An Adaptive Data Association Scheme for Multi-Target Tracking in Radar

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Abstract

This paper introduced a scheme for finding the relationships between the measurements and tracks in multi-target tracking (MTT). We considered the relationships between targets and measurements as MRF and assumed a priori as a Gibbs distribution. An energy function is defined over the measurement space , as accurately as possible so that it may incorporate most of the important natural constraints. To find the minimizer of the energy function, we derived a new equation of closed form.

1 Introduction

The primary purpose of a multi-target tracking(MTT) system is to provide an accurate estimate of the target position and velocity from the measurement data in a field of view. MTT system consists of three blocks: acquisition, association, and prediction.

The purpose of the acquisition is to determine the initial starting position of the tracking. After this stage, the association and prediction interactively determine the tracks. Our primary concern is the association part that must determine the actual measurement and target pairs, given the measurements and the predicted gate centers.

This chapter addresses an optimal adaptive data association scheme. First of all, we derives the minimization process for data association energy equation. Then proposed the parameter updating scheme to automize the system. Finally review the computational complexity. We show the simulation results.

2 Problem Formulation and Energy Function

Let m and n be the number of measurements and targets respectively, in a surveillance region. Then, the relationships between the targets and measurements are efficiently represented by the *validation matrix* Ω [3]:

$$\Omega = \{\omega_{jt} | j \in [1,m], t \in [1,n]\}, \tag{1}$$

where the first column denotes clutter and always $\omega_{j0} = 1$ $(j \in [1,m])$. For the other columns, $\omega_{jt} = 1$ $(j \in [1,m], t \in [1,n])$, if the validation gate of target t contains the measurement j and $\omega_{jt} = 0$, otherwise.

Based on the validation matrix, we must find *hypothesis matrix* [3] $\hat{\Omega}(= \{\hat{\omega}_{jt} | j \in [1,m], t \in [1,n]\})$ that must obey the data association hypothesis:

$$\begin{cases} \sum_{j=1}^{m} \hat{w}_{jt} = 1 & \text{for } (t \in [1,n]), \\ \sum_{t=0}^{n} \hat{w}_{jt} = 1 & \text{for } (j \in [1,m]). \end{cases} \tag{2}$$

Here, $\hat{\omega}_{jt} = 1$ only if the measurement j is associated with clutter $(t = 0)$ or target t $(t \neq 0)$. Generating hypothesis matrices leads to a combinatorial problem, where the number of data association hypothesis increases exponentially with the number of targets and measurements.

The ultimate goal of this problem is to find the hypothesis matrix $\hat{\Omega} = \{\omega_{jt} | j \in [1,m], t \in [1,n]\}$, given the observation $Y = \{Y_k | k \in [1,m]\}$, which must satisfy (2).

Let's consider that $\hat{\Omega}$ is a parameter space and (Ω, Y, X) is an observation space. Then,*a posteriori* can be derived by the Bayes rule:

$$P(\hat{\Omega}|\Omega, \mathbf{y}, \mathbf{x}) = \frac{P(\Omega|\hat{\Omega})P(\mathbf{y},\mathbf{x}|\hat{\Omega})P(\hat{\Omega})}{P(\Omega,\mathbf{y},\mathbf{x})}. \tag{3}$$

Here, we assumed that $P(\Omega, \mathbf{y}, \mathbf{x}|\hat{\Omega}) = P(\Omega|\hat{\Omega})P(\mathbf{y},\mathbf{x}|\hat{\Omega})$, since the two variables Ω and (X, Y) are separately observed. This assumption makes the problem more tractable as we shall see later.

Given the parameter $\hat{\Omega}$, Ω and (X, Y) are observed. If the conditional probabilities describing the relationships between the parameter space and the observation spaces are available, one can obtain the MAP estimator:

$$\hat{\Omega}^* = \arg\max_{\hat{\Omega}} P(\hat{\Omega}|\Omega, \mathbf{y}, \mathbf{x}). \tag{4}$$

According to [2], one gets

$$\hat{\Omega}^* = \arg\min_{\hat{\Omega}} A\sum_{t=1}^{n}\sum_{j=1}^{m}\mathbf{r}_{jt}^2\hat{w}_{jt} + \frac{B}{2}\sum_{t=1}^{n}\sum_{j=1}^{m}(\hat{w}_{jt}-w_{jt})^2 + \sum_{t=1}^{n}(\sum_{j=1}^{m}\hat{w}_{jt}-1) + \sum_{j=1}^{m}(\sum_{t=0}^{n}\hat{w}_{jt}-1), \quad (5)$$

In (5), the first term favors associations which locates near the velocity line by weighted validation matrix. The second term tends to discourage unrealistic association by comparing the generated matched events with the validation matrix. The third term represent the constraints as explained in (2).

