결합형 유한요소-경계요소기법을 사용한 PZT4 구형 셀 형태의 하이드로폰 시뮬레이션

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PZT4 spherical shell-typed hydrophone simulation using a coupled FE-BE method

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Abstract

This paper describes the application of a coupled finite element-boundary element method to obtain the steady-state response of a hydrophone. The particular structure considered is a flooded piezoelectric spherical shell. The hydrophone is three-dimensionally simulated to transduce an incident plane acoustic pressure onto the outer surface of the sonar spherical shell to electrical potentials on inner and outer surfaces of the shell. The acoustic field formed from the scattered sound pressure is also simulated. And the displacement of the shell caused by the externally incident acosutic pressure is shown in temporal motion. The coupled FE-BE method is described in detail.

1. Introduction

Most of hydrophones have the structure of a thin shell sphere. It is because we usually want to have omnidirectivity in its pressure-sensitive characterisitcs. Because of its simple type of structure, the behaviour of the spherical hydrophone is well known in analysis. Even so, any numerical method for simulating the hydrophone is often required because the numerical method could be further extended to other complicated types of structure for better performance. Since a hydrophone is used in water, modelling of the hydrophone must satisfy both internal materialistic transduction and externally radiating condition. In these aspects the finite element method (FEM) and the boundary element method (BEM) is perhaps the most suitable numerical techniques for the solution. Both methods were developed for the numerical solution of partial differntial equations (PDE) with boundary conditions. Since both methods solve the PDEs by numerically elemental integration, they are compatible each other and therefore they can be coupled together [1,2]

Different types of in-air piezoelectric transducers have been simulated by the FEM [3-5]. And also modified FEMs such as the mixed FE perturbation method [6] or the mixed

FE plane-wave method [7] have been developed in order to simulate an array of transducers or composite sonar transducers. Further developments have been made so as to include the effects of infinite fluid loading on transducer surface. For example, Bossut et. al. [8] and Hamonic et. al. [9] used fluid finite elements as an extension to structural finite elements with the condition that outer boundary of the fluid elements represents continued radiation. Others used 'infinite' fluid elements for infinite acoustic radiation [10,11]. The BEM is probably accepted as the most suitable method for the radiation problem because the BEM directly solves the Helmholtz PDE with the radiation condition [1,2].

The main aim of this paper is to simulate the structural behaviour of the flooded piezoelectric spherical shell when the sonar shell is driven by external incident acoustic presure. The directivity pattern of the scattered acoustic pressure is shown in temporal motion and compared with that of a rigid steel sphere.

2. Coupled FE-BE Method

The coupled FE-BEM algorithm is well descrived by S.S.Jarng [12]. The sonar transducer model could be modelled in matrix equations as follows;

$$\begin{aligned} &\{F\} + [L](A^{\oplus})^{-1}\Psi^{\oplus}_{inc} \\ &= [K_{uu}]\{a\} + [\rho_f\,\omega^2[L](A^{\oplus})^{-1}B^{\oplus}]\{a\} \\ &+ [K_{u\phi}]\{\phi\} - \omega^2[M]\{a\} + j\omega[R]\{a\} \\ &-\{Q\} = [K_{\phi u}]\{a\} + [K_{\phi\phi}]\{\phi\} \end{aligned} \tag{1}$$

And the acoustic pressure in the far field is determined as

$$\begin{aligned} \Psi(p_i) = &\sum_{m=1}^{nt}\sum_{j=1}^{8} A^i_{m,j}\Psi_{m,j} \\ &- \rho_f\,\omega^2 \sum_{m=1}^{nt}\sum_{j=1}^{8} B^i_{m,j}a_{m,j} \\ &- (A^{\oplus})^{-1}\Psi^{\oplus}_{inc} \end{aligned} \tag{2}$$

where

{F}	Applied Mechanical Force
$\{F_I\}$	Fluid Interaction Force
{Q}	Applied Electrical Charge
{a}	Elastic Displacement
{Φ}	Electric Potential
$[K_{uu}]$	Elastic Stiffness Matrix
$[K_{u\phi}]$	Piezoelectric Stiffness Matrix $[K_{\phi u}] = [K_{u\phi}]'$
$[K_{\phi\phi}]$	Permittivity Matrix
[M]	Mass Matrix
[R]	Dissipation Matrix
ω	Angular Frequency

