a) shows a physical situation during a planar CA task. The optimum twist lies in the repelling motion space. Fig.3(b) shows the optimum within the differential twist vector space in 3 dimensional planar CA problem. The optimum twist t^* is determined from (4)-(7) as follows.

$$t^* = d. \tag{8}$$

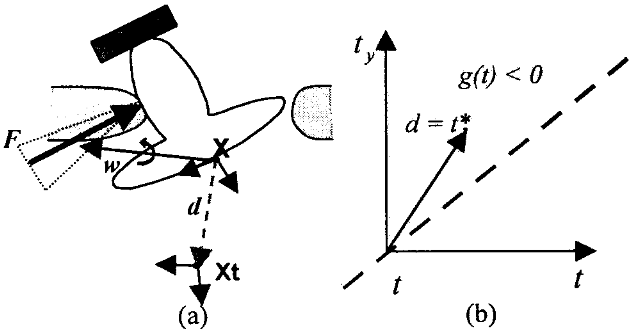

Fig.3 Optimum by repelling motion
(a) Corresponding contact state in planar CA task
(b) Optimum twist t^* in the repelling motion space

In the other case that $g(t^*) = 0$, in which the inequality constraint is active at the contact state such as shown in Fig.4(a), the optimum twist is in the reciprocal motion space as shown in Fig.4(b). In this case, the optimum twist t^* can be solved from (4) -(7) as follows.

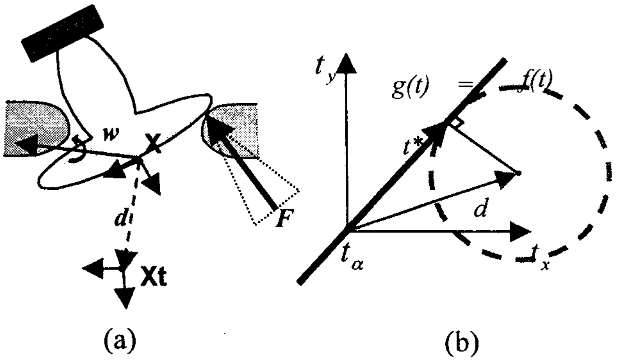

Fig.4 Optimum by reciprocal motion
(a) Corresponding contact state in CA task
(b) Optimum twist t^* in the reciprocal motion space

$$t^* = d + u^* \cdot w \tag{9}$$

where $u^* = -w^T d / \|w\|^2 \ge 0$.

It is important to note that the Lagrangian multiplier u^* maps the sensed resultant wrench into the target approaching twist using the dynamically updated target twist *d.* Thus the u^* has accommodation characteristics similar to the conventional damping control law. However, In contrast to the conventional accommodation matrices, which are usually linear mappings and determined in the least-square sense, the Lagrangian multiplier u^*, which is a scalar, determines an optimum target approaching twist dynamically using the sensed current resultant wrench and the current target twist *d.* Therefore, the optimum twist equation (9) has characteristics of an adaptive accommodation, which dynamically changes the mapping depending on the current contact wrench and the target twist. Thus the new control law (9) is called *adaptive accommodation law* in this study.

The condition of u^* that should be greater than or equal to 0, is a necessary condition for determining whether a target approaching twist exists. If $u^* < 0$, there is no optimum solution and thus a target approaching twist does not exist. Therefore, the u^* is an important index to check if the target approachability condition is satisfied at a contact state. Thus in this study, it is called *"accommodation index."*

Perturbation Twist in the Virtual Admissible Motion Space

In the adaptive accommodation control law, the optimum twist determined from the resultant wrench may not exist in the AMS(admissible motion space) but in the VAMS(virtual admissible motion space). Therefore, if it does not exist in the AMS, the optimum twist obtained from the Kuhn-Tucker conditions can not guarantee the target approachability. Rather, it can even make the part exert a large wrench to the environment making a dangerous contact situation. To prevent this problem, the bounded wrench condition should be added to the optimization. That is, when the magnitude of the sensed resultant wrench exceeds the wrench limit in case of an ill-conditioned contact state, a twist perturbation in a random direction within the VAMS is generated to find a direction making the sensed wrench bounded. As a result of repetitive perturbations in VAMS, the part can be moved to a different contact state(or a non-contact state) which may be possible to proceed insertion. Especially in the case of a jamming (or wedging) situation due to a multi-contact with large friction, the rotational perturbation in the same direction as that of the sensed torque vector is effective since it tends to change the contact state to a different one or to a non-contact state. This concept is similar to the conventional damping control law. The translational perturbation twist vector δ_Γ is modeled as

