

## **Determination of J-Resistance Curves of Nuclear Structural Materials by Iteration Method**

Thak Sang Byun, Bong Sang Lee, Ji Hyun Yoon, Il Hiun Kuk,  
and Jun Hwa Hong

Korea Atomic Energy Research Institute  
Nuclear Materials Technology Development  
P.O. Box 105, Yusong, Taejon 305-600, Korea

### **Abstract**

*An iteration method has been developed for determining crack growth and fracture resistance curve (J-R curve) from the load versus load-line displacement record only. In this method, the hardening curve, the load versus displacement curve at a given crack length, is assumed to be a power-law function, where the exponent varies with the crack length. The exponent is determined by an iterative calculation method with the assumption that the exponent varies linearly with the load-line displacement. The proposed method was applied to the static J-R tests using compact tension (CT) specimens, a three-point bend (TPB) specimen, and a cracked round bar (CRB) specimen as well as it was applied to the quasi-dynamic J-R tests using CT specimens. The J-R curves determined by the proposed method were compared with those obtained by the conventional testing methodologies. The results showed that the J-R curves could be determined directly by the proposed iteration method with sufficient accuracy in the specimens from SA508, SA533, and SA516 pressure vessel steels and SA312 Type 347 stainless steel.*

### **1. Introduction**

Since the J-integral has been regarded as the most important parameter for characterizing the elastic-plastic fracture resistance of structural materials, significant efforts have been devoted to develop more simplified methodologies of determining J-integral versus crack extension curve (J-R curve) [1-7]. In the J-R fracture testing the crack length measurement is the most cumbersome procedure and needs a high degree of accuracy to obtain a correct J-R curve. The most successful experimental methods using a single specimen are the elastic unloading compliance method and potential drop method [8,9]. In static fracture testing they are now used routinely in many laboratories. However, the unloading compliance method can not be applied to the dynamic loading conditions because the unloading-reloading cycles to obtain the elastic compliance need a quasi-static loading condition. Also, the potential drop method requires much sophisticated equipment such as a high current power supplier and a high speed data acquisition system, and there is a difficulty in determining the crack growth initiation point in dynamic fracture testing.

To eliminate these limitations in the experimental measurement of crack length, several direct methods have been attempted to obtain the J-R curve from the load versus displacement record only without additional equipment. The most important approaches are the key curve method [1], normalization method [2-5], and load ratio method [6,7]. In these direct methods, the final result depends on how to determine the required curves; such as the key curve, normalization function, reference hardening curve. To obtain more accurate curves, many modified methods have been suggested and successfully applied to various materials [1,5,7]. However, most methods developed are only for compact tension (CT) or bending specimens.

This paper presents a new direct method (we call it 'iteration method') which can be applied to any types of specimen and to any loading rate. For a given crack length, the hardening curve is expressed by a power-law function of displacement without any explicit variables for specimen type or loading rate. The exponent in the hardening curve is assumed to be a function of displacement and is determined by iterative calculations. The other coefficients are directly determined from the experimental load versus displacement data. As a result, the iteration method can be applied to any specimen type as long as the J-integral is formulated for the specimen. This paper includes the application results for CT specimen cases including static tests and quasi-dynamic tests and the results for the three-point bend (TPB) specimen and cracked round bar (CRB) specimen cases.

## 2. Iteration Method

Based on the definition of energy release rate, the J-integral is expressed by the following equation:

$$J = - \frac{1}{B} \left. \frac{\partial U}{\partial a} \right|_{v=const.} \quad (1)$$

where B is the specimen thickness, a is the crack length, v is the load-line displacement, and U is the elastic-plastic energy measured as the area under the hardening curve. The experimental load versus displacement curve and hardening curves are represented in Fig. 1. For a fixed displacement  $v_i$ , the difference between the energies for the crack lengths  $a_i$  and  $a_{i+1}$  is given by

