Comparisons of Various DEM Interpolation Techniques

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Abstract

Extracting a Digital Elevation Model (DEM) from spaceborne imagery is important for cartographic applications of remote sensing data. The procedure for such DEM generation can be divided into stereo matching, sensor modelling and DEM interpolation. Among these, DEM interpolation contributes significantly to the completeness and accuracy of a DEM and, yet, this technique is often considered "trivial". However, na?e DEM interpolation may result in a less accurate and sometimes meaningless DEM. This paper reports the performance analysis of various DEM interpolation techniques. Using a manually derived DEM as reference, a number of sample points were created randomly. Different interpolation techniques were applied to the sample points to generate DEMs. The performance of interpolation was assessed by the accuracy of such DEMs. The results showed that kriging gave the best results at all times whereas nearest neighborhood interpolation provided a fast solution with moderate accuracy when sample points were large enough.

1. Introduction

Extracting a Digital Elevation Model (DEM) from spaceborne imagery is a very essential procedure for cartographic applications of remote sensing data. This involves a series of image processing and geo-information engineering techniques, which in general can be represented by the following three steps: stereo matching, sensor modelling and DEM interpolation. Stereo matching produces conjugate points from a stereo image pair. These points are in image coordinates and converted into ground coordinates by sensor modelling. DEM interpolation creates a DEM by interpolating the conjugate points in ground coordinates. Among these, DEM interpolation contributes significantly to the completeness and accuracy of a DEM and, yet, this technique is often considered "trivial". However, na?e DEM interpolation may result in a less accurate and sometimes meaningless DEM.

Given these considerations, there is a need to analyze various DEM interpolation techniques and their implications on the quality of a DEM. This paper will address this issue. Assuming that a DEM has a grid structure, four grid interpolation techniques (bin averaging, moving window averaging, nearest neighbor interpolation, kriging) will be investigated. Section 3 will describe the performance analysis of the various DEM interpolation techniques implemented here. For this, a manually derived DEM was used as a reference DEM. A number of sample data were randomly collected from the reference DEM. Various interpolation techniques were applied to these sample data and DEMs were created through such interpolation. The performance of an interpolation technique was estimated by analyzing the accuracy of the interpolated DEM. The results of performance assessment showed that kriging recovered the surface shape very accurately with even very small number of sample data and that nearest neighborhood interpolation produced acceptable results when sample data was large enough.

2. Principles of various DEM interpolation techniques

There are many different interpolation methods proposed and the exact number can hardly be figured out. It is, however, not intended here to include all such methods. Rather, only a few well-known interpolation techniques have been investigated. These are bin averaging, moving window averaging, nearest neighborhood interpolation and kriging.

Bin averaging

Bin averaging is the simplest of the four interpolation techniques implemented here. The input data are stored in a structure called "bin". A bin is defined as a rectangle centered at a grid of an output DEM. The width of a bin is dependent on the DEM resolution and the number of total bins is equal to that of pixels in the output DEM. Bin averaging assigns height to a bin by averaging the height of points stored in the bin. In this method, height is not assigned to a bin when the bin has no points.

Moving window averaging

Moving window averaging assigns height to a grid (or a location) by defining a circle of a given radius and averaging the height of input points within the circle. This method will create a DEM with smooth height distributions. However, this is not an exact interpolation method[1] [Wackernagel, 1995] and always has residual errors.

Nearest neighborhood interpolation

In this, height of a grid is assigned as the height of the input point whose distance to the grid point is the shortest among input points. It is often practiced to use a circle of a given radius and to search for the nearest point within the circle.

Kriging

Kriging is an interpolation method based on the theories developed in Geostatistics [Wackernagel, 1995]. Fundamental theoretical backgrounds of kriging can be founded in the reference and this paper concentrates only on its essence.

The problem of kriging is to find the optimal weights, ω_i, for interpolation of the height *(H)* at x_o in the following equations.

$$H(x_o) = \sum_{i=1}^{n} \omega_i H(x_i)$$

First of all, input points are assumed to have the second order stationarity, i.e., the expectation of height is independent of the location and the variance between heights at two different positions is only related to the distance between the two points. In particular, the property on the variance is very important. Using this property, a variogram, $\gamma(h)$, can be defined describing variance between any two points with a given distance *h* as

$$\mathrm{var}\left[H(x_i + h) - H(x_i)\right] = 2\gamma(h).$$

Interpolation can be said "optimal" if this is unbiased and produces the minimum estimation error. For an unbiased estimation, the following equation will hold.

$$E\left[\sum_{i=1}^{n} \omega_i H(x_i) - H(x_o)\right] = 0$$

In order to have the minimum estimation error, the following expression is to be minimized.

