

# 최소평균자승에러 알고리즘을 이용한 non-parametric 검파기 설계

공 형 윤

울산대학교 전기 전자 및 자동화공학부 (전자공학과)

Tel : 052-259-2194, Fax : 052-259-1685

e-mail : hkong@uou.ulsan.ac.kr

## Design of Non-Parametric Detectors with MMSE

Hyung Yun Kong

School of Electrical, Electronic & Automation

e-mail : hkong@uou.ulsan.ac.kr

### (Abstract)

*A class of non-parametric detectors based on quantized  $m$ -dimensional noise sample space is introduced. Due to assuming the nongaussian noise as a channel model, it is not easy to design the detector through estimating the unknown functional form of noise; instead equiprobably partitioning  $m$ -dimensional noise into a finite number of regions, using a VQ and quantiles obtained by RMSA algorithm is used in this paper to design detectors. To show the comparison of performance between single sample detector and system suggested here, Monte-Carlo simulations were used. The effect of signal pulse shape on the receiver performance is analyzed too.*

### I. Introduction

In many radio communication environments frequency selectivity caused by multipath propagation degrades the performance of digital communication channel by causing ISI which imposes limitations of the data transmission rate [1],[3]. Due to unknown characteristics of multipath channel in wireless communication, the objective of the receiver is to

identify the channel characteristics in order to reduce the ISI components and additive noise. Most often the functional form of the noise distribution is assumed to be known, with only a finite number of unknown parameters. Much research has been concentrated on these "parametric" detection problems, but in many practical digital communication systems, the functional form of the noise distribution is unknown, and the detection problem is then "nonparametric" [4]-[5]. The main advantage of the nonparametric detector is its robustness, i.e., the detector performs relatively well in a broad class of noise environments. In this paper, a new class of nonparametric detectors are proposed. Assuming that the detected symbols are correct then we can observe the pure noise samples during the training and transmitting mode, which is equiprobably partitioned into finite number of independent regions based on vector quantization (VQ) and quantiles which is estimated by robbins monro stochastic approximation (RMSA) [2]. Based on the equiprobably partitioned noise samples and hypothesis, the received signals will be grouped and detected through the nonparametric detectors which have estimated threshold. Fading severely degrades the BER performance. Diversity reception is one of the most powerful techniques to combat fading. In postdetection diversity, diversity outputs are weighted according to

each branch's channel conditions before combining so that the contribution of the weaker signal branch is minimized. In this paper, we take a postdetection phase combining necessary in predetection combining is not required and because switching noise due to an abrupt phase is not produced.

## II. Structure of Non-Parametric Detectors

We can observe the pure noise samples  $n_{n-k}$ ,  $k=1,2,3,\dots,m$  during the training mode based on assuming that the detected symbols are correct. An  $m$ -dimensional noise sample space is equiprobably partitioned into a finite number of regions, using a VQ and quantiles obtained by RMSA algorithm in the training mode. Having the past noise samples partitioned for each hypothesis, the received signal is subsequently partitioned(see section III). Noise sample  $n_{n-m}$  is partitioned, continuing the process until sample  $n_{n-1}$  is partitioned. Using the RMSA procedure, we will get the estimated value of mean  $\xi_{i,k}$  under the each hypothesis and partitioned noise sample

$$\xi_{i,k} = E\{y_n | VQ(w_{n-1}, w_{n-2}, \dots, w_{n-r}) \in \rho, H_i\} \quad (1)$$

where  $i=1,2$  (for hypothesis) and  $k$  : level of partition. The estimated threshold under each partition  $\rho$  and each hypothesis can be expressed as

$$\zeta_{j,k} = \frac{\xi_{1,k} + \xi_{0,k}}{2}$$

where  $j=1,2$  represent the number of channel path. The average value of estimated threshold can be

$$th_k = \frac{\zeta_{1,k} + \zeta_{2,k}}{2} \quad (2)$$

This system use the minimum mean square error(MMSE) equalizer with respect to the estimated mean not the transmitted information and the outputs of equalizer pass through non-parametric detectors having average value of the thresholds. The conditional MSE can be expressed by

$$MSE = E\left[\xi_{a,k} - (\beta_{qk} + \gamma_{qk}y_n + \sum_{j=1}^m \alpha_{qk,j}n_{n-j})\right]^2 | S_k,$$

$$l=1,2,\dots,m \text{ and } a=0,1 \quad (3)$$

Using the orthogonality principle and solve the linear

equations, we will generate the coefficients  $\alpha_{qk}$ ,  $\beta_{qk}$ ,  $\gamma_{qk}$ . The output of the equalizer is

$$\hat{\xi}_k^* = \sum_{j=1}^m \chi_{S_j}[y_n, n_{n-1}, \dots, n_{n-m}] [\beta_{qk} + \gamma_{qk} + \sum_{j=1}^m \alpha_{qk,j}n_{n-j}] \quad (4)$$

