한국항만학회 '98 추계학술대회논문집, pp. 159~165

論 文 B3-3

Two-Stage Control of a Container Crane: Time Optimal Travelling and Nonlinear Residual Sway Control

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Key Words: crane control, feedback linearization, underactuated mechanical system, variable structure control

Abstract

In this paper the sway-control problem of a container crane is investigated. The control loop is divided into two stages. The first stage is a modified time optimal control for trolley traversing. The velocity command for trolley traversing consists of three components; a reference velocity and two feedback signals for compensating the deviations of trolley and sway angle from their desired trajectories. For trolley's exact positioning the trolley dynamics is identified via an error equation identifier structure. The second stage is a nonlinear residual sway control that starts at the end of first stage. The control design for the second stage is investigated from the perspective of controlling an underactuated system, and the control law combines the feedback linearization and variable structure control.

1. Introduction

There are two methods to control the motor drives in crane systems. One is speed control in which the reference speed is tracked, and the other is torque control in which the armature current reference is tracked and no speed feedback is used. Concerning the speed control method, (Manson, 1977) investigated an analytic solution to the time-optimal control for overhead cranes. (Mita and Kanai, 1979) divided the traversing interval into three parts (acceleration zone, maximum-speed zone, deceleration zone) and investigated a bang-bang control assuming a constant rope length. (Sakawa and Shindo, 1982) divided the crane motion into five different sections and an optimal speed reference which minimized a quadratic cost function was derived. (Auernig and Troger, 1987) investigated the time-optimal control for the diagonal movement with a simplified model which was linearized in the swing angle. In (Hong et al., 1997a,b) five velocity patterns for trolley movement with constant rope length were analyzed and compared

Compared to the speed control method, the results on the torque control method are rare. This is because the speed control is widely used in industries, and the dry friction in the motor derives and transmission may cause some significant problems in practical implementation with torque control. However, the torque control method is more attractive from the theoretical point of view. (Boustany and D'Andrea-Novel, 1992) investigated an indirect adaptive control scheme using dynamic feedback linearization and estimation technique for an overhead crane.

For a given desired location of the load, the trolley needs to travel as fast as it can. On the other hand the trolley movement should not result in any residual sway motion of the load at the end of trolley stroke. However, due to the disturbances like winds and the changes of rope length some residual sway always exists at the end of each trolley stroke. In this paper the control loop is divided into two stages. The first stage is a modified time optimal control for trolley traversing. The main velocity command to the trolley motor is a velocity profile that is derived from the time-optimal control. However, a modification is made when either the trolley displacement or/and the sway angle of the load deviates from its desired trajectory. The second stage is a nonlinear residual sway control that starts at the end of first stage. However, the second stage controller should start when the residual sway angle is larger than a specified value. The control design for the second stage is investigated from the perspective of controling an under-actuated mechanical system, and the control law combines the feedback linearization and variable structure control.

2. Path Planning and Modeling

Fig. 1 shows a schematic diagram of the container crane. Fig. 2 shows a transportation sequence of the container (Sakawa and Shindo, 1982; Hamalainen et al., 1995). The sway control problem is divided into two stages. The first stage is the time optimal control for the trolley traversing which covers from B to E. In this stage trolley traversing and load hoisting are independently controlled. A simplified model is used in deriving an optimal control law for the trolley traversing. The second stage is from E to F. A nonlinear control strategy which combines feedback linearization and variable structure control is applied for suppressing the remaining swing motion of the load.

2.1 Modeling of Mechanical Parts

Using the Lagrange mechanics, the equations of motion are obtained as

$$(m_1 + m_T + m_L)\ddot{x} + m_L \sin\phi \ddot{l} - m_L l \dot{\phi}^2 \sin\phi + 2m_L \cos\phi \dot{l} \cos\phi \dot{\phi} + m_L l \cos\phi \ddot{\phi} = F_1 - b_1 \dot{x} \quad (1)$$

$$\frac{1}{2} m_L \ddot{x} \sin\phi + 2(m_2 + \frac{1}{4} m_L)\ddot{l} - \frac{1}{2} m_L l \dot{\phi}^2 - \frac{1}{2} m_L g \cos\phi = F_2 - b_2 \dot{l} \quad (2)$$

