ASSESSING THE RISK-POOLING EFFECT OF WAREHOUSE INVENTORY IN A ONE-WAREHOUSE N-RETAILER DISTRIBUTION SYSTEM

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Abstract

This paper suggests the "infinite-retailer model" to approximate expected backorders per cycle of the One-warehouse N-retailer distribution system where the warehouse holds back some of the replenishment quantity to satisfy retailer backorders at the end of the cycle through direct shipping to customers. The main objective is to show the functional relationship between the warehouse inventory and the expected backorders per cycle. We illustrate the relationship using a uniform demand case.

1. Introduction

The other day I visited one of the discount stores in my area, Service Merchandise to buy a humidifier. I decided to buy one of the models exhibited in the store, filled out an order form, and submitted it to a cashier. But the cashier told me that the model I chose was out of stock. After checking its availability at the central warehouse, she recommended a mail order through the store saying that there is no additional shipping and handling charge. I did so and the humidifier arrived in about a week after the order. Through this experience, I got to know that this type of mail orders is very common at many stores in the United States.

This paper examines the multi-echelon distribution system that consists of one warehouse and multiple retailers, and allows direct shipments from the warehouse to an individual customer; that is, part of each system-replenishment quantity is kept at the warehouse for direct shipments. If at the end of cycle a retailer is out of stock and the warehouse has available stock, demand is met from the warehouse inventory. The system with direct shipments requires fewer inventory to attain a specified service level (= portion of demand met from inventory on-hand both at the retailers and at the warehouse) than the equivalent inventory system without direct shipments. But it incurs additional shipping and handling costs. We want to assess the risk-pooling value of warehouse inventory in a one-warehouse N-retailer distribution system. In particular, we want to identify the functional relationship between portion of each system-replenishment quantity kept at the warehouse and expected system backorders per cycle.

2. Model and Assumptions

This paper studies a one-warehouse N-retailer system facing stochastic demand and operating in a periodic-review mode. In the specific system examined, the warehouse places a system-replenishment order every period, and receives it after a fixed leadtime L. At that time the warehouse makes allocation decision; that is, the warehouse retains part of the system-replenishment quantity, and allocates the rest to the N retailers. The allocations to the retailers are delivered after a fixed leadtime l.

At the end of each cycle, retailer backorders are met with the warehouse inventory through direct shipments to customers. We assume that the transshipments between retailers are not allowed. If total retailer backorders at the end of cycle is greater than the warehouse inventory, the difference is backordered. The objective of the paper is to develop a model to estimate the expected system backorders per cycle of the system with direct shipments (System 2), which can be compared to that of the system without direct shipments (System 1) to assess the risk-pooling value of the warehouse inventory. The additional assumptions are (i) Period demand is i.i.d. across periods and among the retailers, (ii) The optimal system-replenishment policy is a base-stock policy, and (iii) Equal allocation is optimal; that is, at the time of allocation, we replenish every retailer up to the same amount. This is true if we relax the non-negativity constraints on retailer allocations, which is called the "allocation assumption" (Eppen and Schrage (1981)).

3. The N-Retailer Model

Notation

L= outside-supplier's leadtime

l= leadtime between the warehouse and any retailer

y= system base-stock

$s=$ $\frac{y}{N}$, allocation to each retailer in System 1

$\alpha=$ portion of s assigned to each retailer at the time of allocation in System 2 ($\alpha=1$ means no stock retained at the warehouse, that is, System 1)

$\alpha s=$ allocation to each retailer in System 2

$(1-\alpha)s=$ (stock kept at the warehouse at the time of allocation) $\div N$

$\delta_i=$ random period demand at retailer i, $i=1,\ldots,N$, with p.d.f. $\phi(.)$ and c.d.f. $\Phi(.)$

For simplicity of the presentation, without loss of generality, we assume that the leadtimes (L and l) are both zero. If s is assigned to each retailer at the time of allocation, expected system backorders per cycle of system 1 will be

$$EB_1 = N\int_s^{\infty}(\delta - s)\phi(\delta)d\delta \tag{1}$$

If $(1-\alpha)s$ per retailer is kept at the warehouse at the time of allocation, with direst shipments allowed, expected system backorders per cycle of system 2 will be

$$EB_2 = \int_0^{\infty}\ldots\int_0^{\infty}\max\{0,\sum_{i=1}^{N}\max\{0,\delta_i - \alpha s\} - N(1-\alpha)s\}\phi(\delta_N)d\delta_N\ldots\phi(\delta_1)d\delta_1 \tag{2}$$