## III. RMSA Algorithm for Partitioning & Calculating Partition Moments in Sample Space

In this section we consider the method of partitioning the sample space and evaluating the partition moments used in this paper. Using the training mode, we estimate quantiles which are used in VQ procedure by applying RMSA procedure and then partition the observation space into  $m$  regions using VQ. For each partition, the partition moments which are required to calculate the coefficients of MMSE equalizer are estimated by RMSA. We look for an estimate the quantile  $v$  that satisfies the following conditions based on assuming the  $r_n$  as sample space which has probability distributed function (PDF) satisfying the conditions:

$$p_r [r_n < v] = F(v) = p, \quad p : \text{given value} \quad (5)$$

The proposed recursive estimation is for  $n=0,1,2,\dots$

$$e_{n+1} = e_n - q_n [u(e_n - r_{n+1}) - p] \quad (6)$$

where  $u(r)$  is Heaviside step function and  $q_n$  is the sequence of positive numbers having the following conditions :

$$q_n \downarrow 0, \quad n$$

$$\Rightarrow \infty; \quad \sum_{n=0}^{\infty} q_n = \infty; \quad \sum_{n=0}^{\infty} q_n^2 < \infty \quad (7)$$

Now, we are considering of establishing a recursive estimation scheme for estimating the partition moment defined by

$$m_i(a,b) = E[\chi_{(a,b)}(r)r^i], \quad i=1,2,\dots \quad (8)$$

where  $\chi_{(a,b)}$  is the indicator function of the interval  $(a,b]$ , and  $\chi_{(a,b)}r^i$  will also be denoted by  $H$ . From the available sequence  $[r_n]$ ,  $n=0,1,2,\dots$  the proposed partition moments are

$$\mu_{n+1} = \mu_n + q_n [H_{n+1} - \mu_n] \quad (9)$$

where  $H_{n+1} = \chi_{(a,b)}(\gamma) r^i$  and  $q_n$  satisfies the previous conditions described. Let's consider the simple example for the MMSE estimation of the transmitted signal,  $a_n$ , from  $(m+1)$  present and past received signals. We replace the conditioning vector  $(r_n, r_{n-1}, \dots, r_{n-m})$  by the output of a VQ. The VQ maps from  $S^{m+1}$  to  $\mathcal{Q} = \rho_1, \rho_2, \dots, \rho_l$  where  $\mathcal{Q}$  is a finite set with elements called pseudo-states of  $(r_n, r_{n-1}, \dots, r_{n-m})$ . Based on  $SNR (dB) > 0$ , we can observe the pure noise samples during the training and transmitting mode and express the MMSE estimation of  $a_n$ ,

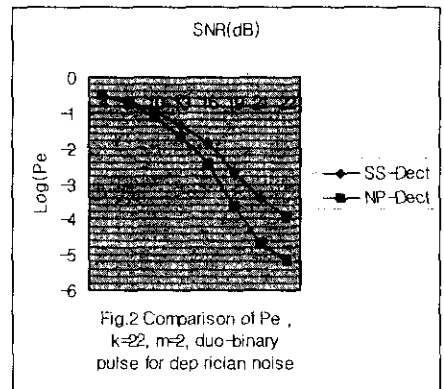
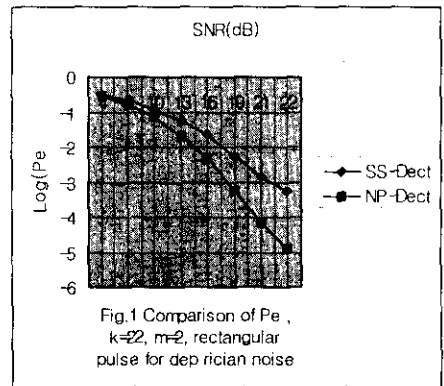
$$MSE = E \left[ \left| a_n - (\beta_{ak} + \gamma_{ak} r_n + \sum_{j=1}^m a_{ak, n-j}) \right|^2 \mid S_n \right] \quad (10)$$

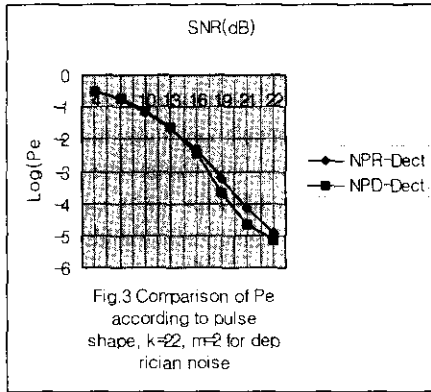
In here, we use the weight proportional to the second power of the  $R_k(nT)$  which is the output envelop of the equalizer.

#### IV. Discussion of Results & Conclusion

Here we use the dependent noise model generated by the auto-regressive (AR) model as a noise component in a rician fading channel and use the 6<sup>th</sup> order Chebychev filter to generate ISI and we choose a component having a rayleigh amplitude distribution as a fading component. Through the computer simulation, we demonstrate that the performance of system are much better than single sample detector for dependent rician noise case for signal to noise ratio ( $SNR(dB) = 4, 7, 10, 13, 16, 19, 21, 22$ ) in terms of  $Log_{10}(Pe)$  as shown in Fig.1-Fig.2 under using the different signaling pulse shape, i.e., rectangular & duo-binary pulse. Fig.3 shows that the comparison of  $Pe$  between NPD-Dect(duo-binary pulse case) and NPR-Dect(rectangular pulse), the effects of ISI was smaller for duo-binary pulse than that for the rectangular pulse so the performance results for the duo-binary pulse point to a improvement as compared to that of a rectangular pulse under dependent rician noise. For this case we use 625 as total partition number, we can expect that as the number of partition increases the system performance will improve, until a saturation point is reached. In

this paper we suggest a new system having the non-parametric detectors with non-linear equalizer to suppress the ISI and additive nongaussian noise causing errors in the receiver and present the performance of the system by using the Monte-Carlo simulation method. Due to design the system under assuming a nongaussian noise environment, it will be able to apply to many applications in wireless communication.





## V. References

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