Leaky-Wave Radiation from a Coaxial Waveguide with Periodic Circumferential Slots

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Abstract The analysis of a coaxial cable with periodic slots as a leaky-wave antenna is considered. The mode matching method and method of moments with Galerkin testing procedure is then used to obtain the determinantal equation for the unknown leaky-wave propagating constant. Both technigues allow the fields to be obtained in any region, and the radiation characteristics of the structure. By resorting to a suitable numerical techniques, it is possible to calculate the leaky-wave propagating constants and the radiation patterns. Numerical results demonstrate the validity of the technique.

I. INTRODUCTION

Surface waves on a coaxial cable with periodic slots are considered by Tran [1] for two cases : the cable in free space and centered in a cylindrical tunnel. J. R.Wait [2] develops the theory for a plane wave incident on a slotted cable in free space, and Hill and Wait [3] present numerical data for surface waves on a lossless slotted cable in free space. J. H.Richmond [4] present the theory for the slotted coaxial cable buried in an eccentric cylindrical earth, but no numerical solution has been given, at least to the author's knowledge, to analyze the structure in terms of leaky-waves. In this paper, we will focus on a type of leaky-wave antenna, see Fig.1. The structure under consideration has been a subject of the [1] - [5]. We choose the same rudimentary model as before [2]. The basic cylindrical structure is again shown in Fig.1.and Fig.2. In the present work, a simple and accurate mode expansion formulation that takes into account higher order space harmonics is applied to the analysis of the infinite extent structure with periodic slots of finite and zero thickness. For this case, by applying the boundary conditions and MOM method, the fields in each region are expressed as a summation of mode and space harmonics and a homogeneous matrix equation is obtained. The eigen value problem is solved for the leakage-constant and phase-constant of the space harmonic of order zero by using the Newton's method [6]. The proposed techniques are accurate when a sufficient number of terms is retained. The influences of the slot width, slot period, slot angle and radiation characeristics(leakage constant, phase constant, and radiation pattern) are investigated. We introduced the first kind of Chebyshev polynomial as a basis function for the magnetic current in the slot region. Also, the mode expansion technique, for zero thickness slot case can be applied to more complicated geometries including dielectric coated structures. Our results can be compared with those of the infinite extent structure with circulary symmetric and dielectric coated case [7].

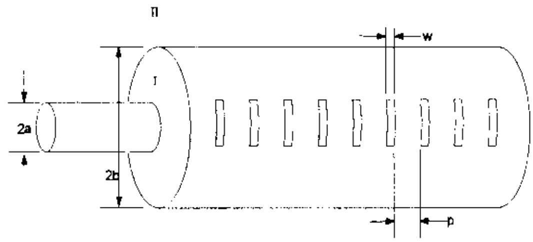

Fig. 1. Side view of coaxial cable with periodic rectangular slots.

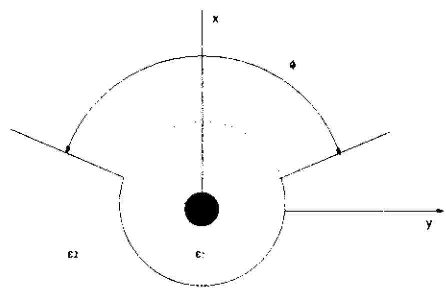

Fig. 2. Cross section of coaxial cable with periodic rectangular slots.

II. FORMULATION

The geometry of the problem is sketched in Fig.1 and 2. As usual in problems of this type, we express the fields in terms of electric and magnetic vector potentials that have only z components, F_z and A_z, respectively.