$$\mathrm{var}\left[\sum_{i=1}^{n} \omega_i H(x_i) - H(x_o)\right] = -\sum_{i=1}^{n}\sum_{j=1}^{n} \omega_i \omega_j \gamma(x_i - x_j) + 2\sum_{i=1}^{n} \omega_i \gamma(x_i - x_o)$$

Based on these two expressions, a Lagrange object function can be defined as

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[1] i.e. even though there is an input point for the exact location of a grid, the interpolated height is not the same as the height of such input point.

$$\varphi(\omega_i,\mu) = \mathrm{var}\left[\sum_{i=1}^{n}\omega_i H(x_i) - H(x_o)\right] - 2\mu\left(\sum_{i=1}^{n}\omega_i - 1\right).$$

Optimal weights for an unbiased, minimum variance estimation can be achieved by partial differentiating the above equations with each ω_i and μ. The optimal weights for kriging can be found by the following equations

$$\sum_{j=1}^{n}\omega_j\gamma(x_i - x_j) + \mu = \gamma(x_i - x_o),\ i = 1,\cdots,n$$

$$\sum_{j=1}^{n}\omega_j = 1$$

3. Comparisons of the performance of DEM interpolation techniques

This section describes the performance analysis of the four DEM interpolation algorithms explained earlier. For the analysis, a manually generated DEM were used as a reference DEM. The size of this DEM was 464 by 749 pixels and contained 347536 grid points. Height at each grid was encoded as the short integer type. Figure 1 shows the reference DEM. Height is represented as a brightness value in the figure.

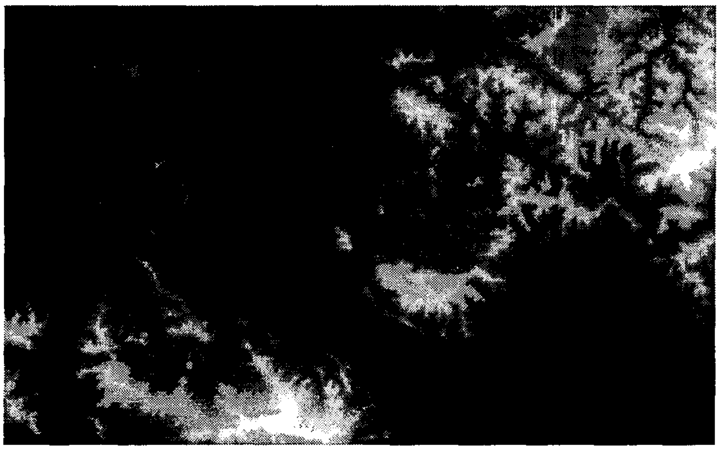


Figure 1. The reference DEM

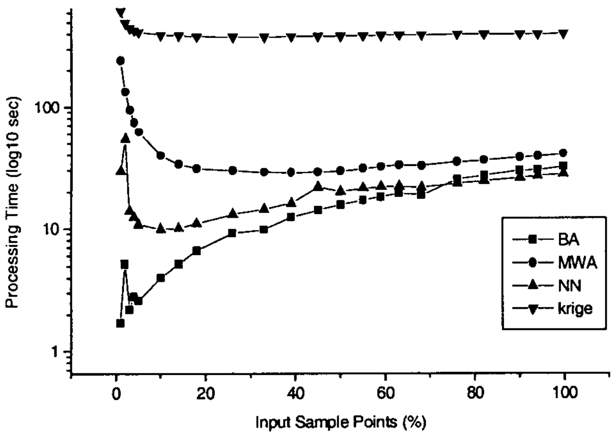
From the reference DEM sample points were extracted randomly. Using these random points DEMs were created through interpolation. These DEMs were compared with the reference DEM to assess the accuracy of interpolation. In order to assess the effect of the number of sample points, interpolation repeated 22 times with different numbers of sample points. The actual number of sample points used for interpolation varied from 3455 points (1% of the total interpolation points) to 346407 points (99.7%).

For each interpolation method and for each different number of sample points, a DEM was created and compared with the reference DEM. The performance of interpolation was assessed by the processing times and the root mean square error (RMSE) of the interpolated DEM. Table 1 summarizes the performance analyses. In the table, the RMSE of DEMs from bin averaging were not included because only one point was assigned to a bin and hence DEMs from bin averaging were replica of input points.

Figure 2 shows comparisons of processing times. As can be seen in the figure, kriging required the largest processing time whereas bin averaging did the smallest. The processing time for nearest neighborhood interpolation was similar to that for bin averaging. The processing time for bin averaging increased as the number of input points. This was because the time required to input points into bins was the dominant factor. Processing times for all others were quite independent of the number of input points.