3. Results

The coupled FE-BE method has been programmed with Fortran language running at SUN Ultra Workstation. Calculation is done with double precision and the program is made for three dimensional structures. Because each structural node has 4 DOF, the size of the globally assembled coefficient matrices of the matrix equation are 4*ng by 4*ng. The particular structure considered is a flooded piezoelectric (PZT4) spherical shell. Fig. 2 shows a part of the whole shell. The inner and the outer radii of the shell are 3cm and 4cm respectively. The shell could be divided into either 32 (Fig. 1(b)) isoparametric elements or 24 elements (Fig.1(c)). Global node numbers are attributed at 20 nodes of each element. It is desired to have more elements representing smaller local regions for higher frequency analysis. However, calculation with more number of nodes cost more time. Therefore meshing of elements depends on the maximal limit of interest frequency. The type of element meshing is often important. For example, the 32 elements of Fig. 1(b) have the disadvantage of concentrating on the top point of the sphere in their global nodes, while the 24 elements of Fig. 1(c) do not have such a concentrating problem even though they have smaller number of elements. If a global node is shared with too many adjacent element, it might cause calculational error during matrix solution.

Table 1 shows the material properties of the PZT4 piezoelectric ceramic. The actual ceramic shell is radially polarized and therefore the electrode is coated radially on inner and outer surfaces. Hence, the axially polarized property values of Table 1 is to be converted to its radial polling direction by the tensor theory [13].

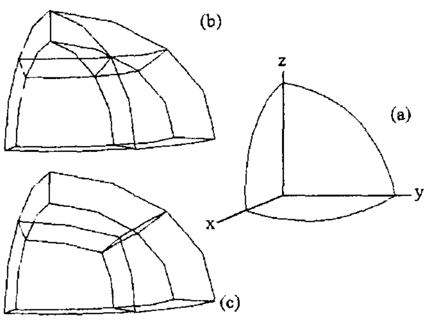

Fig. 1 A structure is discretized into finite structural elements. A piezoelectric shell can be divided into either 32 elements (b) or 24 elements (c).

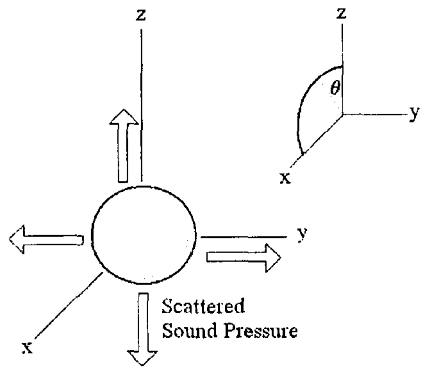

Fig. 2 The acoustic pressure in the far field is calculated along the circle with the directivity angle θ.

The present modelling of the SONAR transducer is a pressure sensitive hydrophone. So, the hydrophone is three-dimensionally simulated to transduce an incident plane acoustic pressure onto the outer surface of the sonar spherical shell to electrical potentials on inner and outer surfaces of the shell. This acoustical energy drives the piezoelectric shell as a receiver. The incident acoustic pressure is of cause scattered forwardly and backwardly after it is struck on the outer surface of the shell sphere. From equation (2) the acoustic pressure in the far field is calculated along the circle with the directivity angle θ (Fig. 2). After normalizing the far field pressure, the averaged value of the pressure is calculated. This normalized and averaged value of the far filed pressure is then used as the quantitative degree of the omnidirectional directivity.