$$l\ddot{\phi} + 2\dot{l}\dot{\phi} + g\sin\phi + \ddot{x}\cos\phi = 0 \quad (3)$$

where x is the trolley displacement and l is the rope length. These nonlinear equations (1)-(3) are used in Section 4 in designing a nonlinear controller for the residual sway. With the assumption that ϕ is small, the following approximations are made; $\cos\phi \cong 1$, $\sin\phi \cong \phi$, and $\dot{\phi}^2 \cong 0$. Apply the fact from (3) that $m_L(l\ddot{\phi} + 2\dot{l}\dot{\phi} + \ddot{x}) = -m_L g\phi$ and neglect the terms involving ϕ. Assume also that $\dot{l} = 0$. Then, a simple model for trolley traversing control is obtained

$$m\ddot{x} + b\dot{x} = F \quad (4)$$

$$l\ddot{\phi} + g\phi = -\ddot{x} \quad (5)$$

where $m = (m_1 + m_T)$, $b = b_1$ and $F = F_1$. Note that i)during trolley traversing the hoist motor is independently controlled, and ii)the control input for sway suppression is the trolley acceleration. Since (5) neglects a lot of nonlinear dynamics, there always exists initial sway of the load, and disturbances might enter during traversing, it is expected that the control action based upon (4)-(5) is not sufficient, and some modification of the control law based upon (4)-(5) is needed as discussed in Section 3. Laplace transforming (6a, b) yields

$$H_x(s) = V(s)/F(s) = 1/(ms + b). \quad (6)$$

$$H_\phi(s) = \phi(s)/X(s) = -s^2/l \,/\, (s^2 + g/l). \quad (7)$$

2.2 An Identified Model for Velocity Servo Control

A model for adjusting the velocity command when the trolley displacement deviates from the desired trajectory is investigated. The crane system is powered with AC servo motors with vector controllers. Since the sway control is exerted through the trolley movement, the analysis in this section is focused on the trolley servo system. Since the electrical time constant of the vector controller is much smaller than the mechanical time constant, the electrical dynamics is neglected. That is, the control force F in (6) is assumed to be of the form (Lee et al., 1997)

$$F = k_m i_m \cong k_t u \quad (8)$$

where k_m and i_m are the trolley motor constant and motor current , respectively; u is the input to the trolley motor, and k_t is a constant which depends on the trolley motor constant and the power transmission (gear ratio) of the trolley system. Note that the exact value of k_t is not known. Note also that in the torque control mode i_m is the control input to be designed. Since the speed control with velocity feedback is pursued for trolley traversing, the input u to the trolley motor should be of the form

$$U(s) = K_V(s)(V_c(s) - V(s)) \quad (9)$$

where $K_V(s)$ denotes the controller transfer function to be specified. V_c and V are the velocity command to the drive and actual output out of the trolley, respectively. Combining (6) and (9) yields the transfer function from V_C to V as

$$H_V(s) = \frac{V(s)}{V_c(s)} = k_t K_V(s) H_x(s) \Big/ 1 + k_t K_V(s) H_x(s) \quad (10)$$

In this paper let the velocity servo controller be a PI controller, i.e. $K_V(s) = (k_{vp} + k_{vi}/s)$, where k_{vp} and k_{vi} are proportional and integral gains, respectively. Then (10) is of the form.

$$H_V(s) = \frac{k_t k_{vp}/m\, s + k_t k_{vi}/m}{s^2 + (b + k_t k_{vp}/m)s + k_t k_{vi}/m} = \frac{b_1 s + b_0}{s^2 + a_1 s + a_0} \quad (11)$$

In (11) the exact values of a_0, a_1, b_0 and b_1 are not known. In this paper these are identified via the method of an equation error identifier. The following identified equation for the pilot crane of Fig. 5 is used for the design of traversing control law

$$H_V(s) = (4.4s + 78.1)/(s^2 + 16.4s + 80.1)\,. \quad (12)$$

3. Travelling Control: Time-Optimal

Fig. 3 shows the block diagram of the trolley traversing control. The velocity command V_c to the trolley drive system consists of three components as follows;

$$V_c = V_r + (k_{\phi p} + k_{\phi d}s)(\phi_r - \phi) + (k_{xp} + \frac{k_{xi}}{s})(x_r - x) \quad (13)$$