3 Designing of Adaptive Data Association Scheme

The optimal solution for (5) is hard to find by any deterministic method. Instead, one can convert the present constrained optimization problem to an unconstrained problem by introducing Lagrange multipliers and using the local dual theory[6, 7]. This section addresses how to solve the equation and also how to determine the related parameters automatically.

3.1 Minimizing the Energy Function

The problem is to find $\hat{\omega}^*$ such that $\hat{\omega}^* = \arg\min_{\hat{\omega}\geq 0} L(\hat{\omega},\lambda,\epsilon)$, where

$$L(\hat{\omega},\lambda,\epsilon) = \alpha\sum_{t=1}^{n}\sum_{j=1}^{m} r_{jt}\hat{\omega}_{jt} + \frac{\beta}{2}\sum_{t=1}^{n}\sum_{j=1}^{m}(\hat{\omega}_{jt}-\omega_{jt})^2 + \sum_{t=1}^{n}\lambda_t(\sum_{j=1}^{m}\hat{\omega}_{jt}-1) \quad (6)$$
$$+\sum_{j=1}^{m}\epsilon_j(\sum_{t=0}^{n}\hat{\omega}_{jt}-1). \quad (7)$$

Here, λ_t and ϵ_j are just Lagrange multipliers. Note that (6) includes the effect of the first column of the association matrix, which represents the clutter as well as newly appearing targets. In general setting, we assume $m > n$, since most of the multitarget problem is characterized by many confusing measurements that exceed far over the number of original targets.

Let's modify (6) so that each term has equal elements:

$$L(\hat{\omega},\lambda,\epsilon) = \alpha\sum_{t=0}^{n}\sum_{j=1}^{m} r_{jt}\hat{\omega}_{jt}(1-\delta_t) \quad (8)$$
$$+\frac{\beta}{2}\{\sum_{t=0}^{n}\sum_{j=1}^{m}(\hat{\omega}_{jt}-\omega_{jt})^2 - d_{mn} - 1\}$$
$$+\sum_{t=0}^{n}\lambda_t\left\{\sum_{j=1}^{m}\hat{\omega}_{jt} - 1 - d_{mn}\delta_t\right\} \quad (9)$$
$$+\sum_{j=1}^{m}\epsilon_j(\sum_{t=0}^{n}\hat{\omega}_{jt}-1), \quad (10)$$

where $d_{mn} \triangleq m-n-1$.

(8) is a convex function which guarantees the extrema. Using the convex analysis for the local duality[6], the optimal solution can be obtained by

$$(\hat{\omega}^*,\lambda^*,\epsilon^*) = \arg\max_{\epsilon}\max_{\lambda}\min_{\hat{\omega}\geq 0} L(\hat{\omega},\lambda,\epsilon). \quad (11)$$

The necessary conditions[6] for achieving extreme in (8) are

$$\begin{cases} \nabla_{\hat{\omega}_{jt}} L(\hat{\omega},\lambda,\epsilon) = 0, \\ \nabla_{\lambda_t} L(\hat{\omega},\lambda,\epsilon) = 0, \\ \nabla_{\epsilon_j} L(\hat{\omega},\lambda,\epsilon) = 0. \end{cases} \quad (12)$$

First from $\nabla_{\hat{\omega}_{jt}} L(\hat{\omega},\lambda,\epsilon) = 0$, one obtains

$$\hat{\omega}_{jt}^* = \frac{\beta\omega_{jt} - \alpha r_{jt}(1-\delta_t) - \lambda_t - \epsilon_j}{\beta}, = -\frac{\lambda_t + f_{jt}(\epsilon_j)}{\beta}. \quad (13)$$

where $f_{jt}(\epsilon_j) \triangleq -\beta\omega_{jt} + \alpha r_{jt}(1-\delta_t) + \epsilon_j$.

Putting (13) into (8) and rearranging the related terms yields

$$L(\lambda,\epsilon) = -\frac{1}{2\beta}\sum_{t=0}^{n}\sum_{j=1}^{m}\Big[\lambda_t^2 + 2\lambda_t f_{jt}(\epsilon_j) \quad (14)$$
$$+\frac{2\beta\lambda_t}{m}(1+d_{mn}\delta_t) + f_{jt}(\epsilon_j)^2 - \beta^2\omega_{jt}^2$$
$$+\frac{\beta^2(d_{mn}+1)}{mn} + 2\beta\frac{\epsilon_j}{n+1}\Big]. \quad (15)$$

Next, from $\nabla_{\lambda_t} L(\lambda,\epsilon) = 0$, one obtains the solution:

$$\lambda_t^* = -\frac{\beta}{m}(1+d_{mn}\delta_t) - \frac{1}{m}\sum_{j=1}^{m} f_{jt}(\epsilon_j). \quad (16)$$

Substituting (16) into (14) results in

$$\epsilon_j^{\tau+1} = \bar{\epsilon}_j^{\tau} + \mu\frac{1}{n+1}\sum_{t=0}^{n}\{\beta(\omega_{jt}-\bar{\omega}_t) \quad (17)$$
$$-\alpha(1-\delta_t)(r_{jt}-\bar{r}_t)\}, \quad (18)$$

where μ is an updating constant and τ an iteration index.