$$\delta_\Gamma = diag(C_\Gamma, C_\Gamma, C_\Gamma) \cdot f \qquad (10)$$

In the equation (10), the $diag(C_\Gamma, C_\Gamma, C_\Gamma)$ is a diagonal matrix (3x3) whose diagonal components are all C_V's. Based on the damping control method, C_Γ is a constant accommodation parameter offering a translational compliance in contact. The magnitude of the C_Γ can be determined from the inverse of the tolerable translational stiffness K_Γ at the robot end-point and the sampling time T_S for the controller as follows,

$$C_V = \frac{1}{K_V T_S}\,. \qquad (11)$$

The tolerable stiffness K_Γ can be estimated from the desired robot end point stiffness and the measurable range of the FT sensor.

On the other hand, the rotational perturbation twist vector δ_ω is also modeled by the sensed resultant torque vector $\tau = (\tau_x, \tau_y, \tau_z)$ as

$$\delta_\omega = diag(C_\omega, C_\omega, C_\omega) \cdot \tau \qquad (12)$$

Similar to the equation (3.13), the $diag(C_\omega, C_\omega, C_\omega)$ is a diagonal matrix (3x3) whose diagonal components are all C_ω's. The C_ω is also a constant accommodation parameter for rotational compliance. The magnitude of the C_ω can be designed in the same way as (11),

$$C_\omega = \frac{1}{K_\omega T_S}\,. \qquad (13)$$

Finally, six dimensional perturbation twist vector δ is determined in VAMS as follows,

$$\delta = (\delta_\Gamma^T, \delta_\omega^T)^T \qquad (14)$$

Consequently, the perturbation twist δ is generated from the sensed resultant wrench to proceed insertion maintaining the tolerable magnitude of the resultant contact wrench, when the value of the accommodation index is less than 0 or the sensed resultant wrench is larger than the prescribed wrench bound during CA.

3. Controller Design

Based on the damping (or accommodation) control approach which maps the contact wrench into the twist to produce an accommodation effect (or an admittance), the adaptive accommodation controller is designed. When contact is detected during insertion, TAF module is activated and computes a target approaching twist in either optimum twist generator or perturbation twist generator. The block diagram of the TAF module is as shown in Fig.5

The proposed TAF control system has three control modes, i.e., the non-contact mode, the optimum twist mode and perturbation twist mode. In the non-contact twist mode, the controller performs like a free-space position controller to approach the commanded target pose. When contact is detected, the optimum twist mode or perturbation twist mode is activated. If the accommodation index is zero or positive and the bounded resultant wrench condition are satisfied, the optimum twist mode is activated in the TAF module. On the other hand, either the accommodation index or the bounded resultant wrench condition is not satisfied, the perturbation twist mode is activated to determine the perturbation twist vector.

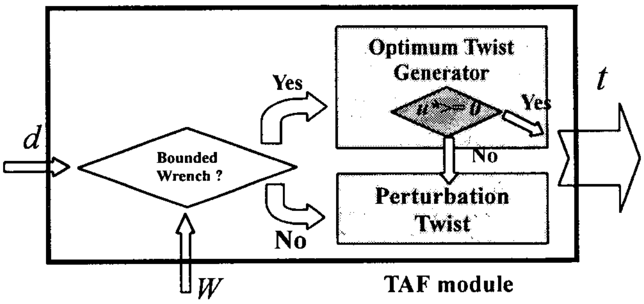

Fig.5 Target approachable force-guided (TAF) module

Unlike the accommodation matrix[2] based on linear mapping, the mapping in our damping controller has nonlinear characteristics as represented in (9). However, the TAF control system can be simply implemented to an existing high stiffness position controller by adding an external wrench feedback loop. The algorithm for solving an optimum twist which is analytically derived in (8) and (9), and the perturbation generation module can be simply implemented as an external feedback loop. The accommodation parameters C_Γ and C_ω used to generate perturbations should be designed carefully by taking into account the tolerable stiffness of the part and environment, and the bandwidth of the robot. The overall block diagram of the proposed TAF control system for CA is shown in Fig.6.

