Delay Analysis for Dynamic Multiplexing Scheme in Connection-oriented Wireless Cellular Networks

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Abstract

We consider connection-oriented wireless cellular networks. These networks employ dedicated radio channels for the transmission of signaling information. A forward signaling channel is a common signaling channel assigned to carry the multiplexed stream of paging and channel allocation (virtual circuit allocation) packets from a base station to mobile stations. The delay levels experienced by paging and channel allocation packets have serious effect on the utilization level of the limited radio channel capacity. While a slotted mode operation is used to reduce the power consumption level at mobile stations, it may induce an increase in packet delay levels. In this paper, we thus consider a multiplexing scheme for paging and channel allocation packets under which slots are dynamically allocated for the paging packet transmission. For this dynamic scheme, we develop an analytical method for deriving the delay characteristics exhibited by paging and channel allocation packets, and investigate the effect of network parameters on the delay level by using this method.

1 Introduction

We consider connection-oriented wireless cellular networks, including second generation wireless cellular networks, e.g., IS-54 (Electronic Industry Association Interim Standard 54), IS-95 and GSM (European Global System for Mobile Communications), and wireless ATM networks [2] [3] [6]. These networks employ dedicated radio channels for the transmission of signaling information. A forward signaling channel is a common signaling channel assigned to carry the multiplexed stream of paging and channel allocation packets (virtual circuit allocation packets in wireless ATM networks) from a base station to mobile stations. Paging packets are broadcasted by the base station across the forward signaling channel to alert a mobile station to an incoming call. Upon receipt of a channel request from a mobile station, the base station selects a traffic channel to be assigned to the mobile station (when the request is admitted), and transmits a channel allocation packet across the forward signaling channel. In designing a multiplexing scheme for the forward signaling channel, it is necessary to investigate the following factors.

(1) The delay level experienced by channel allocation packets must be properly limited. For a mobile station initiating a call, the channel allocation packet serves as a positive acknowledgement message. If the channel allocation packet is not received in time, the channel request packet is retransmitted, leading to the degradation of throughput efficiency in signaling channels.
(2) To avoid time-out of the call initiating party and to yield an acceptable circuit (virtual circuit) set-up time, it is necessary to limit the delay level experienced by paging packets.
(3) To reduce power consumption at mobile stations, a slotted mode operation is available. Under such an operation, an inactive mobile station only listens to the forward signaling channel during prescribed time slots.

A multiplexing scheme must be designed to properly integrate these features. In this paper, we consider a multiplexing scheme for paging and channel allocation packets under which slots are dynamically assigned for the paging packet transmmission, whereby the paging packet delay level can be reduced. For this scheme, we develop an approximation method for calculating the delay levels experienced by paging and channel allocation packets. Using this analytical method, we investigate the effect of network parameters on the delay performance and the power consumption level at mobile stations.

In Section 2, we describe the candidate dynamic multiplexing scheme. In Section 3, we present an approximation method for the calculation of the distributions for paging and channel allocation packet delay times under the dynamic multiplexing scheme described in Section 2. Section 5 is devoted to numerical examples and performance comparisons.

2 Dynamic Multiplexing Scheme for Forward Signaling Channel

The region covered by a wireless cellular network is patitioned into location areas (LA), and a LA is also partitioned into cells. A base station (BS) is located in each cell. A forward signaling channel is used for the transport of multiplexed channel allocation (CA) and paging (PG) packets from a BS to the mobile stations (MS) residing in the cell supported by the BS. We assume that time is divided into frames and a frame is subdivided into m slots. We also assume that CA and PG packets have the same fixed length and the packet transmission time is equal to a slot duration time. For reducing power consumption level at MS's, the MS's in a LA are divided into a number of groups. We set the number of groups to be equal to the number of slots per frame. The BS is logically equipped with m queues of PG packets (identified as PG queues) and a queue of CA packets (identified as CA queue). A PG packet which joins the PG queue k is transmitted in the kth slot of a frame for $k = 1, \cdots, m$. When a PG packet destined to a MS in group k arrives at the BS, the packet joins the PG queue k if the number of packets in the queue which are waiting or in service is less than a threshold level c. Otherwise, the packet joins a PG queue which has the lowest system-size among the PG queues except the PG queue k. The PG packets in each queue are served in a FCFS service discipline. Every CA packet joins the CA queue. The CA packet at the top of the CA queue is transmitted in the earliest slot in which the corresponding PG queue is empty. In the CA queue, every packet is served in a FCFS service discipline. If a PG packet destined to a MS in group k joins PG queue k, the MS's in group k only listen to the forward signaling channel during the kth slot of each frame. Otherwise, such MS's must always listen to the forward signaling channel. This control can be realized by using a broadcast channel from the BS to MS's.