$$-\Delta U_i = U_i - U_{i+1} = U_i(1 - R_i) \quad (2)$$

where the energies,  $U_i$  and  $U_{i+1}$ , are the area under the hardening curves for  $a_i$  and  $a_{i+1}$ , respectively:

$$U_i = \int_0^{v_i} P(a_i, v) dv \quad (3a),$$

$$U_{i+1} = \int_0^{v_i} P(a_{i+1}, v) dv \quad (3b),$$

and  $R_i$  is the ratio between the energies;  $R_i = U_{i+1}/U_i$ . Then the amount of crack extension,  $\Delta a_i (=a_{i+1} - a_i)$ , is expressed by

$$\Delta a_i = \frac{U_i(1 - R_i)}{BJ_i} \quad (4)$$

Fig.1 shows the relationship between the hardening curves for fixed crack lengths and the experimental load versus displacement curve for growing crack. Assuming the deformation theory of plasticity, the hardening curve for a crack length  $a_i$  should intercept with the experimental load versus displacement curve at the point  $(v_i, P_i)$ . For a given crack length, the power-law function form has been used for describing the relation between load and

displacement, where the exponent in the power-law function is usually assumed to be constant [3]. However, in this paper, it is assumed that the exponent varies with crack length. It means that the shape of hardening curve may change with crack growth. Therefore, the hardening curves for the given crack lengths  $a_i$  and  $a_{i+1}$ , respectively, can be expressed by

$$P(v, a_i) = P_i \left( \frac{v}{v_i} \right)^{n_i} \quad (5a)$$

$$P(v, a_{i+1}) = P_{i+1} \left( \frac{v}{v_{i+1}} \right)^{n_{i+1}} \quad (5b)$$

Inserting these equations into eqs. (3a) and (3b), respectively, the energy ratio is expressed as follows:

$$R_i = \left( \frac{n_i + 1}{n_{i+1} + 1} \right) \left( \frac{P_{i+1} v_{i+1}}{P_i v_i} \right) \left( \frac{v_i}{v_{i+1}} \right)^{n_{i+1} + 1} \quad (6)$$

To describe the shape change of hardening curve during crack extension, a linear variation of the hardening exponent with displacement is assumed:

$$n_i(a_i) = n_0(a_0) + \alpha v_i(a_i) \quad (7)$$

In this equation the values of  $n_0$  and  $\alpha$  should be determined to obtain the value of  $n_i$ . The  $n_0$ -value can be obtained from the power-law regression for the experimental load-displacement data in the initial displacement region assuring no crack extension.

## 2.1. Crack growth in the CT and TPB specimens

For both the CT specimen and the TPB specimen, the J-integral has been evaluated by the following form [9]:

$$J_i = \frac{\eta_i U_i}{B b_i} \quad (8)$$

where  $b_i$  is the uncracked ligament ( $=W-a_i$ ;  $W$  is the specimen width) and  $\eta_i$  is a dimensionless parameter depending on the crack length and specimen configurations;  $\eta_i = 2$  for the TPB specimens and  $2 + 0.522 \times b_i/W$  for the CT specimens. Using eqs. (4) and (8), the crack extension from  $i$  to  $i+1$  is obtained as

$$\Delta a_i = \frac{b_i}{\eta_i} (1 - R_i) \quad (9)$$

## 2.2. Crack growth in the CRB specimens

To estimate J-integral from the load versus displacement curve, we used the expression developed by Rice et al. [10]:

$$J_i = \frac{1}{2\pi r_i^2} \left[ 3 \int_0^{v_i} P dv - P_i v_i \right] \quad (10)$$

where  $r$  is the radius of uncracked ligament. Inserting eqs. (5a) into this equation,  $J_i$  becomes:

$$J_i = \frac{(2 - n_i)}{2\pi r_i^2} U_i \quad (11)$$

Since the increase in the crack surface is defined by  $B\Delta a_i = 2\pi r_i \Delta r_i$ , eqs. (4) and (11) give