The first term V_r is the reference trolley velocity which assumes zero sway angle of the load when it is applied to system (5). In subsection 3.1 a stepped pattern which was derived via bang/off-bang control is introduced. x_r and ϕ_r are the reference trolley displacement and the sway angle of the load when the reference trolley velocity V_r is applied to (5). Therefore, the second and the third terms modify the velocity command when the trolley displacement and the sway angle of the load are deviated from their desired trajectories, respectively.

3.1 A Reference Velocity Profile

In (Hong et al., 1997b) five velocity patterns with constant rope length are derived and compared. These include one trapezoidal, two stepped, one notched, and one double acceleration/deceleration patterns. The shortest traversing time is provided by the notched pattern which is derived from the bang/bang control. However, a stepped pattern is used in this paper because it excites the crane structure less compared with the notched one. Fig. 4 shows a reference velocity profile, which is a stepped pattern. V_{max} and α stand for the maximum velocity and acceleration of the motor, respectively. During acceleration l_C is the rope length referenced in determining T_2, and during deceleration l_E is the rope length referenced in calculating T_4 and T_6. $T_1 \sim T_7$ are:

$T_1 = V_{max}/2\alpha$;

$T_2(l_C) = 2\arctan\left[\sin\left(T_1\sqrt{g/l_C}\right)/\left(1 - \cos\left(T_1\sqrt{g/l_C}\right)\right)\right]/\sqrt{g/l_C}$;

$T_3 = T_1 + T_2$; $T_5 = T_4 + T_1$; $T_6 = T_4 + T_2(l_E)$;

$T_7 = T_6 + T_1$; T_4 = time to decelerate if

$$x_d - x = \alpha T_1 (T_1 + T_2(l_E)) \quad (14)$$

3.2 Trolley Position Control

The upper closed loop in Fig. 3 represents the feedback control action to locate the trolley at its target position. An integral control has been included to reject the effects of the disturbance D_V on the output x in the low frequency region. The examples of this type of disturbance include the slip of the crane wheels and the offsets of the vector servo controller. The loop shaping method has been adopted for the design of the position servo compensator $K_x(s)$ in Fig. 3 (Lee et al., 1997). The magnitude of the open loop transfer function is made sufficiently large for good command tracking and disturbance rejection in the low frequency region, and sufficiently small for robust stability and sensor noise attenuation in the high frequency region. The compensator dynamics for the pilot crane is introduced as

$$K_x(s) = k_{xp} + k_{xi}/s = 2.6 + 0.4/s \quad (15)$$

Therefore, the closed loop transfer function is of the form

$$H_x(s) = \frac{11.4s^2 + 203.2s + 3.1}{s^4 + 16.4s^3 + 91.7s^2 + 203.2s + 3.1}\,. \quad (16)$$

3.3 Sway Angle Compensation

The lower closed loop in Fig. 3 represents the feedback control action for the sway angle compensation when the angle deviates from its desired trajectory. Let $K_\phi(s)$ in Fig. 3 be a PD controller, and set $K_{\phi p}/K_{\phi d}=20$. Then, the open loop transfer function for the pilot crane becomes

$$H_{\phi o}(s)=K_{\phi d}(s+K_{\phi p}/K_{\phi d})H_V(s)H_\phi(s)/s$$
$$=\frac{-K_{\phi d}s^2(4.4s^2+166.1s+1562)}{6.5s(s^4+16.4s^3+81.9s^2+24.76s+121.2)} \quad (17)$$

where $l=6.5m$ has been used. Using the root locus method the compensator dynamics is selected as

$$K_\phi(s)=10.6+0.53s \quad (18)$$

which provides the damping ratio of 0.7 to the closed loop system.