For example, when N=2, the following is expected system backorders per cycle;

$$\begin{aligned} EB_2 = &\int_0^{\alpha s}\int_{(2-\alpha)s}^{\infty}(\delta_2-(2-\alpha)s)\phi(\delta_2)d\delta_2\phi(\delta_1)d\delta_1 + \int_{\alpha s}^{(2-\alpha)s}\int_{2s-\delta_1}^{\infty}(\delta_1+\delta_2-2s)\phi(\delta_2)d\delta_2\phi(\delta_1)d\delta_1 \\ &+ \int_{(2-\alpha)s}^{\infty}\int_0^{\alpha s}(\delta_1+(2-\alpha)s)\phi(\delta_2)d\delta_2\phi(\delta_1)d\delta + \int_{(2-\alpha)s}^{\infty}\int_{\alpha s}^{\infty}(\delta_1+\delta_2-2s)\phi(\delta_2)d\delta_2\phi(\delta_1)d\delta \end{aligned} \tag{3}$$

As you can see from the two-retailer case, when N is large, it will be very complicated to write the expression like (3). Therefore, it will be very nice to have a good approximation of expected system backorders per cycle in the N-retailer case.

4. The Infinite-Retailer Case

If we assume that there are infinite numbers of retailers in the system, we can easily write the expression for backorders per cycle per retailer even in System 2. When there are infinite number of retailers in the system, backorders per cycle per retailer of System 1 is

$$EBPR_1 = \int_s^{\infty}(\delta - s)\phi(\delta)d\delta, \tag{4}$$

and that of System 2 is

$$EBPR_2 = \int_{(\alpha+\beta)s}^{\infty}(\delta - \alpha s)\phi(\delta)d\delta, \tag{5}$$

where β satisfies

$$\int_{\alpha s}^{(\alpha+\beta)s}(\delta - \alpha s)\phi(\delta)d\delta = (1-\alpha)s. \tag{6}$$

β determines how much we can reduce backorders per cycle per retailer by carrying inventory at the warehouse. If the right hand side of (6) is greater than the left hand side for $\beta = \infty$, there will be no backorders at the end of cycle.

<u>Uniform Demand Case</u>

Assume that δ~U(0,1). Then,

$$\phi(\delta) = \begin{cases} 1, \text{ if } 0 \le \delta \le 1 \\ 0, \text{ otherwise} \end{cases}$$

<u>System 1</u>

When δ~U(0,1), backorders per cycle per retailer in System 1 is

$$EBPR_1 = \int_s^{\infty}(\delta - s)d\delta \cdot$$

(i) If $s \geq 1$, then,

$$EBPR_1 = 0 \cdot \tag{7}$$

(ii) If $0 \leq s < 1$, then,

$$EBPR_1 = \frac{1}{2}(s-1)^2 \cdot \tag{8}$$

System 2

β satisfies the following equation,

$$\int_{\alpha s}^{(\alpha+\beta)s}(\delta - \alpha s)d\delta = (1-\alpha)s \cdot \tag{9}$$

Solving (9) for β, we get

$$\beta = \sqrt{\frac{2(1-\alpha)}{s}} \cdot \tag{10}$$

(i) If $\alpha s + \sqrt{2(1-\alpha)s} \geq 1$, then,

$$EBPR_2 = 0 \,. \tag{11}$$

(ii) If $0 \leq \alpha s + \sqrt{2(1-\alpha)s} < 1$, then

$$\begin{aligned} EBPR_2 &= \int_{\alpha s+\sqrt{2(1-\alpha)s}}^{1}(\delta - \alpha s)d\delta \\ &= \frac{1}{2}(\alpha^2 s^2 - 2s + 1) \end{aligned} \tag{12}$$

By comparing (8) and (12), we can tell that given a certain set of system parameters under which (8) and (12) are true, the benefits from carrying warehouse inventory, i.e., the reduction in backorders per cycle per retailer, is

$$\frac{1}{2}(1-\alpha^2)s^2 \cdot \tag{13}$$

We know that expected backorder function of System 1 is convex regardless of demand distribution. In System 2, since $\frac{\partial^2 (EBPR_2)}{(\partial\alpha)^2} > 0$, backorders function (12) is also convex when δ~U(0,1). This convexity of backorders of System 2 can be a useful one in the future analyses (i.e., minimization of total expected costs).

5. Future Study

This paper suggests one possible approach, the infinite-retailer approximation, which can be used to estimate expected backorders per cycle of the periodic-review inventory system with direct shipments from the warehouse to customers allowed. The main objective was to show the functional relationship between warehouse inventory and expected backorders per cycle in the infinite-retailer case.

The further analysis would be needed in the following areas;

(i) Check the validity of the infinite-retailer approximation through a numerical study.

(ii) Construct an optimization problem (i.e., find α that minimizes the sum of expected holding, backordering, and extra shipping and handling costs).

(iii) Determine s and α simultaneously.

(iv) Model the case in which only certain portions of customers want direct shipments when no stock is available at a retailer. A few modifications will suffice to include this case in the current model.

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