$$\Delta r_i = \frac{r_i}{2 - n_i} [1 - R_i] \quad (12).$$

### 3. Computational Procedure and Applications

The input data are the experimental load versus displacement data ( $P_i$  vs.  $v_i$  data), initial and final crack lengths (or initial and final crack radii), and specimen dimensions. The first step in the calculation is to determine the value of  $n_0$  from the initial part of the experimental load versus displacement curve. Since the  $n_0$ -value is for the initial crack length, the regression range should be selected carefully to assure no crack growth. The second step is an iterative calculation to determine the exponent  $n_i$  for each load-displacement point ( $v_i$ ,  $P_i$ ). A trial value for  $\alpha$  in eq. (7) is assumed and the total crack extension is calculated by using the equations in the previous section. The criterion in the iterative calculation is that the calculated crack extension must be the same as the measured value. If the total crack extension calculated can not satisfy this criterion, a value of  $\alpha$  is reassumed and crack extension lengths are calculated again. This iterative calculation will be continued until the result satisfies the criterion. The third step is to calculate the J-R curve with the hardening curves determined in the previous step.

The proposed iteration method was applied to 5 cases as listed in Table 1 [11-13]. For all cases, the J-R curves determined by the iteration method were compared with those measured by the conventional methods. In the quasi-static fracture testing with the CT and TPB specimens the crack length was measured by the unloading compliance method. Otherwise, in the quasi-dynamic loading tests the crack length was measured by the direct current potential drop (DCPD) method [9]. The analyses for determining J-R curves were carried out in accordance with the standard method. For the CRB specimen, the load-displacement data were read from a figure in the reference [11].

## 4. Application Results

### 4.1. CT and TPB specimen cases under static loading conditions

Figs. 2 and 3 present the static J-R curves from the CT specimens. Regardless of the test materials, agreements are found between the iteration method and the standard unloading compliance method [9]. Some J-R curves determined by the unloading compliance method reveal relatively larger data scatters. The iteration method, however, gives smoother J-R curves.

The J-R curves for the TPB specimen are compared in Fig. 4. This result also shows an agreement between the two methods. Although the amount of crack extension is relatively small in the small TPB specimen; less than 2 mm, the J-R curve determined by the iteration method traces accurately the shape of J-R curve from the unloading compliance method.

### 4.2. CRB specimen case under static loading condition

For the CRB specimens, no standard method for determining the J-R curve has been established. Therefore, the J-R curve has been obtained by the multi-specimen method [11]. In Fig. 5 the J-R curve determined by the iteration method is compared with the experimental data points obtained from four specimens. The load versus displacement curve used for the present calculation was obtained from the case that had revealed the largest crack extension. In Fig. 5,  $J_{EXP}$  is the experimental value from the multi-specimen method [11] and  $J_1$  and  $J_2$  are the data from the iteration method. In the calculations of  $J_{EXP}$  and  $J_1$  the experimental load versus displacement curve was regarded as the hardening curve for all crack lengths. However, when calculating the  $J_2$ -values, different hardening curves were used for different crack lengths. Comparing the  $J_1$  curve with the  $J_{EXP}$ -values, it is concluded that the iteration method can be applied to the CRB specimens with sufficient accuracy.

On the other hand, the hardening curves determined by iterative calculations were used for the calculation of  $J_2$ . Fig. 5 shows that, as the crack extends, the  $J_2$ -values become smaller than the  $J_{EXP}$ - and  $J_1$ -values. This result is because the crack growth effect on the hardening curve has been ignored in the calculations of  $J_{EXP}$  and  $J_1$ . When considering the original definition of J-integral, as given by eq. (1), the  $J_2$  curve is regarded as a more reasonable crack resistance curve.