4. Nonlinear Control of Residual Sway

4.1 Feedback Linearization

Note that equations (1)-(3) are in the form of (A1)-(A2) in the Appendix. Specifically, those terms in (A3) are

$$M_{11}=m_Ll^2,\quad M_{12}=M_{21}^T=[m_Ll\cos\phi \quad 0], \quad (19)$$
$$M_{22}=\begin{bmatrix} m_1+m_T+m_L & m_L\sin\phi \\ m_L\sin\phi/2 & 2(m_2+m_L/4)\end{bmatrix},$$
$$C_1=2\dot{l}\dot{\phi}m_Ll,$$
$$C_2=\begin{bmatrix} -m_Ll\dot{\phi}^2\sin\phi+2m_L\dot{l}\dot{\phi} & m_Ll\dot{\phi}^2/2\end{bmatrix}^T,$$
$$G_1=m_Lgl\sin\phi,$$
$$G_2=[0 \quad -m_Lg\cos\phi/2]^T$$

Following the procedure of Section A.1 the partial linearization is performed as follows: Take the input to the trolley f as

$$\begin{bmatrix} f_1 \\ f_2\end{bmatrix}=\overline{M}_{22}\begin{bmatrix} u_1 \\ u_2\end{bmatrix}+\overline{C}_2+\overline{G}_2 \quad (20)$$

where

$$\overline{M}_{22}=\begin{bmatrix} m_1+m_T+m_L-m_L\cos^2\phi & m_L\sin\phi \\ m_L\sin\phi/2 & 2(m_2+m_L/4)\end{bmatrix},$$
$$\overline{C}_2=\begin{bmatrix} -m_Ll\dot{\phi}^2\sin\phi+2m_L\dot{l}\dot{\phi}-2\dot{l}\dot{\phi}m_L\cos\phi \\ -m_Ll\dot{\phi}^2/2\end{bmatrix},$$
$$\overline{G}_2=\begin{bmatrix} -m_Lg\cos\phi\sin\phi \\ -m_Lg\cos\phi/2\end{bmatrix}$$

and u_1 and u_2 are the additional control inputs to be decided in the sequel, then the resulting partially linearized system is obtained as

$$\ddot{\phi}=-\frac{1}{l}(2\dot{l}\dot{\phi}+g\sin\phi)-\frac{\cos\phi}{l}u_1 \quad (21)$$
$$\ddot{x}=u_1$$
$$\ddot{l}=u_2$$

The state variables are defined as $x=[x_1,x_2,x_3,x_4,x_5,x_6]^T=[\phi,x,l,\dot{\phi},\dot{x},\dot{l}]^T$. Then (21) is converted into the state equation as

$$\dot{x}=f(x)+B(x)u$$
$$=\begin{bmatrix} x_4 \\ x_5 \\ x_6 \\ -(2x_4x_6+g\sin x_1)/x_3 \\ 0 \\ 0\end{bmatrix}+\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ -\cos x_1/x_3 & 0 \\ 1 & 0 \\ 0 & 1\end{bmatrix}\begin{bmatrix} u_1 \\ u_2\end{bmatrix} \quad (22)$$

Considering that the control action for ϕ is provided by x, while the hoisting motion interferes the ϕ-x dynamics as disturbance, the following sliding surface is defined

$$\sigma=C(x)\tilde{x}$$
$$=\begin{bmatrix} c_1(x_3) & c_2 & 0 & c_3(x_1,x_3) & c_4 & 0 \\ 0 & 0 & c_5 & 0 & 0 & 1\end{bmatrix}\tilde{x} \quad (23)$$

where $x_d=[0 \quad x_2 \quad x_3 \quad 0 \quad 0 \quad 0]^T$, $\sigma\in R^2$, $C\in R^{2\times6}$, $\tilde{x}=x-x_d\in R^6$, and the matrix C will be determined in such a way that the linearization of the closed loop system at an operating point has the eigenvalues at specified locations.

