LMI를 이용한 연속/이산 불확실성 시간지연 시스템의 견실 신뢰 H^{∞} 제어

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Robust Reliable H^{∞} Control of Continuous/Discrete Uncertain Time Delay Systems: LMI Approach

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Abstract

In this paper, we present robust reliable H^{∞} controller design methods of continuous and discrete uncertain time delay systems through LMI(linear matrix inequality) approach, respectively. Also the existence conditions of state feedback control are proposed. Using some changes of variables and Schur complements, the obtained sufficient conditions are transformed into LMI form. We show the validity of the proposed method through numerical examples.

1. Introduction

The robust H^{∞} controller design method of parameter uncertain time delay systems has attracted the attention of many control researchers [1,2,3,4,5]. Recently Seo *et al.*[6] and Veillet *et al.*[7] consider the problem of reliable H^{∞} control design. Especially, Seo *et al.*[6] considered the problem of robust and reliable H^{∞} control design for linear uncertain systems with time-varying norm-bounded parameter uncertainty in the state matrix and also with actuator failures among a prespecified subset of actuators. However they did not deal with time delay. Gu *et al.*[8] and Wang[9] treated the problem of robust H^{∞} reliable control for linear state delayed systems with parameter uncertainty through algebraic Riccati equation approach. Their works were considered in continuous time case. Also, LMI(linear matrix inequality) Toolbox by convex optimization algorithms has been developed. Therefore our objective is to find static state feedback controller in continuous time case and discrete time case through LMI technique, respectively.

In this paper, we present state feedback controller satisfying quadratic stability with H^{∞}-norm bound for all admissible uncertainties and all actuator failures occurred within the prespecified subset in continuous and discrete time case. The sufficient conditions and the controller design method are proposed. Also, examples are demonstrated.

2. Main results

Consider the system described by uncertain time delay systems

$$
\begin{aligned}
\delta x(t) &= [A+\Delta A(t)]x(t)+[A_d+\Delta A_d(t)]x(t-d) \\
&\quad +[B_u+\Delta B_u(t)]u(t)+[B_w+\Delta B_w(t)]w(t) \\
z(t) &= [C+\Delta C(t)]x(t)+[C_d+\Delta C_d(t)]x(t-d) \\
&\quad +[D_u+\Delta D_u(t)]u(t)+[D_w+\Delta D_w(t)]w(t) \\
x(t) &= 0, \quad t\le 0
\end{aligned}
\tag{1}
$$

where $x(t)\in\mathbf{R}^n$ is the state, $u(t)\in\mathbf{R}^m$ is the control input, $w(t)\in\mathbf{R}^p$ is the square integrable disturbance input vector and $z(t)\in\mathbf{R}^r$ is the controlled output. All matrices have appropriate dimensions and we assume that all states are measurable. In here,

$$
\delta x(t)=\begin{cases} \dot{x}(t) : \text{(CT)} \\ x(t+1) : \text{(DT)} \end{cases}
\tag{2}
$$

where CT and DT mean continuous and discrete time, respectively. And the parameter uncertainties are defined as follows:

$$
\begin{aligned}
&\begin{bmatrix} \Delta A(t) & \Delta B_u(t) & \Delta B_w(t) & \Delta A_d(t) \\ \Delta C(t) & \Delta D_u(t) & \Delta D_w(t) & \Delta C_d(t) \end{bmatrix} \\
&=\begin{bmatrix} H_x \\ H_z \end{bmatrix} F(t)[\,E_x \;\; E_u \;\; E_w \;\; E_d]
\end{aligned}
\tag{3}
$$

where H_x, H_z, E_x, E_u, E_w, E_d are known real matrices and $F(t)$ is an unknown matrix function which is bounded by

$$F(t) \in \Omega := \{F(t):\ F(t)^T F(t) \le I, \text{ the elements of } F(t) \text{ are Lebesgue measurable}\}. \tag{4}$$