Table 1. Comparisons of the performance of DEM interpolation algorithms

number of input points (percentage)	bin average	moving window average		nearest neighborhood		kriging	
	time	time	RMSE	time	RMSE	time	RMSE
3455 (1%)	1.7s	242s	57.77m	29.6s	52.8m	617s	39.98m
6880 (2%)	5.2s	133s	46.72m	54.5s	41.37m	495s	30.03m
10268 (3%)	2.2s	94s	41m	14s	35.85m	448s	25.19m
13615 (4%)	2.8s	74s	37.02m	12.4s	32.30m	431s	22.08m
17377 (5%)	2.6s	62s	33.87m	10.8s	29.42m	415.5s	19.69m
34754 (10%)	4s	39.8s	25.57m	9.9s	21.98m	393s	13.40m
48464 (14%)	5.2s	33.7s	22.05m	10.1s	18.92m	388s	10.87m
63101 (18%)	6.6s	30.9s	19.49m	11s	16.75m	382s	9.12m
90222 (26%)	9.2s	29.9s	16.35m	13.1s	14.07m	378s	6.98m
114711 (33%)	9.8s	28.9s	14.42m	14.4s	12.43m	378s	5.7m
136756 (39%)	12.4s	28.7s	12.97m	16.1s	11.34m	381s	4.85m
156791 (45%)	14.1s	29s	11.89m	21.8s	10.47m	384s	4.2m
175097 (50%)	15.6s	.29.4s	11.25m	20s	9.79m	384s	3.73m
191476 (55%)	17s	31s	10.86m	21.3s	9.19m	387s	3.36m
206299 (59%)	18.1s	31.9s	10.64m	21.9s	8.68m	389s	3.06m
219947 (63%)	19.3s	32.9s	10.50m	22s	8.19m	390s	2.78m
237698 (68%)	18.7s	32.6s	10.31m	21.6s	7.57m	392m	2.48m
265185 (76%)	25.2s	35s	9.85m	23.3s	6.54m	397s	2.02m
285717 (82%)	27.1s	36s	9.28m	24.5s	5.66m	394s	1.69m
312627 (90%)	29.7s	38.1s	8.22m	25.9s	4.25m	395s	1.22m
327992 (94%)	30.2s	39s	7.5m	27.1s	3.18m	396s	0.88m
346407 (99.7%)	32.1s	40.7s	6.51m	27.9s	0.75m	400s	0.22m



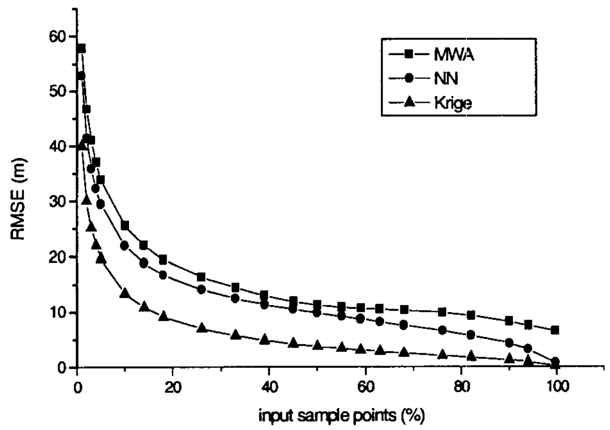


Figure 2. A line graph for processing time

Figure 3. A line graph of DEM accuracy

(In the figures, BA refers to bin averaging, MWA moving window averaging, NN nearest neighborhood interpolation)

On the contrary, when the number of input points were small, interpolation required more time. This was because it took more time to search for the nearest points for a small number of input points than for a large one. One other strange observation was the smaller processing time for nearest neighborhood interpolation than for bin averaging when the number of sample points reached to 100%. This was due to the way the two interpolation was implemented. The time taken for bin search was longer than the time taken for nearest neighbor point search.

Figure 3 shows the accuracy of each interpolation with the different number of input points. As expected, root mean square errors decreased as the number of input points increased. In all cases, kriging produced the highest accurate DEM, nearest neighborhood interpolation the next highest, moving window averaging the lowest. Moving window averaging had a significant amount of errors even with input points of nearly 100% of the total grid points. This was due to the fact that moving window averaging was not an exact interpolation method.

DEMs produced with different numbers of input points and by each interpolation are shown in figures 4-9. Even with the 1% input points, the DEM created by kriging recovers some patterns of the reference DEM. The DEM created by nearest neighborhood interpolation has very severe artifacts of circular patterns. With the 5% input points, kriging recovered reference DEM patterns almost completely whereas the DEMs by the other two remained troublesome. With the 45% input points, all three DEMs resemble the reference DEM[2].

From the performance analysis of each interpolation method and visual inspections of DEMs interpolation, it was shown that kriging worked at all times better than the other three methods. However, kriging required significant computation time. The second best interpolation seemed nearest neighborhood interpolation. This interpolation worked very fast and produced accurate DEMs with moderate accuracy when there were enough input sample points (at least 45% of the total interpolation points).