The closed loop system in the sliding mode is of the form

$$\dot{x}=\left[I-B\left(\frac{\partial\sigma}{\partial x}B\right)^{-1}\frac{\partial\sigma}{\partial x}\right]f(x) \quad (24)$$
$$=\begin{pmatrix} x_4 \\ x_5 \\ x_6 \\ \frac{c_1\cos x_1}{x_3}x_4+\frac{c_2\cos x_1}{x_3}x_5-\frac{1}{x_3}(2x_4x_6+g\sin x_1)-Term1 \\ -c_1x_4-c_2x_5+\frac{c_3}{x_3}(2x_4x_6+g\sin x_1) \\ -c_5x_6\end{pmatrix}$$

where $Term1 = (c_4 - 1)(2x_4x_6 + g\sin x_1)/x_3$. Linearize (24) on an operating point $x_o = [0 \;\; x_2 \;\; x_3 \;\; 0 \;\; 0 \;\; 0]^T$, then

$$\dot{x} = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ -c_4g/x_3 & 0 & 0 & c_1/x_3 & c_2/x_3 & 0 \\ c_3g/x_3 & 0 & 0 & -c_1 & -c_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & -c_5 \end{pmatrix} x \quad (25)$$

is obtained. Locate the closed loop eigenvalues at λ = -2, -2.5, -2.43 $\pm$ 3*i*. Then the matrix C in (23) is determined as $c_1 = c_2x_3 + 6.87$, $c_2 = -29.88/g$, $c_3 = (c_4 - 1)x_3/\cos x_1$, $c_4 = -24.6/g$, $c_5 = 2.5$.

5. Simulations

In this section, we present a series of simulations with the proposed controller: combined partial feedback linearization and VSC. The parameters used in the simulations are as follows: $m_1 = 10kg$, $m_2 = 10kg$, $m_t = 30kg$, $m_L = 100kg$, $g = 9.81m/s^2$. The trolley is assumed to have reached its desired position ($x_d = 5m$). It is assumed that the residual sway angle is 5°, and the rope length changes from 3m to 5m. It is shown in Fig. 6 that the swing motion dissipates in 3 seconds.

Aknowledgement

This work was supported in part by the Center for Intelligent and Integrated Port Management Systems(CIIPMS). The CIIPMS is a Regional Research Center supported by the Korea Science and Engineering Foundation.

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Appendix

A.1 Feedback Linerization

An underactuated mechanical system is a system in which the number of actuators is less than the degrees of freedom that the system possesses. Consider an n degrees of freedom open loop mechanism with joint variables $q^1,\dots,q^n$. Each joint is assumed to have a single degree of freedom and only $m < n$ joints are active. The joint which is capable of actuation is called active joint. The remaining $l = n - m$ joints with no actuation are called passive or free joints depending on the availability of brakes.

Using the Lagrange formalism, the equations of motion are obtained. Without loss of generality the equations can be rearranged so that the coordinates for passive joints are grouped in $q_1 \in R^l$ and the coordinates for active joints are grouped in $q_2 \in R^m$.

$$M_{11}\ddot{q}_1 + M_{12}\ddot{q}_2 + C_1(q,\dot{q}) + G_1(q) = 0 \qquad (A1)$$

$$M_{21}\ddot{q}_1 + M_{22}\ddot{q}_2 + C_2(q,\dot{q}) + G_2(q) = f \qquad (A2)$$

where the vector functions $C_1(q,\dot{q}) \in R^l$ and $C_2(q,\dot{q}) \in R^m$ contain Coriolis and centripetal terms, the vector functions $G_1(q) \in R^l$ and

$G_2(q) \in R^m$ contain gravitational terms, and $f \in R^m$ represents the input generalized force produced by the m actuators at the active joints. Note that f appears only on the active joints q_2. Hence like a fully actuated robot, the dynamic equation itself for an underactuated system can also be written as

$$M(q)\ddot{q}+C(q,\dot{q})+G(q)=Bf \quad \text{(A3)}$$

where

$$q=[q_1{}^T, q_2{}^T]^T,\ M(q)=\begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix},\ B=\begin{bmatrix} O_{l\times m} \\ I_{m\times m} \end{bmatrix},$$