#### 4.3. CT specimen cases under quasi-dynamic loading conditions

For the static or quasi-dynamic cases, the DCPD method may be applicable to the measurement of crack length with sophisticated equipment [9]. In Fig. 6, the J-R curves determined by the iteration method are compared with the data points estimated by the DCPD method. For the DCPD method, the data points at small crack extensions were excluded from the J-R curves. The crack length at early J-R curve is strongly dependent on the critical value of the potential drop determined as the crack growth initiation point. At larger crack extension values, however, the two methods show an agreement in the J-integral values.

#### 4.4. The exponent $n$

Table 2 contains the calculated exponents of hardening curves as the functions of displacement. In the present work the  $n_0$ -values were evaluated by power-law fits of load-displacement data in the range of 0 to  $0.5v_p(P_{max})$ , where  $v_p(P_{max})$  is the plastic load-line displacement at maximum load. The exponent has been known to be similar to the work hardening exponent of the Hollomon type flow curve:  $q$  [14]. Table 2 also shows that the  $n_0$ -values are similar to the values of the work hardening exponent obtained from tensile tests; the values of these parameters are in the range of 0.15 to 0.18.

Table 2 also shows that the  $n$ -value increases with displacement at the slope of  $\alpha$ . This  $\alpha$ -value represents the shape change of the hardening curve during crack growth. The load can be separated into the functions of  $v$  and  $a$  when the exponent of the power-law hardening curve is constant;  $\alpha=0$  [3]. When the load is separable, the shape of the hardening curve is independent on the crack growth, and consequently the hardening curve is independent on the crack growth history. However, the exponent dependent on crack growth causes the inseparable behavior of load. As seen in Table 2, the  $\alpha$ -values calculated are always larger than zero. This result indicates that the hardening curve determined by the present method is dependent on the crack growth history.

## 5. Summary and Conclusion

An iteration method has been developed for determining the J-R curves of nuclear structural steels from the load versus load-line displacement record only. The iteration method and application results are summarized as follows:

(1) In the iteration method, the hardening curve is described by a power-law function, in which the exponent is given as a linear function of load-line displacement. For each crack length, the hardening curve is determined by iterative calculation method. The iterative calculation is continued until the total amount of crack extension becomes equal to the measured crack extension. Finally, the J-integral values are calculated from the hardening curves and crack lengths are determined in the iterative calculation step.

(2) The method developed was successfully applied to the static J-R tests using CT, TPB, and CRB specimens and to the quasi-dynamic J-R tests using CT specimens. The iteration method can be regarded as a general method that can be applied to any specimen types and to any loading rates.

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## References

- [1] J.A. Joyce, H.A. Ernst and P.C. Paris, *Fracture Mechanics: Twelfth Volume, ASTM STP 700*, 222-236 (1980).
- [2] J.D. Landes and R. Herrera, *Int. J. of Fracture* **36**, R9-R14 (1988).
- [3] M.H. Sharobeam and J.D. Landes, *Int. J. of Fracture* **47**, 81-104 (1991).
- [4] J.D. Landes, Z. Zhou, K. Lee and R. Herrera, *J. of Test. and Eval.* **19**, 305-311 (1991).
- [5] C.J.F.O. Forters and F.L. Bastian, *J. of Test. and Eval.* **25**, 302-307 (1997).
- [6] J.M. Hu, P. Albrecht, and J. A. Joyce, *Fracture Mechanics: Twenty-Second Symposium (Vol.1), ASTM STP 1311*, 880-903 (1992).
- [7] B.S. Lee, J.H. Yoon, and J.H. Hong, Submitted to *Engi. Frac. Mech.*, December (1997).
- [8] A. Saxena and S.J. Hudak, Jr., *Int. J. of Fracture* **14**, 453-468 (1978).
- [9] ASTM E 1737-96, Standard Test Method for J-Integral Characterization of Fracture Toughness, 67-90 (1996).
- [10] J.R. Rice, P.C. Paris, and J.C. Merkle, *ASTM STP 536*, ASTM, Philadelphia, 231-245 (1973).
- [11] J.H. Giovanola, R.W. Kloop, J.E. Croker, D.J. Alexander, W.W. Corwin, and R.K. Nanstad, *ASTM STP 1329*, W.R. Corwin, S.T. Rosinski, and E. van Walle, Eds., in press, (1997).
- [12] J.H. Hong et al., Fracture resistance (J-R curve) characteristics of primary piping materials for Yong-Gwang 3/4 nuclear power plants, Report no: KAERI/CR-SH91089/91, Korea Atomic Energy Research Institute, (1991).
- [13] B.S. Lee et al., Evaluation of DSA effects on SA516-Gr.70 steel for reactor coolant piping elbow material (dynamic and quasi-static J-R characteristics), Report no: KAERI/CR-043/97, Korea Atomic Energy Research Institute, (1997).
- [14] Y. Liu, X. Li and X. Tao, *Int. J. of Pres. Ves. & Piping* **31**, 157-164 (1988).