$$A_z(\rho,\phi z)=\sum_n\sum_m A_{mn}B_n^a(k_{\rho m}\rho)\exp(-jn\phi)\exp(-j\beta_m z) \tag{1}$$

$$F_z(\rho,\phi z)=\sum_n\sum_m F_{mn}B_n^f(k_{\rho m}\rho)\exp(-jn\phi)\exp(-j\beta_m z) \tag{2}$$

where $\beta_m=\beta o+2\pi m/p, k_{\rho m}=\sqrt{k^2-(\beta_m)^2}, \beta_o=\beta-j\alpha$
The summations here over m and n extend from $-\infty+\infty$

through all integers including zero. Our objective is to determine the leaky-wave propagating constant $\beta - j\alpha$
In region I ($a \leq \rho \leq b$)

$$E_\phi^{(I)}(\rho,\phi,z) = \frac{-1}{j\omega\mu\epsilon_0\epsilon_{r1}\rho}\sum_n\sum_m A_{mn}^{(I)} n\beta_m B_n^{a(I)}(k_{\rho m}^{(1)}\rho)\exp(-jn\phi)\exp(-j\beta_m z) + \frac{1}{\epsilon_0\epsilon_r 1}\sum_n\sum_m F_{mn}^{(I)} k_{\rho m}^{(1)} B_n^{f(I)'}(k_{\rho m}^{(1)}\rho)\exp(-jn\phi)\exp(-j\beta_m z) \quad (3)$$

$$E_z^{(I)}(\rho,\phi,z) = \frac{1}{j\omega\mu\epsilon_0\epsilon_{r1}}\sum_n\sum_m A_{mn}^{(I)} k_{\rho m}^{(1)^2} B_n^{a(I)}(k_{\rho m}^{(1)}\rho)\exp(-jn\phi)\exp(-j\beta_m z) \quad (4)$$

$$H_\phi^{(I)}(\rho,\phi,z) = \frac{-1}{\mu}\sum_n\sum_m A_{mn}^{(I)} k_{\rho m}^{(1)} B_n^{a(I)'}(k_{\rho m}^{(1)}\rho)\exp(-jn\phi)\exp(-j\beta_m z) + \frac{-1}{j\omega\mu\epsilon_0\epsilon_r 1\rho}\sum_n\sum_m F_{mn}^{(I)} n\beta_m B_n^{f(I)}(k_{\rho m}^{(1)}\rho)\exp(-jn\phi)\exp(-j\beta_m z) \quad (5)$$

$$H_z^{(I)}(\rho,\phi,z) = \frac{1}{j\omega\mu\epsilon_0\epsilon_{r1}}\sum_n\sum_m F_{mn}^{(I)} k_{\rho m}^{(1)^2} B_n^{f(I)}(k_{\rho m}^{(1)}\rho)\exp(-jn\phi)\exp(-j\beta_m z) \quad (6)$$

The radial functions are expressed in terms of Hankel functions and their derivatives as follows :

$$B_n^{a(I)}(k_{\rho m}^{(1)}\rho) = H_n^{(2)}(k_{\rho m}^{(1)}\rho) - \frac{H_n^{(2)}(k_{\rho m}^{(1)}a)}{H_n^{(1)}(k_{\rho m}^{(1)}a)} H_n^{(1)}(k_{\rho m}^{(1)}\rho)$$
$$B_n^{f(I)}(k_{\rho m}^{(1)}\rho) = H_n^{(2)}(k_{\rho m}^{(1)}\rho) - \frac{H_n^{(2)'}(k_{\rho m}^{(1)}a)}{H_n^{(1)'}(k_{\rho m}^{(1)}a)} H_n^{(1)}(k_{\rho m}^{(1)}\rho) \quad (7)$$

These forms permit the satisfaction of the boundary conditon $E_\phi = E_z = 0$ at $\rho = a$

In region II ($\rho \geq b$)

$$B_n^{a(II)}(k_{\rho m}^{(2)}\rho) = H_n^{(2)}(k_{\rho m}^{(2)}\rho) \quad (8)$$
$$B_n^{f(II)}(k_{\rho m}^{(2)}\rho) = H_n^{(2)}(k_{\rho m}^{(2)}\rho) \quad (9)$$