$C=[C_1^T, C_2^T]^T, G=[G_1^T, G_2^T]^T$. It is noted that the inertia matrix M defined in (A3) is not necessarily equal to the conventional inertia matrices of mechanical manipulators. However, M still preserves the important properties such as symmetry, positive definiteness of the original inertia matrix. The term M_{11} is an invertible $l\times l$ matrix as a consequence of the uniform positive definiteness of M in (A3). Therefore

$$\ddot{q}_1=-M_{11}{}^{-1}(M_{12}\ddot{q}_2+C_1+G_1)\,. \quad \text{(A4)}$$

Substituting (A4) into (A2) yields

$$\overline{M}_{22}\ddot{q}_2+\overline{C}_2+\overline{G}_2=f \quad \text{(A5)}$$

A partial feedback linearizing controller can therefore be defined for equation (A5) according to

$$f=\overline{M}_{22}u+\overline{C}_2+\overline{G}_2 \quad \text{(A6)}$$

where $u \in R^m$ is an additional control input yet to be defined, and the $m\times m$ matrix $\overline{M}_{22}$ is itself symmetric and positive definite. Hence

$$M_{11}\ddot{q}_1+C_1+G_1=-M_{12}u \quad \text{(A7)}$$

$$\ddot{q}_2=u \quad \text{(A8)}$$

A.2 Variable Structure Control

In this section a VSC for enhancing robustness against unmodeled dynamics and disturbances is investigated. With the state variables defined below (A7) and (A8) become

$$\dot{x}(t)=f(x)+B(x)u$$

$$=\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ -M_{11}^{-1}(C_1+G_1) \\ 0_{m\times m} \end{bmatrix}+\begin{bmatrix} 0_{l\times m} \\ 0_{m\times m} \\ -M_{11}^{-1}M_{12} \\ I_{m\times m} \end{bmatrix}u \quad \text{(A9)}$$

where $x=\begin{bmatrix} q_1^T, q_2^T, \dot{q}_1^T, \dot{q}_2^T \end{bmatrix}^T \in R^{2n}$, $u \in R^m$, and $B(x) \in R^{2n\times m}$ Associated with the system there is a (n-m)-dimensional switching surface (also called a discontinuity or equilibrium manifold).

$$S=\{x \in R^n \mid \sigma(x)=[\sigma_1(x),\cdots,\sigma_m(x)]^T=0\} \quad \text{(A10)}$$

The sliding surface is often referred to $\sigma(x)=0$. This surface should be designed to have the asymptotic stability. Once the surface is defined, control law must guarantee that the sliding surface is reachable, and that the plant's state trajectory is maintained on that sliding surface for all subsequent time. The control input is divided into two parts:

$$u=u_{eq}+u_N \quad \text{(A11)}$$

where u_N is the reaching control which forces the trajectory toward the sliding surface and u_{eq} is the equivalent control which assures the trajectory to remain on the surface once it gets there. Using the chain rule, the equivalent control u_{eq} can be introduced as

$$\dot{\sigma}=\partial\sigma/\partial x\,\dot{x}=\partial\sigma/\partial x\,f(x)+\partial\sigma/\partial x\,B(x)u_{eq}=0 \quad \text{(A12)}$$

Assuming that the matrix product $\partial\sigma/\partial x\,B(X)$ is nonsingular for all x, (A12) yields

$$u_{eq}=-\left[\partial\sigma/\partial x\,B(x)\right]^{-1}(\partial\sigma/\partial x\,f(x)) \quad \text{(A13)}$$

Therefore, substituting (A12) into (A9) the closed loop dynamics becomes

$$\dot{x}(t)=\left[I-B(x)\left[\partial\sigma/\partial x\,B(x)\right]^{-1}\partial\sigma/\partial x\right]f(x) \quad \text{(A14)}$$

In deriving the reaching control law the Lyapunov approach is used. Let $V(x,\sigma)=\sigma^T(x)\sigma(x)/2$ be a Lyapunov candidate function. The control u must be chosen so that the time derivative of $V(x,\sigma)$ is negative definite for $\sigma \neq 0$. To this end, consider

$$\dot{V}(x,\sigma)^{\,T}=\sigma^T\{\partial\sigma/\partial x\,f+\partial\sigma/\partial x\,B(u_{eq}+u_N)$$
$$=\sigma^T\,\partial\sigma/\partial x\,Bu_N$$