Table 1. Material Properties of PZT4 (Axially Polarized Properties, Dielectric coefficients at 100 KHz)

		Unit
ρ	7500	Kg/m^3
C_x^x	1.39E+11	N/m^2
C_y^x	7.78E+10	N/m^2
C_z^x	7.43E+10	N/m^2
C_y^y	1.39E+11	N/m^2
C_z^y	7.43E+10	N/m^2
C_z^z	1.15E+11	N/m^2
C_{yz}^{yz}	2.56E+10	N/m^2
C_{zx}^{zx}	2.56E+10	N/m^2
C_{xy}^{xy}	3.06E+10	N/m^2
$e_{p,z}^x$	-5.2	$(N/m^2)/(V/m)$
$e_{p,z}^y$	-5.2	$(N/m^2)/(V/m)$
$e_{p,z}^z$	15.1	$(N/m^2)/(V/m)$
$e_{p,z}^{yz}$	12.7	$(N/m^2)/(V/m)$
$e_{p,z}^{zx}$	12.7	$(N/m^2)/(V/m)$
ε_x^x	6.4605E-9	F/m
ε_y^y	6.4605E-9	F/m
ε_z^z	5.6198E-9	F/m

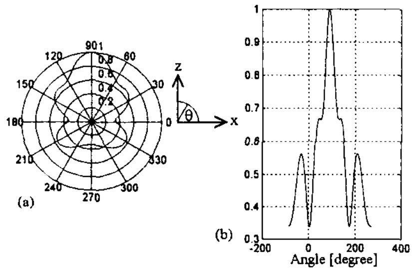

Fig. 3 Normalized directivity pattern of scattered acoustic press of a solid steel sphere (a) In polar form (b) In Rectangular form, IR=0cm, OR=4cm, 24 elements, ka=π

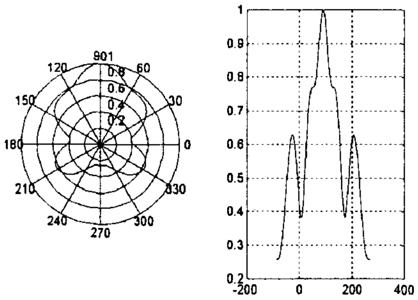

Fig. 4 Normalized directivity pattern of scattered acoustic press of a solid PZT4 sphere (a) In polar form (b) In Rectangular form, IR=0cm, OR=4cm, 24 elements, ka=π

Fig. 3 shows the directivity pattern of a solid steel sphere in polar form (a) and in rectangular form (b) for 740.6Hz input frequency. This particular figure is often used to confirm the calculation of the coupled FE-BEM algorithm to be correct [12]. And Fig. 4 shows the directivity pattern of a solid PZT4 sphere for the same 740.6Hz input frequency which is equivalent to ka=π (a is radius). Fig. 5 and Fig. 6 show the directivity patterns of the PZT4 spherical shell for the same ka=π but with 24 elements (Fig. 5) and with 32 elements (Fig. 6) respectively. These two figures show that the number of total elements and the type of the element meshing do not make any difference as the input frequency is small enough to be distinguished. In both figures, the inner and outer surfaces of the shell are not electrically electroded, that is, both inner and outer surfaces do not have any equipotential. However, the practical hydrophone need to be electroded in order to be connected to electric wires. Fig. 7 is similar to Fig. 5 except that the inner and outer surfaces of the PZT4 shell has been electroded in Fig. 7. The electroding of the PZT4 shell is simply done by manipulating of the global coefficient matrix [12]. The resulting potentials of the excited PZT4 shell are V_{outer} = -4.8357 + i9.8881 , V_{inner} = -4.8141 + i9.8806 , so that thier corresponding potential difference is 16.2 [mV] in magnitude for 740.6 Hz.

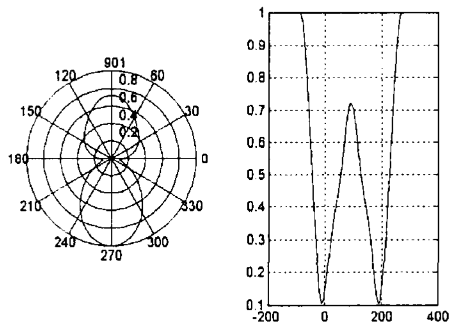

Fig. 5 Normalized directivity pattern of scattered acoustic press of a PZT4 spherical shell (a) In polar form (b) In Rectangular form, IR=3cm, OR=4cm, 24 elements, ka=π