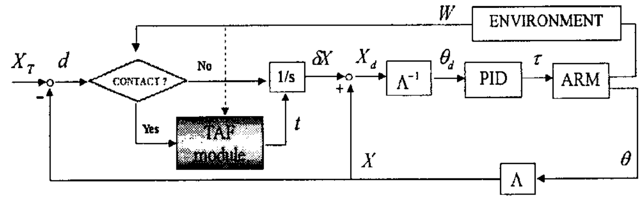

Fig.6 Block diagram of target approachable control system for CA

4. Experiment

Robot control system for CA

A variety of CA tasks are experimented by using the VME based real-time robot control system(Fig.7) called HUROCOS (Humanoid Robot Control System). The HUROCOS, which runs in a real-time OS (VxWorks) is a general-purpose robot control system aiming at building a humanoid robot called CENTAUR in Korea Institute of Science and Technology (KIST).

T-insertion

As a typical example of planar assembly, which uses rotational motion effectively to insert a T-shaped part into the target pose, T-insertion/outsertion task is implemented and experimented by the robot control system described above. The specifications required to execute the insertion task is listed Table.1.

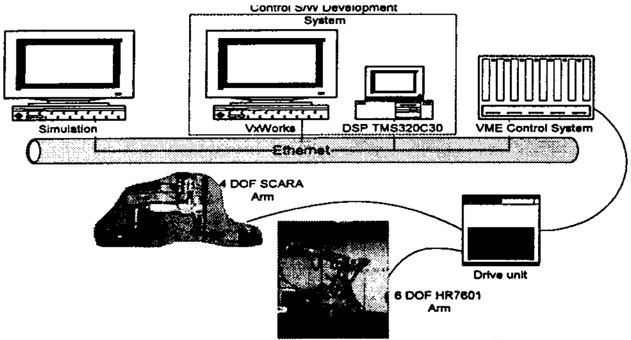

Fig.7 VME-based real time robot control system

The trajectory(twist) update rate is assigned by considering the computation time for the twist generation and the inverse kinematics of the robot. In the case of the planar robot, the inverse kinematics is so simple that the trajectory update rate could be increased up to 2 msec. The servo rate in the PID controller is also set by considering the steady state response of the robot dynamics.

Table.1 Task specification data for T-insertion

Task specification		Value
Initial pose in base frame (*x*, *y*, *roll*)		(620 mm, -0.5 mm, 0 rad)
Target pose in TCP frame (*x*, *y*, *roll*)		(50 mm, 0.5 mm, 0 rad)
Trajectory update rate (Ts)		2 msec
Servo rate (ts)		1 msec
Tolerable Stiffness at TCP (Kd)	Translational	150 N/mm
	Rotational	200,000 N*mm/rad
Desired Twist (t_d)	Translational	50 mm/sec
	Rotational	1.2 rad/sec

The robot used in the T insertion is Goldstar GHR350-II SCARA robot which is similar to an Adept robot. To make the robot move in X, Y and α direction in TCP frame during T insertion, it has been reconstructed to a 3-DOF by freezing the third prismatic joint. The result of the experiment is illustrated in sequental photographs in Fig.8.

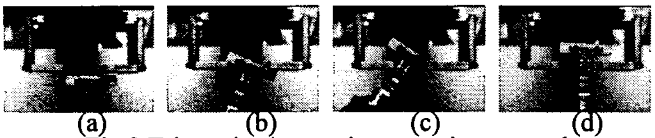
(a) (b) (c) (d)

Fig.8 T-insertion/outsertion experiment result

The result of the T-insertion experiment is shown sequentially from Fig.8(a) to Fig.8(d). Fig.8(a) shows an initial position of T. The initial position is set to ensure an approaching direction to the target position where the insertion is completed. There is no limitation to the initial position unless it does make a sticking state which the target approachability is not satisfied in view of the accommodation index. From the contact mode plot shown in Fig.10, almost contact modes are optimum twist mode "1" thanks to the adaptive accommodation property. In the perturbation twist mode "2", which is usually corresponds to a multi-contact case, the perturbed motion helps to disengage the jamming condition by changing the current contact state to another state. Fig.9 show the resultant forces and torque measured by F/T sensor during insertion. From the tolerable stiffness specified in Table.1, the measured forces and torques satisfy the bounded wrench property which offers a compliance in contact.