Now, we classify actuators of the system (1) into two groups similar to the works[6,7]. One is a selected subset of actuators susceptible to failures, which is denoted by $\Omega \subseteq \{1, 2, \ldots, m\}$. This set of actuator is redundant in terms of the stabilization of the system, while it may contribute to and is necessary for improving control performance. The other is a set of actuators robust to failures, which is denoted by $\overline{\Omega} \subseteq \{1, 2, \ldots, m\} - \Omega$. We assume these actuators never fail and also they are required in order to stabilize a given system. Introduce a decomposition

$$B_u = B_\Omega + B_{\overline{\Omega}} \tag{5}$$

where B_Ω and $B_{\overline{\Omega}}$ are formed from B_u by zeroing out columns. In the following, we let $\alpha \subseteq \Omega$ denote a particular subset of susceptible actuators that actually fail and adopt the following notation

$$B_u = B_\alpha + B_{\overline{\alpha}} \tag{6}$$

where B_α and $B_{\overline{\alpha}}$ have meanings analogous to those of B_Ω and $B_{\overline{\Omega}}$, respectively. From definitions of B_α, $B_{\overline{\alpha}}$, B_Ω, and $B_{\overline{\Omega}}$, we can obtain the following facts

$$\begin{aligned} B_\Omega B_\Omega^T &= B_\alpha B_\alpha^T + B_{\Omega-\alpha} B_{\Omega-\alpha}^T \\ B_{\overline{\Omega}} B_{\overline{\Omega}}^T &= B_{\overline{\alpha}} B_{\overline{\alpha}}^T + B_{\Omega-\alpha} B_{\Omega-\alpha}^T. \end{aligned} \tag{7}$$

Our objective is to find a memoryless state feedback controller

$$u(t) = Kx(t) \tag{8}$$

that stabilizes the linear time delay system (1) with a given H^∞ norm constraint on disturbance attenuation, for all admissible uncertainties and all actuators failures occurred within the prespecified subset Ω.

Lemma 1. For given $\gamma > 0$ and $\lambda > 0$, the system (1) is QSH^∞-AF(quadratically stabilizable with H^∞ norm bound for all admissible uncertainties and all actuator failures occurred within the subset Ω) by state feedback control (8) if and only if the system

$$\begin{aligned} \delta x(t) &= Ax(t) + A_d x(t-d) + B_{\overline{\Omega}} u(t) \\ &\quad + [\,B_w \ \ \gamma\lambda H_x \ \ B_\Omega\,]\, \widehat{w}(t) \\ \hat{z}(t) &= \begin{bmatrix} C \\ \frac{1}{\lambda} E_x \end{bmatrix} x(t) + \begin{bmatrix} C_d \\ \frac{1}{\lambda} E_d \end{bmatrix} x(t-d) \\ &\quad + \begin{bmatrix} D_{\overline{\Omega}} \\ \frac{1}{\lambda} E_u \end{bmatrix} u(t) + \begin{bmatrix} D_w & \gamma\lambda H_z & D_\Omega \\ \frac{1}{\lambda} E_w & 0 & 0 \end{bmatrix} \widehat{w}(t) \end{aligned} \tag{9}$$

is QSH^∞-AF for the same state feedback control (8). Therefore the original system (1) can be transformed into the system without parameter uncertainties and particular subset of the susceptible actuators.

Proof. Omitted due to space limit. ■

For simplicity of manipulation, rewrite the system (9) as follows:

$$\begin{aligned} \delta x(t) &= Ax(t) + A_d x(t-d) + Bu(t) + \widehat{B}\widehat{w}(t) \\ \hat{z}(t) &= \widehat{C}x(t) + \widehat{C_d}x(t-d) + D_1 u(t) + D_2 \widehat{w}(t) \end{aligned} \tag{10}$$

where

$$\begin{aligned} &B = B_{\overline{\Omega}},\ \ \widehat{B} = [B_w \ \ \gamma\lambda H_x \ \ B_\Omega],\ \ \widehat{C} = \begin{bmatrix} C \\ \frac{1}{\lambda} E_x \end{bmatrix}, \\ &\widehat{C_d} = \begin{bmatrix} C_d \\ \frac{1}{\lambda} E_d \end{bmatrix},\ D_1 = \begin{bmatrix} D_{\overline{\Omega}} \\ \frac{1}{\lambda} E_u \end{bmatrix},\ D_2 = \begin{bmatrix} D_w & \gamma\lambda H_z & D_\Omega \\ \frac{1}{\lambda} E_w & 0 & 0 \end{bmatrix} \\ &\hat{z}(t) = \begin{bmatrix} z(t) \\ \tilde{z}(t) \end{bmatrix},\ \ \widehat{w}(t) = \begin{bmatrix} w(t) \\ \widetilde{w}(t) \\ v(t) \end{bmatrix} \end{aligned} \tag{11}$$

Here $\widetilde{w}(t)$ and $\tilde{z}(t)$ are additional input and output, and $v(t)$ is the output of faulty actuators. When we apply the control (8) to the system (10), the closed loop system from $\widehat{w}(t)$ to $\hat{z}(t)$ is given by

$$\begin{aligned} \delta x(t) &= A_K x(t) + A_d x(t-d) + \widehat{B}\widehat{w}(t) \\ \hat{z}(t) &= \widehat{C_K}x(t) + \widehat{C_d}x(t-d) + D_2 \widehat{w}(t) \end{aligned} \tag{12}$$

where $A_K = A + BK$ and $\widehat{C_K} = \widehat{C} + D_1 K$.