$$E_\phi^{(II)}(\rho,\phi,z) = \frac{-1}{j\omega\mu\epsilon_0\epsilon_{r2}\rho}\sum_n\sum_m A_{mn}^{(II)} n\beta_m H_n^{(2)}(k_{\rho m}^{(2)}\rho)\exp(-jn\phi)\exp(-j\beta_m z) + \frac{1}{\epsilon_0\epsilon_{r2}}\sum_n\sum_m F_{mn}^{(II)} k_{\rho m}^{(2)} H_n^{(2)'}(k_{\rho m}^{(2)}\rho)\exp(-jn\phi)\exp(-j\beta_m z) \quad (10)$$

$$E_z^{(II)}(\rho,\phi,z) = \frac{1}{j\omega\mu\epsilon_0\epsilon_{r2}}\sum_n\sum_m A_{mn}^{(II)} k_{\rho m}^{(2)^2} H_n^{(2)}(k_{\rho m}^{(2)}\rho)\exp(-jn\phi)\exp(-j\beta_m z) \quad (11)$$

$$H_\phi^{(II)}(\rho,\phi,z) = \frac{-1}{\mu}\sum_n\sum_m A_{mn}^{(II)} k_{\rho m}^{(2)} H_n^{(2)'}(k_{\rho m}^{(2)}\rho)\exp(-jn\phi)\exp(-j\beta_m z) + \frac{-1}{j\omega\mu\epsilon_0\epsilon_{r2}\rho}\sum_n\sum_m F_{mn}^{(II)} n\beta_m H_n^{(2)}(k_{\rho m}^{(2)}\rho)\exp(-jn\phi)\exp(-j\beta_m z) \quad (12)$$

$$H_z^{(II)}(\rho,\phi,z) = \frac{1}{j\omega\mu\epsilon_0\epsilon_{r2}}\sum_n\sum_m F_{mn}^{(II)} k_{\rho m}^{(2)^2} H_n^{(2)}(k_{\rho m}^{(2)}\rho)\exp(-jn\phi)\exp(-j\beta_m z) \quad (13)$$

As a suitable trial function, let the magnetic currents in the slot as follows :

$$\underline{M} = M_\phi\hat{\phi} + M_z\hat{z} \quad (14)$$

$$M_\phi(\phi,z) = \sum_p\sum_q\sqrt{1-\left(\frac{2\phi}{\phi_0}\right)^2}(-j)^p\cos\omega_p(\phi+\frac{\phi_0}{2})\exp(-j\beta_0 z)\frac{T_q(\frac{2z}{w})}{\sqrt{1-(\frac{2z}{w})^2}}M_{\phi,pq}, \omega_p = \frac{p\pi}{\phi_0} \quad (15)$$

$$M_z(\phi,z) = \sum_r\sum_s\frac{T_r(\frac{2\phi}{\phi_0})}{\sqrt{1-(\frac{2\phi}{\phi_0})^2}}\exp(-j\beta_0 z)\sqrt{1-(\frac{2z}{w})^2}(-j)^s\cos A_s(z+\frac{w}{2})M_{z,rs}, A_s = \frac{s\pi}{w} \quad (16)$$

where the Chebyshev functions of the first kind ($T_n(\cos\theta) = \cos(n\theta)$) is used as basis function.
We match the magnetic currents in the slot with $E_\phi^{II}(b,\phi,z)$ and $E_z^{II}(b,\phi,z)$ in (10) and (11) at $\rho = b$
and impose the continuity of electric and magnetic fields at the interfaces. The resulting equations for the electric field are multiplied by the space harmonic functions and integrated over the period. The equations for the magnetic field are multiplied by the mode functions and integrated over the aperture. After considering the orthogonality properties of modes and space harmonics, and introducing matrix formulations, we obtain the following matrix equations.

$$\begin{bmatrix}\bar{R}_{11} & \bar{R}_{12}\\ \bar{R}_{21} & \bar{R}_{22}\end{bmatrix}\begin{pmatrix}M_\phi\\ M_z\end{pmatrix} = 0. \quad (17)$$