Double-peg assembly

As a final example of the complex assembly to verity the adaptive accommodation controller, a double-peg assembly task has been experimented.

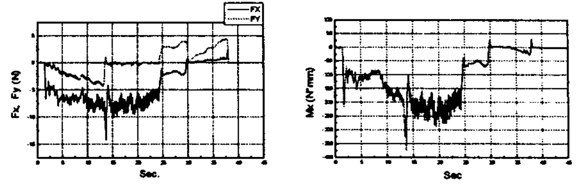

Fig.9 Sensed resultant wrench

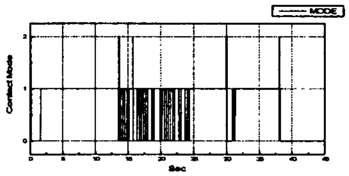

Fig.10 Contact modes during insertion

This experiment can be thought of as a simple example for the wheel assembly operation in the car manufacturing line which inserts a wheel with 4 holes into a hub with 4 pegs. However, the radial tolerance of this experiment is set to so small to ensure that the proposed controller can be applied to various precision assembly tasks involving complex geometry. This double-peg assembly experiment focuses on the robustness against the initial misalignment for the successful insertion. This experiment is to show that the assembly is possible as long as the resultant wrench can represent the admissible motion approaching to the target position even though the parts geometry are complex. In the double-peg assembly example, if the initial pose is set so that the two pegs are partially inserted into the holes, and thus the resultant wrench can easily find an admissible motion to the target position, the assembly operation is successful regardless the geometric complexity of the parts.

The task specification for the double-peg assembly with initial misalignment is listed in Table.2. As a result of generating the optimum twist and the perturbation twist to approach the target position, the assembly was finally succeeded in spite of the misalignments. Fig.11 shows the sequential result of the assembly operation and Fig.12 shows the bounded wrench result subject to the prescribed tolerable stiffness.

Table.2 Task specification data for double-peg assembly

Task specification		Value
Initial pose in base frame (*x*, *y*, *z*, *yaw*, *pitch*, *roll*)		(-44 mm, 790 mm, 1453 mm, 85 deg, 0 deg, 180 deg)
Target pose in base frame (*x*, *y*, *z*, *yaw*, *pitch*, *roll*)		(-37 mm, 890 mm, 1454 mm, 85 deg, -15 deg, 180 deg)
Trajectory update rate (Ts)		6 msec
Servo rate (ts)		1 msec
Tolerable Stiffness at TCP (Kd)	Translational	200 N/mm
	Rotational	10,000 N*mm/rad
Desired Twist (t_d)	Translational	25 mm/sec
	Rotational	15 rad/sec

Fig.13 shows the contact modes experienced during the insertion. Due to the small tolerance (0.1 mm), the free non-contact modes are scarcely shown during the insertion stage after the mating stage overcoming the misalignment is completed. The double hole disk tries to be inserted against the misalignment by interchanging its contact mode among free, optimum and perturbation mode. This motion can be seen from 0 until 5 sec. Once the disk is inserted after 5 sec, the inserting motion interchanges between the optimum mode and the perturbation mode. It means that during the time the disk can be inserted by a simple compliant motion just like a single peg-in-hole task.