Lemma 2. For given $\gamma > 0$ and $\lambda > 0$, the system (1) is QSH^∞-AF with the controller (8) if there exist positive definite matrices P and R such that

(i) CT case

$$\begin{bmatrix} A_K^TP+PA_K+R & PA_d & P\hat{B} & \widehat{C}_K^T \\ * & -R & 0 & \widehat{C}_d^T \\ * & * & -\gamma^2 I & D_2^T \\ * & * & * & -I \end{bmatrix} < 0 \qquad (13)$$

(ii) DT case

$$\begin{bmatrix} -P^{-1} & A_K & A_d & \hat{B} & 0 \\ * & -P+R & 0 & 0 & \widehat{C}_K^T \\ * & * & -R & 0 & \widehat{C}_d^T \\ * & * & * & -\gamma^2 I & D_2^T \\ * & * & * & * & -I \end{bmatrix} < 0 \qquad (14)$$

hold for time delay and all actuators failures occurred within the subset Ω. Here $*$ mean symmetric terms.

Proof. From the Lyapunov functional

$$V(x(t)) := \begin{cases} x(t)^TPx(t) + \int_{t-d}^{t} x(\tau)^TRx(\tau)d\tau : \ \mathrm{CT} \\ x(t)^TPx(t) + \sum_{i=t-d}^{t-1} x(i)^TRx(i) : \ \mathrm{DT} \end{cases} \qquad (15)$$

and the performance measure

$$J := \begin{cases} \int_0^{\infty} [\hat{z}(t)^T\hat{z}(t) - \gamma^2 \hat{w}(t)^T\hat{w}(t)]dt : \ \mathrm{CT} \\ \sum_{t=0}^{\infty} [\hat{z}(t)^T\hat{z}(t) - \gamma^2 \hat{w}(t)^T\hat{w}(t)] : \ \mathrm{DT} \end{cases} \qquad (16)$$

the sufficient conditions (13) and (14) can be derived. ■

However the conditions (13) and (14) is not an LMI form in terms of each finding variable P, R, K. It is shown that the (13) and (14) are transformed into LMI form in the following theorem.

Theorem 1. Consider closed loop system (12). For given $\gamma>0$ and $\lambda>0$, if there exist a matrix M and positive definite matrices Q, S such that

(i) CT case

$$\begin{bmatrix} QA^T+AQ+M^TB^T+BM+A_dSA_d^T & \hat{B} & M^TD_1^T+Q\,\hat{C}^T+A_dS\widehat{C}_d^T & Q \\ * & -\gamma^2 I & D_2^T & 0 \\ * & * & -I+\widehat{C}_dS\widehat{C}_d^T & 0 \\ * & * & * & -S \end{bmatrix} < 0 \qquad (17)$$

(ii) DT case

$$\begin{bmatrix} -Q+A_dSA_d^T & AQ+BM & \hat{B} & A_dS\widehat{C}_d^T & 0 \\ * & -Q & 0 & Q\widehat{C}^T+M^TD_1^T & Q \\ * & * & -\gamma^2 I & D_2^T & 0 \\ * & * & * & -I+\widehat{C}_dS\widehat{C}_d^T & 0 \\ * & * & * & * & -S \end{bmatrix} < 0 \qquad (18)$$

holds for time delay and all actuators failures occurred within the subset Ω.

Proof. Using Schur complements and the changes of variables

$$M = KP^{-1}, \ Q = P^{-1}, \ S = R^{-1}, \qquad (19)$$

the obtained sufficient conditions (13) and (14) are changed to (17) and (18), respectively. ■

Remark 1. The (17) and (18) are LMI form in terms of changed variables. Therefore robust reliable H^{∞} state feedback controller K can be calculated from the $M=KP^{-1}$ after finding the LMI solutions Q, M, and S from the (17) and (18). Using LMI toolbox[11], the solutions can be easily obtained at a time.