Putting (13), (16), and (17) together, one obtains the final representations of the solution:

$$\left\{\begin{array}{lcl} \hat{\omega}^*_{jt} & = & \frac{\beta\omega_{jt}-\alpha r_{jt}(1-\delta_t)-\lambda_t-\epsilon_j}{\beta}, \\ \lambda^*_t & = & -\frac{\beta}{m}(1+d_{mn}\delta_t) - \frac{1}{m}\sum_{j=1}^{m} f_{jt}(\epsilon_j), \\ \epsilon^*_j & = & \bar{\epsilon}^\tau_j + \mu\Big[\frac{1}{n+1}\sum_{t=0}^{n}\{\beta(\omega_{jt}-\bar{\omega}_t) \\ & & -\alpha(1-\delta_t)(r_{jt}-\bar{r}_t)\}\Big], \end{array}\right. \quad (19)$$

where ϵ^* means optimal value of $\bar{\epsilon}$ at any scan.

3.2 Estimating Parameters

(19) contains two parameters α and β. Remember that α and β are related to the cost functions in (5).

To obtain the parameters, we consider the ML (Maximum Likelihood) estimation: Given $(\omega, \mathbf{y}, \mathbf{x})$, Θ is estimated as a maximum likelihood estimate such that

$$\Theta = \arg_\Theta P(\omega, \mathbf{y}, \mathbf{x}|\hat{\omega}, \Theta), \quad (20)$$

where $\Theta \triangleq [\alpha|\beta]^T$. Unfortunately, although the ML is unique if it exists[1], the ML estimation is computationally prohibitive due to the calculation of the partition function. Therefore, as an alternative of ML, MPL (Maximum Pseudo Likelihood) is considered. In the MPL estimation, $P(\omega, \mathbf{y}, \mathbf{x}|\hat{\omega}, \Theta)$ is represented as a product of local partition function[1]:

$$\begin{aligned} P(\omega, \mathbf{y}, \mathbf{x}|\hat{\omega}, \Theta) &= \frac{P(\mathbf{y}, \mathbf{x}|\omega, \hat{\omega}, \Theta)P(\hat{\omega}|\omega, \Theta)P(\omega|\Theta)}{P(\hat{\omega}|\Theta)}, \\ &\approx \prod_{t=0}^{n}\prod_{j=1}^{m}\frac{1}{Z_{jt}}\exp\{-[\alpha r_{jt}\hat{\omega}_{jt}\delta_t + \frac{\beta}{2}(\hat{\omega}_{jt}-\omega_{jt})^2]\}, \\ &= \prod_{t=0}^{n}\prod_{j=1}^{m}\frac{1}{Z_{jt}}\exp\{-\Theta^T\Phi(\hat{\omega}_{jt})\}, \end{aligned} \quad (21)$$

where Z_{jt} is a local partition function: $Z_{jt} = \sum_{\hat{\omega}_{jt}\in\hat{\omega}}\exp\{-\alpha r_{jt}\hat{\omega}_{jt}\delta_t - \frac{\beta}{2}(\hat{\omega}_{jt}-\omega_{jt})^2]$ and $\Phi(\hat{\omega}_{jt})$ is the cost function in our system:

$$\Phi(\hat{\omega}_{jt}) = \left[\begin{array}{c} r_{jt}\hat{\omega}_{jt}\delta_t \\ \frac{1}{2}(\hat{\omega}_{jt}-\omega_{jt})^2 \end{array}\right]. \quad (22)$$

It is proven that (21) is strictly concave with respect to Θ if and only if the parameters that comprise Θ are linearly independent with each other[1]. Therefore, Θ can be found from the gradient search method:

$$\frac{\partial\Theta}{\partial t} = -\mu\nabla_\Theta \log P(\omega, \mathbf{y}, \mathbf{x}|\hat{\omega}, \Theta). \quad (23)$$

Putting (21) into (23) arrives

$$\begin{aligned} \Theta^{\tau+1} &= \Theta^\tau - \mu\nabla_\Theta \ln P(\omega, \mathbf{y}, \mathbf{x}|\hat{\omega}, \Theta)|_{\Theta=\Theta^\tau} \\ &= \Theta^\tau - \mu\sum_{t=0}^{N}\sum_{j=1}^{N}\Big[\Phi(\hat{\omega}_{jt}) - \frac{1}{Z_{jt}} \quad (24) \\ &\quad \sum_{\hat{\omega}_{jt}\in\hat{\omega}}\Phi(\hat{\omega}_{jt})\exp\big(-\Theta^{\tau T}\Phi(\hat{\omega}_{jt})\big)\Big], \quad (25) \end{aligned}$$

where μ and τ are an updating constant and an iteration index, respectively.