The existence of a nontrivial solution for (17) requires that the coefficient determinant vanish :

$$det[\bar{R}] = 0. \tag{18}$$

The determinant equation (18) defines the dispersion relation, from which the complex propagation constant $\beta_0 = \beta - j\alpha$ can be obtained, and then the radiation characteristics of the leaky-wave antenna can be completely determined. And normalizing the amplitude for the fundamental mode $M_{\phi 00}$, the rest of matrix coefficients are obtained, Thus the fields in any region are computed by (3) - (13). The far-zone magnetic field radiated into the region (II) can be calculated by using the asymptotic evaluation, thus the radiation pattern can be obtained

III. NUMERICAL RESULTS AND DISCUSSION

For the numerical resolution of the problem, the complex zeros of a characteristic equation must be found. The numerical procedure starts sweeping down in β_0 from $k_0\sqrt{\epsilon_r}$ in order to find the zero corresponding to the first mode of the space harmonics of the structure. Once β_0 is in the neighborhood of the zero, the Newton-Raphson method is used to find the complex zero $\beta_0 = \beta - j\alpha$, To save computer time, the program sweeps only for the first computation,and uses the previous value of $\beta_0 = \beta - j\alpha$ as a starting point for successive computations. Figure.3 and Figure.4 shows β/k_2 and α/k_2 versus the normalized period of the slots with two different angles for the single beam scan range. In Figure .3,the β/k_2 curve exhibits sharp peaks around the 0.665 λ in each slot angles. But, in Figure 4., the α/k_2 curve has a sharp minimum around the 0.66λ, where the main beam angle($\theta \approx 90^o$) The effect of the normalized slot width on the radiation characteristics is shown in Fig .5 and .6. For the case of 180^o slot angle, it is found that the phase constant monotonically decreases as the slot width increases but the leakage constant goes to maximum and decreases again as the slot width increases except near the 0.6λ, Since narrow slot width case is corresponding to the periodic slot perturbation of the coaxial waveguide, we can expect that the leakage constant will increase as slot width increases. However, wide slot case can be considered as a periodic metal perturbation of the dielectric coated wire surface waveguide. Therefore, the leakage constant decreases as slot width increases and finally disappears. But, for the case of 90^o slot angle, the β/k_2 curve has a smooth peak around the 0.8, while the α/k_2 curve has the similar behaviour with the case of 180^o slot angle. And, we can compare the value of the β and α for the infinite circumferential periodic slots obtained by using Newton's method [6] with those of the infinite extent structure with circularly symmetric and dielectric coated case [7]. The radiation pattern of the infinite periodic slots is shown in Figure.7, where the main beam radiation angle($\theta \approx 108^o$) and the radiation angle is well matched with the value of the angle, which is calculated by β and α, because leaky-wave main beam angle is determined by β and 3-dB beamwidth of the leaky wave antenna related with α.

IV. CONCLUSION

We have analyzed theoretically the radiation characteristics of a coaxial cable with periodic slots as a leaky-wave antenna. As a boundary value problem, the antenna was analyzed by a rigorous formulation to obtain the complex eigen values, from which the radiation characteristics of the antenna are determined. The numerical solution is very accurate when a sufficient number of mode and space harmonics is retained and provides a faster convergence for α and β. The radiation patterns of the antenna in the azimuth direction and in the E-plane as well as the frequency scanning characteristics of the 3-dB beamwidth and the maximum radiation radiation angle is obtained. Comparing the 180^o slot angle case with 90^o slot angle case, we found that the former has the better efficiency as a leaky-wave antenna due to the fact that the former has the more sharp beam width of the two. And, we observed a few abnormal phenomena near the 0.66λ in Fig .3 and .4, and in Figure .5, a little different behaviour of β/k_2 in each slot angles is observed. For those cases, we have not analzed the abnormality in detail. Thus, future extensions of this work include the numerical results involving the characteristic phenomenon such as radiation and coupling using the k-β and k-α diagrams.