4. Conclusions

This paper reports the performance analysis of various DEM interpolation methods. This paper raised the issue of the importance of DEM interpolation in DEM generation. This paper proved that the use of different interpolation could change the accuracy of an output DEM significantly. The findings in this paper support the author's observation on the importance of DEM interpolation techniques and the danger of na?e interpolation.

Through comprehensive analysis of interpolation performance, this paper concludes that kriging is the best interpolation method among the four. This paper also concludes that if processing time is concerned, nearest neighborhood interpolation can be an alternative but only with a large number of input points. Based on these conclusions, a more efficient DEM generation system can be developed [Kim and Lee, 1998].

Acknowledgment

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REFERENCE

[Wackernagel, 1995] Wackernagel, H., 1995"Multivariate Geostatistics", *Springer-Verlag*
[Kim and Lee, 1998] Kim, T. and Lee, H.K., 1998, "DEM Generation from KOMPSAT-1 Electro-Optical Camera Data", *Proc. of International Symposium on Remote Sensing (14th Fall Symposium of KSRS)*, Kwangju, Korea, September 16-18

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[2] These were not included here for the limitation of space.

Figure 4. A DEM created from the 1% input points by moving window averaging

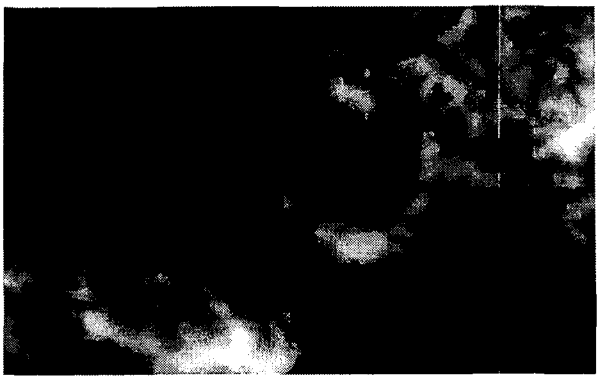

Figure 5. A DEM created from the 5% input points by moving window averaging

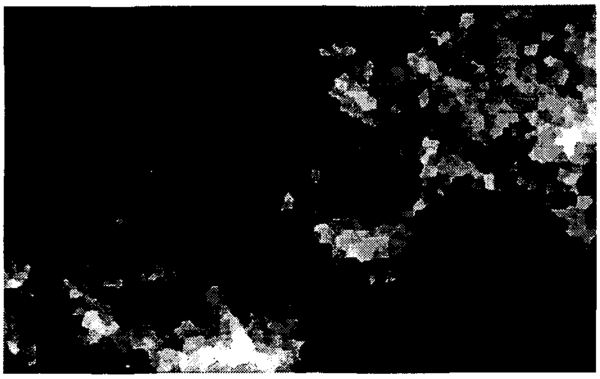

Figure 6. A DEM created form the 1% input points by nearest neighborhood interpolation

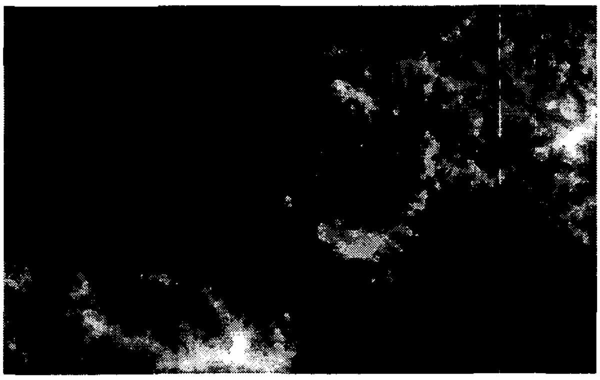

Figure 7. A DEM created from the 5% input points by nearest neighborhood interpolation

Figure 8. A DEM created from the 1% input points by kriging

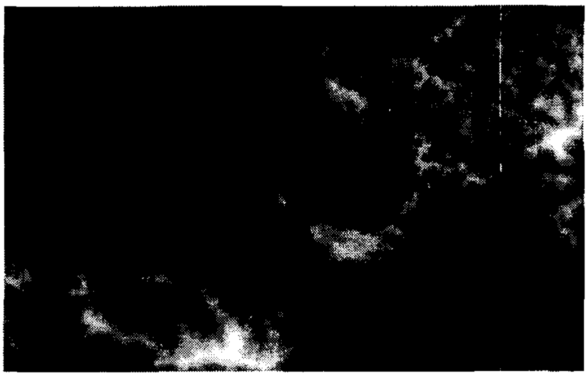

Figure 9. A DEM created from the 5% input points by kriging