4 Experimental Results

For the first three patterns, we present the results of two simulation experiments which demonstrate the performance of an optimal adaptive data association scheme for multi target tracking. In the first experiment, we consider data association capability using the pattern shown in Fig. 1, 2, and 3. we present some results of the experiments comparing the performance of the proposed MAP estimate adaptive data association(MAPODA) with that of the Hopfield Neural PDA(HNPDA) of Sengupta and Iltis [5]. Like as HNPDA, the MAPODA has a good structure for a parallel hardware, currently the algorithm is simulated by a serial computer. Table 2 summarizes the rms position and velocity errors for each targets. The performance of the MAPODA is superior to that of HNPDA. The rms error of HNPDA for the target 8 has not been included since it loses track during the simulation.

Table 1: Initial Positions and Velocities of 10 targets.

Tgt	Pos. (km)		Vel. (km/s)	
i	x	y	$\dot{x}$	$\dot{y}$
1	-4.0	1.0	0.2	-0.05
2	-4.0	1.0	0.2	0.05
3	-6.0	-5.0	0.0	0.3
4	-5.5	-5.0	0.0	0.3
5	8.0	-7.0	-0.4	0.0
6	-8.0	-8.0	0.4	0.0
7	-5.0	9.0	0.25	0.0
8	-5.0	8.9	0.25	0.0
9	0.5	-3.0	0.1	0.2
10	9.0	-9.0	0.01	0.2

Table 2: rms Errors in the case of ten targets

Tgt	Pos. error		Vel. error		Track rate (%)	
i	HN	MAP	HN	MAP	HN	MAP
1	0.64	0.42	0.69	0.18	95	100
2	0.64	0.42	0.42	0.17	95	100
3	0.78	0.42	0.22	0.18	100	100
4	0.60	0.43	0.21	0.18	93	100
5	0.59	0.45	0.67	0.18	85	100
6	0.57	0.45	0.20	0.18	100	100
7	0.57	0.42	0.31	0.49	90	100
8	-	2.95	-	1.18	0	53
9	0.62	0.44	0.27	0.21	80	98
10	0.59	0.45	0.21.	0.18	100	98

5 Conclusions

The purpose of this paper was to explore adaptive data association method as a tool for applying the multi-target tracking. It was shown that it always yields consistent data association, in contrast to the Hopfield Neural PDA, and that these associated data measurements are very effective for multi-target filter. Although the MAPODA find the convergence recursively, the MAPODA is a general method about the solving the data association problems in multi-target tracking. A feature of our algorithm is that it requires only $O(mn)$ storage, where m is the number of candidate measurement associations and n is the number of trajectories, compared to some branch and bound techniques, where the memory requirements grow exponentially with the number of targets. The experimental results show that the MAPODA is superior to the HN-PDA in terms of both rms errors and track maintenance rate. This algorithm has several applications and can be effectively used in radar target tracking system.

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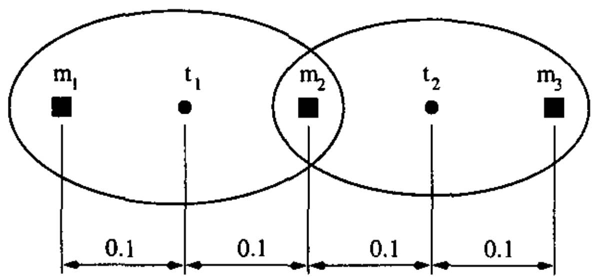

Figure 1: Data association test for equal distance between gates

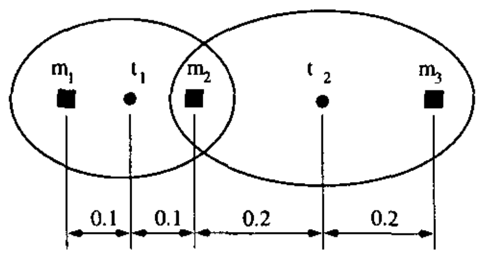

Figure 2: Data association test for double distance between gates

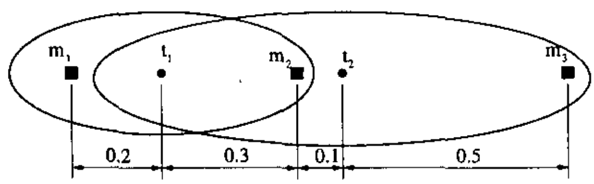

Figure 3: Data association test for random distance between gates