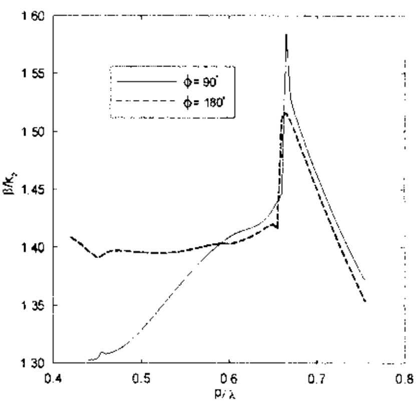

Fig. 3. Phase constant with the normalized period as parameter:$a = 0.05\lambda$, $b = 0.25\lambda$, $w = 0.2\lambda$, $\epsilon_{r1} = 2.5$, $\epsilon_{r2} = 1.0$

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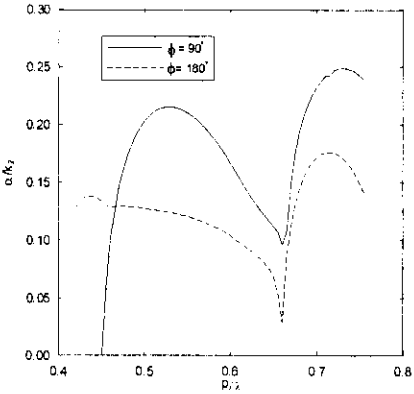

Fig. 4. Leakage constant with the normalized period as parameter:$a = 0.05\lambda$, $b = 0.25\lambda$, $w = 0.2\lambda$, $\epsilon_{r1} = 2.5$, $\epsilon_{r2} = 1.0$

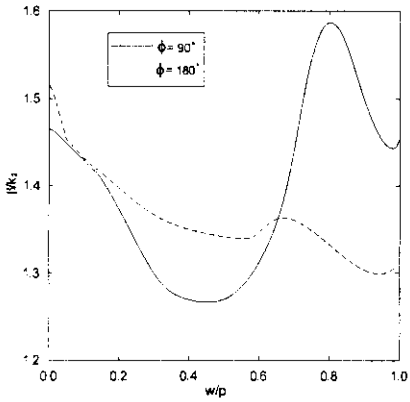

Fig. 5. Phase constant with the normalized slot width as parameter:$p = 0.55\lambda$, $a = 0.05\lambda$, $b = 0.25\lambda$, $\epsilon_{r1} = 2.5$, $\epsilon_{r2} = 1.0$

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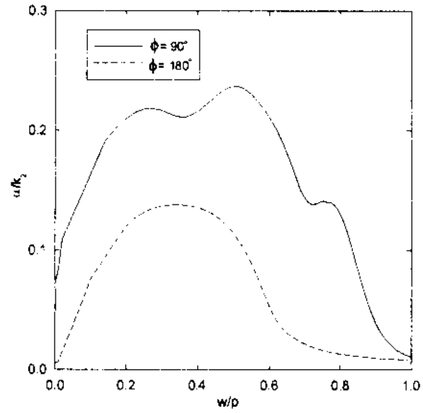

Fig. 6. Leakage constant with the normalized slot width as parameter:$p = 0.55\lambda$, $a = 0.05\lambda$, $b = 0.25\lambda$, $\epsilon_{r1} = 2.5$, $\epsilon_{r2} = 1.0$

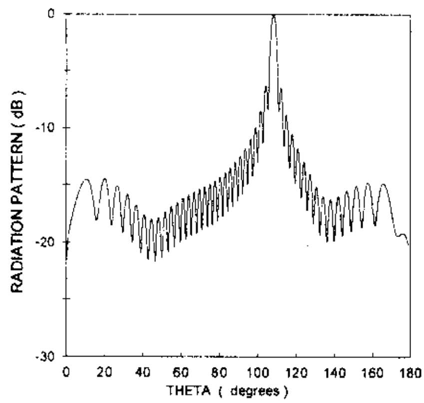

Fig. 7. Radiation pattern in the E - plane :$p = 0.55\lambda$, $a = 0.05\lambda$, $b = 0.25\lambda$,$w = 0.01\lambda$, $\epsilon_{r1} = 2.5$, $\epsilon_{r2} = 1.0$, $\phi = 180^o$