Table 1. Summary of case descriptions [11-13]

Case No.	Material	Specimen Type	Test Temp. [°C]	Loading Condition (Cross-head speed)	W(R)	B	$a_0(r_0)$	$a_f(r_f)$
1	SA508 Gr.3	1T-CT	RT	Quasi-static (1 mm/min)	50.8	25.4	27.2	30.9
2	SA312 Type 347 SS	1T-CT	316	Quasi-static (1 mm/min)	50.8	25.4	31.8	36.6
3	SA533B-1	TPB	RT	Quasi-static (1 mm/min)	10.0	10.0	5.4	7.1
4	HSSI Weld(72W)	CRB	0	Quasi-static (1 mm/min)	16.0		3.2	2.5
5	SA516 Gr.70	1T-CT	316	Quasi-dynamic (2000 mm/min)	50.8	25.4	29.1	38.6

Note: R = specimen radius of CRB specimen,  $r_0$  and  $r_f$  = initial and final uncracked ligament radii of CRB specimen, respectively.

Table 2. The exponent of hardening curves, n, and work-hardening exponent, q

Case No.	Specimen Type	$n_i = n_0 + \alpha v_i$	q
1	1T-CT	$n_i = 0.15 + 0.002v_i$	0.166
2	1T-CT	$n_i = 0.18 + 0.02v_i$	0.169
3	TPB	$n_i = 0.18 + 0.002v_i$	0.160
4	CRB	$n_i = 0.18 + 0.61v_i$	0.151
5	1T-CT	$n_i = 0.16 + 0.07v_i$	0.179

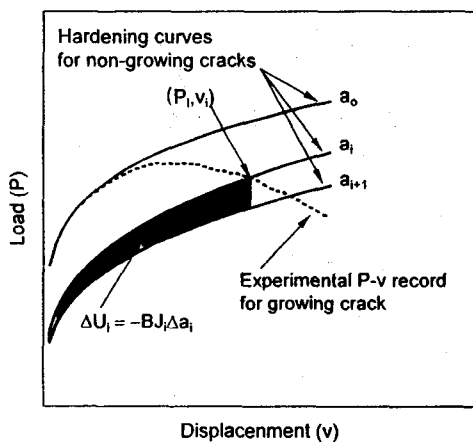


Fig. 1. Schematic of experimental load versus displacement curve and hardening curves

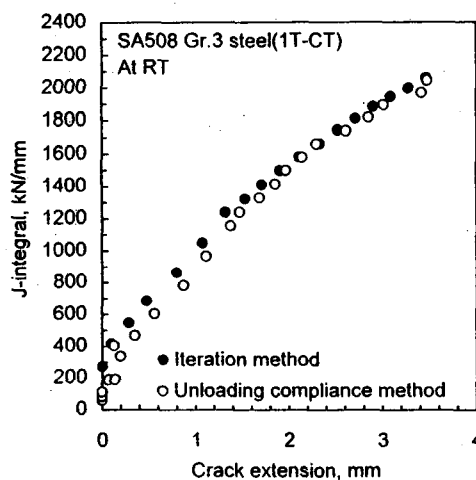


Fig. 2. Comparison of J-R curves for the static test of SA508 Gr.3 steel at room temperature (1T-CT specimen)