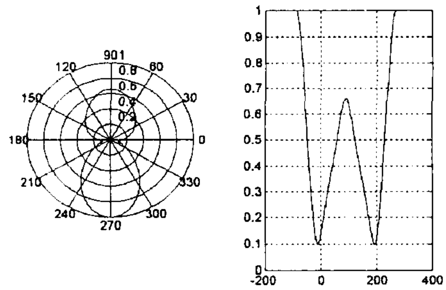

Fig. 6 Normalized directivity pattern of scattered acoustic press of a PZT4 spherical shell (a) In polar form (b) In Rectangular form, IR=3cm, OR=4cm, 32 elements, ka=π

Fig. 8 shows the surface acustic pressure on the outer surface of the PZT shell at a particular instant time. Since the present phydrophone simulation is calculated for steady-state frequency response, its temporal deformation could be figured with different phases. In the figure, the three dimensional displacement has been exagerated to emphasize the form of vibration. And Fig. 9 shows the temporal moving picture of the PZT4 shell for different phases.

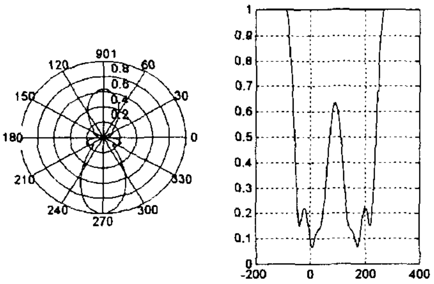

Fig. 7 Normalized directivity pattern of scattered acoustic press of a PZT4 spherical shell (a) In polar form (b) In Rectangular form. The inner and outer surfaces of the ceramic shell are electroded for equipotential surfaces respectively. IR=3cm, OR=4cm, 24 elements, ka=π

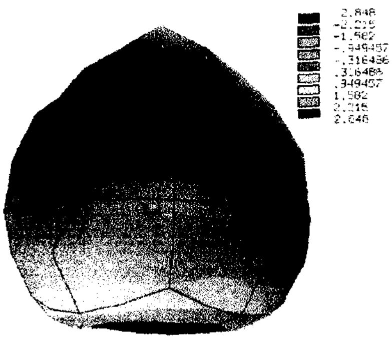

Fig. 8 The surface acustic pressure on the outer surface of the PZT shell at a particular instant time. The three dimensional displacement has been exagerated to emphasize the form of vibration. ka=π

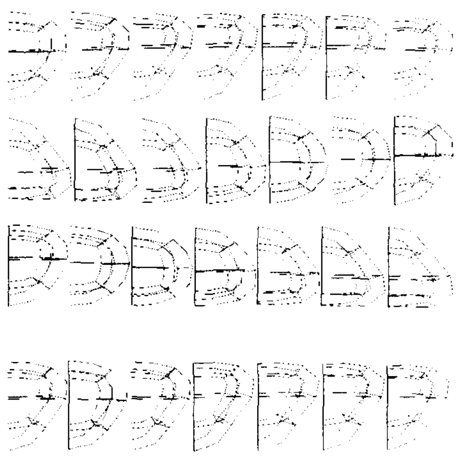

Fig. 9 Vibrational Modes (at 740.6Hz) for different phases

4. Conclusion

A coupled FE-BE method has been developed and applied to simulate a sonar transducer. The particular structure considered is a flooded piezoelectric spherical shell. The transducer is three-dimensionally simulated to transduce an incident plane acoustic pressure onto the outer surface of the shell to electrical potentials on inner and outer surfaces of the shell. The acoustic field formed from the scattered sound pressure is also simulated. And the displacement of the shell caused by the externally incident acosutic pressure is shown in temporal motion. The coupled FE-BE method is very useful for predicting the mechanical and the acoustical behaviour of the sonar transducer.

In general, as the frequency of external loading to the piezoelectric trasnducer is increased, more number of structural finite elements are necessarily required. Most of excuting time of the coupled FE-BEM program is spent in matrix solution in which the size of the matrix is increased to 4ng by 4ng matrix as the number of global nodes are increased to ng.

Therefore the present numerical method need to be linked to a faster matrix solver such as parallel processing for next work.

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