To prevent the chattering phenomenon the reaching control u_N is chosen as

$$\dot{V}(x,\sigma)=-\sigma^T\,\partial\sigma/\partial x\,B\left[\partial\sigma/\partial x\,B\right]^{-1}sat(\sigma)\ =-\sigma^T sat(\sigma)<0$$

where $sat(\sigma)=\begin{cases} \mathrm{sgn}(\sigma) & if\,\sigma\geq\varepsilon \\ \sigma/\varepsilon & if\,\sigma\geq\varepsilon \end{cases}$.

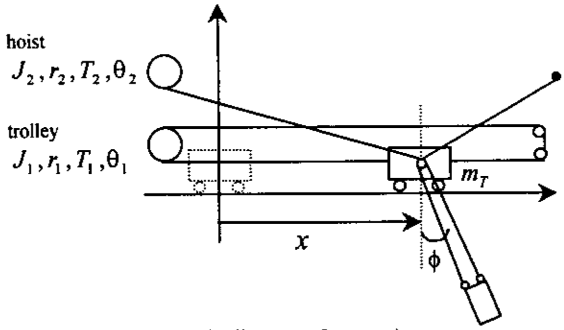

Fig. 1 Schematic diagram of a container crane

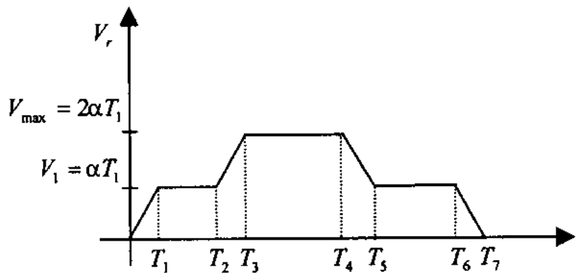

Fig. 4 A stepped velocity pattern

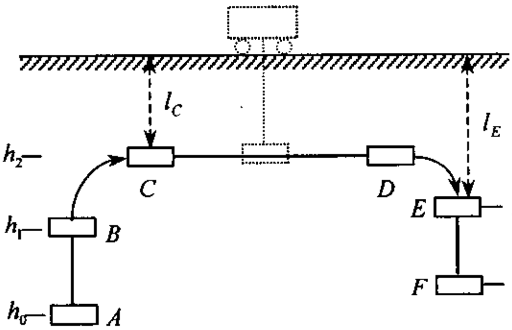

Fig. 2 Transportation sequence of the load

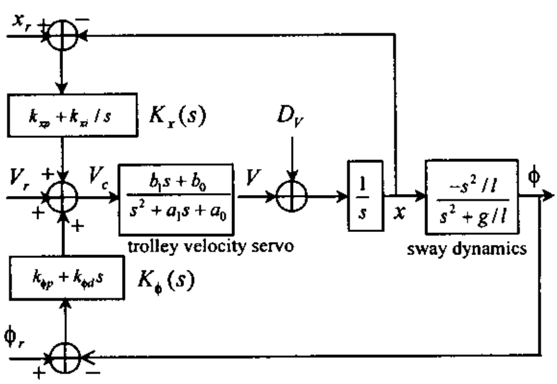

Fig. 3 Block diagram of the traversing control

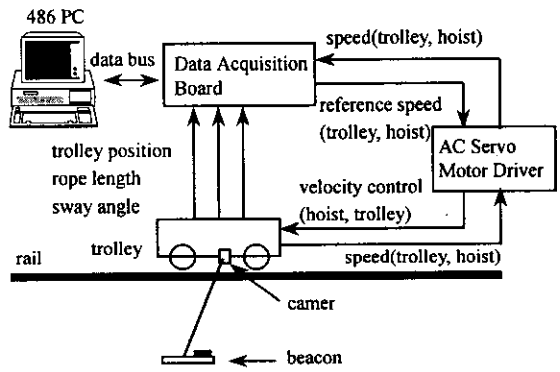

Fig. 5 Configuration of the experimental setup