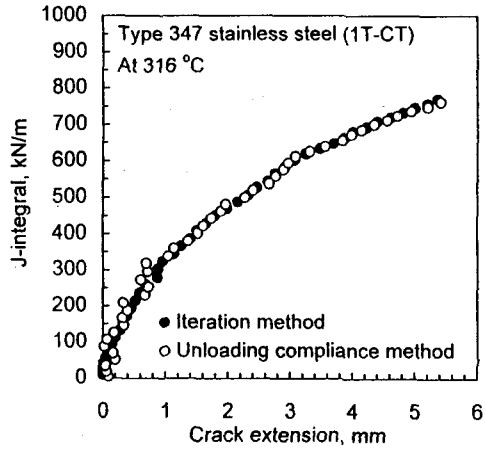


Fig. 3. Comparison of J-R curves for the static test of SA312 type 347 stainless steel at 316 °C (1T-CT specimen).

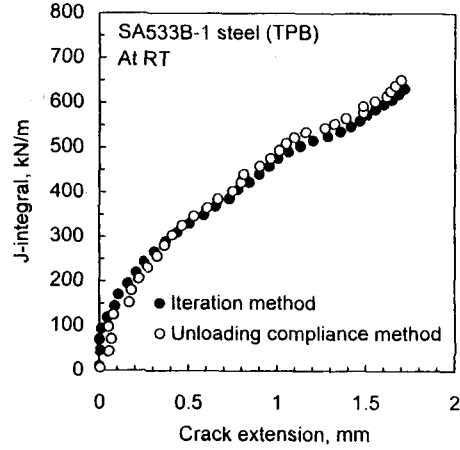


Fig. 4. Comparison of J-R curves for the static test of SA533B-1 steel at room temperature (small TPB specimen).

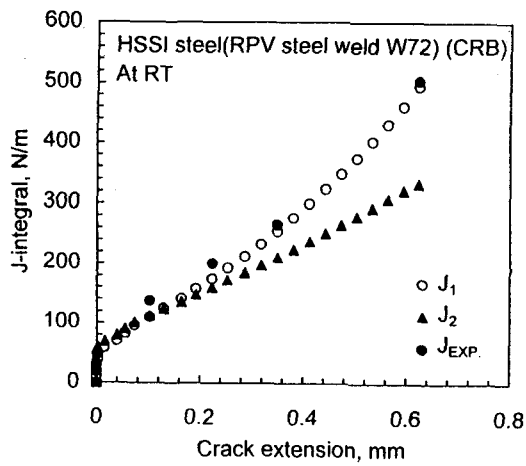


Fig. 5. Comparison of J-R curves for the static test of HSSI weld (W72) at room temperature (CRB specimen). [11]

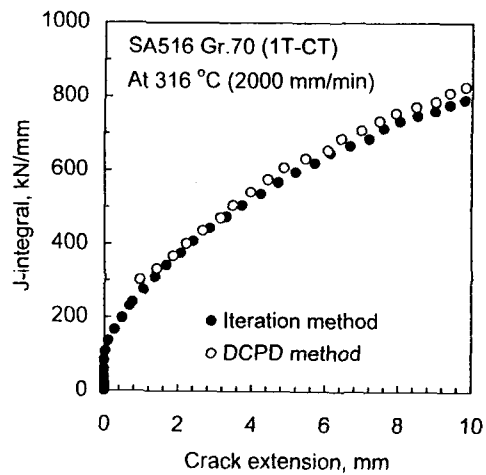


Fig. 6. Comparison of J-R curves for the quasi-dynamic test of SA516 Gr.70 steel at 316 °C (1T-CT specimen, load-line displacement rate: 2000 mm/min).