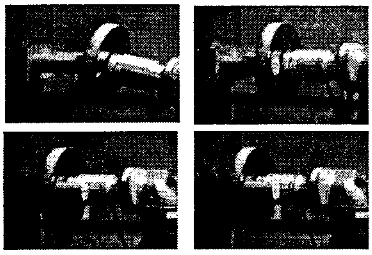

Fig.11 Experiment result : Double-peg assembly

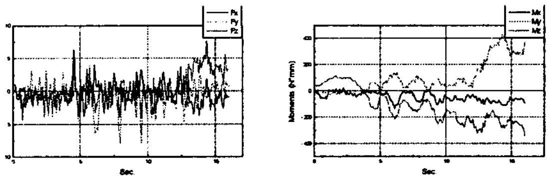

Fig.12 Sensed resultant wrench

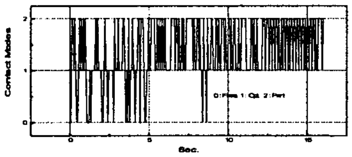

Fig.13 Contact modes during insertion

5. Conclusions

In this work, the complex assembly(CA) is newly defined as a result of the investigation of various assembly tasks encountered in industry and the other automation fields. By examining the operation of a blindfold human for CA, a simple assembly rule called adaptive accommodation law, which imitates human operation is exploited and mathematically formulated using twist and wrench. To attempt automation for the CA tasks using a robot with a force/torque sensor, a new geometry-independent assembly algorithm called adaptive accommodation control is developed by modeling the complex assembly operation as a convex optimization problem, which includes an updated target twist and current contact wrench. The goal of the optimization is to achieve both the target approachability and the bounded wrench property at each contact state during insertion. The contact states are represented as the repelling and the reciprocal constraints derived by screw theory and used as an inequality constraint for the optimization framework. Additionally, the admissible perturbation method which generates a twist in the virtual admissible motion space constructed from the sensed resultant wrench is proposed to make the contact wrench bounded and to disengage the unexpected jamming (or wedging) condition. Since both the optimum twist generator and the perturbation twist generator does not require any geometry information of the parts or the environment, the control algorithm can be easily implemented to various complex assembly tasks.

As well as the peg-in-hole task, a variety of complex assembly tasks such as T-insertion/outsertion and double peg assembly have been successfully implemented and executed in the VME-based real-time robot control system, which is a general purpose robot controller developed for the humanoid robot project at Korea institute of science and technology. This work is expected to be easily extended to a cooperative assembly task using two arms, which is more human-like and efficient in terms of the measure of time and energy, since the adaptive accommodation control algorithm is independent of the geometry of the assembled parts, the number of the contact states and the structure of the robot kinematics.

6. References

[1] Rampersad, H. K., *Integrated and Simultaneous Design for Robotic Assembly*, John Wiley, Chichester, November 1994

[2] Peshkin, M.A., "Programmed Compliance for Error Corrective Assembly", *IEEE Trans. Robotics and Automation,* Vol.6, No.4, 1990, pp.473-482

[3] Shimmels, J.M. and Peshkin, M.A., "Admittance Matrix Design for Force-Guided Assembly", *IEEE Trans. Robotics and Automation*, Vol.8, No.2, 1992, pp.213-227

[4] Shimmels, J.M. and Peshkin, M.A., "Force Assembly with Friction", *IEEE Trans. Robotics and Automation*, Vol.10, No.4, 1994, pp.465-479

[5] Asada, H., "Representation and Learning of Nonlinear Compliance Using Neural Nets", *IEEE Trans. Robotics and Automation*, Vol.9, No.6, 1993, pp.863-867

[6] Xiao, J. and Zhang, L., "Contact Constraint Analysis and Determination of Geometrically Valid Contact Formations from Possible Contact Promitives," *IEEE Trans. of Robotics and Automation*, Vol.13, No.3, 1997, pp.456-466

[7] McCarragher, B. J. and Asada, H., "The Discrete Event Control of Robotic Assembly Tasks," *ASME Journal of Dynamic systems, Meas. And Control*, Vol.117, 1995, pp.384-393

[8] McCarragher, B. J. and Asada, H., "The Discrete Event Modeling and Trajectory Planning of Robotic Assembly Tasks," *ASME Journal of Dynamic systems, Measurement and Control*, Vol.117, 1995, pp.394-400

[9] McCarragher, B. J. and Asada, H., "Qualitative Template Matching Using Dynamic Process Models for State Transition Recognition of Robotic Assembly", *ASME Journal of Dynamic Systems, Measurement and Control,* Vol.115, 1993, pp.261-269

[10] Lee S. and Asada, H., "Assembly Automation using Vibratory End Effector : Modeling and Stability Analysis," *Proc. IEEE Int. Conf. Robotics and Automation,* 1997, pp.1980-1985

[11] Kang, S., Lee, K., Hwang, Y. K., Kim, M. and Lee, C., "A Compliant Motion Control for Insertion of Complex Shaped Objects using Contact", *Proc. IEEE Int. Conf. Robotics and Automation*, 1997, pp.841-846

[12] Ohwovoriole, M.S. and Roth, B., "An Extension of Screw Theory," *Journal of mechanical design*, vol.103, October, 1981, pp.725-735

[13] Arora, *Introduction to Optimum Design*, McGraw-Hill, Singapore, 1989