Remark 2. If the value of γ and λ are given, the inequalities (17) and (18) are LMI form. However, the existence of all solutions depends on the value of λ. The area of future research would be to develop the algorithm obtaining the solutions independently of λ.

3. Numerical Example

Consider the uncertain time delay system of the same example in [9] with

$$A = \begin{bmatrix} 4 & 0.02 & -0.1 \\ -0.3 & 3 & -0.2 \\ 0.3 & -0.1 & 2 \end{bmatrix},$$

$$A_d = \begin{bmatrix} -0.2 & 0.05 & 0.01 \\ 0 & -0.3 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$B_u = \begin{bmatrix} 5 & 0.2 & 0 \\ 0 & 3 & 0.1 \\ 0.1 & 0 & 0.03 \end{bmatrix}, \ B_w = \begin{bmatrix} 0.01 & 0 \\ 0.1 & 0 \\ 0 & 0.1 \end{bmatrix},$$

$$C = \begin{bmatrix} 0.2 & 0 & 0 \\ 0 & 0 & 0.2 \end{bmatrix}$$

$$H_x=\begin{bmatrix}0.08 & 0 & 0\\ 0 & 0.08 & 0\\ 0 & 0 & 0\end{bmatrix},\quad E_x=\begin{bmatrix}0.04 & 0 & 0\\ 0 & 0.02 & 0\\ 0 & 0 & 0.04\end{bmatrix},$$

$$E_d=\begin{bmatrix}0.02 & 0 & 0\\ 0 & 0.02 & 0\\ 0 & 0 & 0.02\end{bmatrix},$$

and other matrices are zero matrices with proper dimensions. For simulation, we take $\gamma=3$, $\lambda=1$, and $\Omega=\{3\}$. we have

$$B_{\bar{\Omega}}=\begin{bmatrix}5 & 0.2 & 0\\ 0 & 3 & 0\\ 0.1 & 0 & 0\end{bmatrix},\quad B_{\Omega}=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0.1\\ 0 & 0 & 0.03\end{bmatrix}$$

In the case of continuous time case, all solutions and state feedback gain are

$$Q=\begin{bmatrix}25.5851 & -0.8625 & -0.8744\\ -0.8625 & 34.7574 & 0.7573\\ -0.8744 & 0.7573 & 0.0723\end{bmatrix},$$

$$S=\begin{bmatrix}79.1138 & -0.0509 & 1.6534\\ -0.0509 & 80.1428 & -0.7005\\ 1.6534 & -0.7005 & 87.1236\end{bmatrix},$$

$$M=\begin{bmatrix}-31.7756 & 4.9475 & 0.0007\\ 0.2205 & -51.0113 & -0.3589\\ 0 & 0 & 0\end{bmatrix},$$

$$K=\begin{bmatrix}-2.6781 & 1.0132 & -43.0195\\ 0.8449 & -2.0233 & 26.4649\\ 0 & 0 & 0\end{bmatrix}.$$

The obtained continuous time state feedback control guarantees QSH^{∞}-AF. Similarly to the continuous time case, all solutions and discrete time state feedback gain is obtained as follows:

$$Q=\begin{bmatrix}17.3768 & 8.7877 & -1.4222\\ 8.7877 & 304.2086 & 13.6712\\ -1.4222 & 13.6712 & 1.0808\end{bmatrix},$$

$$S=\begin{bmatrix}253.9382 & 231.4889 & 26.5253\\ 231.4889 & 624.3931 & -4.4061\\ 26.5253 & -4.4061 & 837.7026\end{bmatrix},$$

$$M=\begin{bmatrix}-13.6346 & 4.2473 & 1.4096\\ -5.8558 & -291.9699 & -11.2058\\ 0 & 0 & 0\end{bmatrix},$$

$$K=\begin{bmatrix}-0.8970 & 0.0795 & -0.8819\\ 0.9999 & -1.3483 & 8.0030\\ 0 & 0 & 0\end{bmatrix}$$

Also the obtained discrete time state feedback controller guarantees QSH^{∞}-AF of the closed loop system.

4. Conclusion

We presented controller design algorithms of continuous and discrete uncertain time delay systems through LMI approach. From the Lyapunov functions and performance measures, the existence conditions of state feedback controller were given. Also, the obtained sufficient conditions were transformed into LMI form using some changes of variables and Schur complements.

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