3 Paging and Channel Allocation Packet Delay Analysis

The region covered by a wireless cellular network is partitioned into cells, and a BS is located in each cell. The BS in a cell delivers CA and PG packets to MS's residing in the cell. These MS's are divided into m groups. Time is divided into frames and each frame is subdivided into slots. A frame consists of m slots. At the BS, the sequence of CA packet arrival times is modeled as a Poisson point process with arrival rate β. For each group $k = 1, \cdots, m$, the sequence of PG packet arrival times is also modeled as a Poisson point process with arrival rate α_k. These arrival processes are assumed to be independent. (The packet arrival rate is measured as the average number of packet arrivals per slot duration time.)

Let $\tilde{X}_t^k$ denote the number of PG packets waiting or in service at PG queue k at time t. Let $\tilde{\alpha}_k$ denote the arrival rate of PG packets which join PG queue k. In [5], (An approximation method for calculating the arrival rate $\tilde{\alpha}_k$ is presented in [5].) Define the completion time of a packet to be the sojourn time of the packet at the top of the queue, i.e., it represents the time elapsed from the moment a packet is placed at the top of the queue to the moment it is transmitted across the channel. Suppose that the completion time for a PG packet destined to a MS in group k is governed by the distribution for a random variable $\tilde{K}^k$ if the packet initiates a busy period associated with PG queue k, and it otherwise follows the distribution for a random variable $\tilde{C}^k$. (The distributions for $\tilde{K}^k$ and $\tilde{C}^k$ are given in [5].) Then, the process $\{\tilde{X}_t^k,\ t \geq 0\}$ can be modeled as the system-size process of an M/G/1 queueing system with exceptional service time for a packet initiating a busy period. Based on this model, the moments of the PG packet delay time at steady-state are calculated for each group $k = 1, \cdots, m$, by using the moments of $\tilde{K}^k$ and $\tilde{C}^k$. Let $\tilde{A}_n^k$ denote the arrival time of the nth PG packet which joins PG queue k. Then, an approximate power consumption level at a MS in PG group k, denoted by γ_k, is expressed as follows [5].

$$\begin{aligned}\gamma_k &= \frac{1}{m} \cdot \lim_{n\to\infty} \mathrm{P}(\tilde{X}^k_{\tilde{A}^k_n -} \leq c-1) \\ &\quad +1 \cdot \lim_{n\to\infty} \mathrm{P}(\tilde{X}^k_{\tilde{A}^k_n -} \geq c), \qquad (1)\end{aligned}$$

for $k = 1, \cdots, m$. Note that the power consumption level does not depend on the CA packet arrivals and departures.

Let A_n and R_n denote the arrival and departure times of the nth CA packet, respectively. Let C_n denote the completion time for the nth CA packet. Then,

$$C_n = R_n - \max\{A_n, R_{n-1}\}.$$

Note that the completion time for a CA packet is finished at the end of a slot duration time.

First, suppose that $R_{n-1} > A_n$, i.e., the nth CA packet does not initiate a busy period associated with CA queue. Set $\Phi(l) = [l \bmod m] + m \cdot \mathrm{I}_{\{l \bmod m = 0\}}$ for integer l. Then, we have

$$\begin{aligned}&\mathrm{P}(C_n = mi + j) \\ &= \sum_{k=1}^{m} \mathrm{P}\Big(\bigcap_{l=1}^{mi+j-1} \{\tilde{X}^{\Phi(k+l)}_{R_{n-1}+l-1} \geq 1\} \bigcap \{\tilde{X}^{\Phi(k+mi+j)}_{R_{n-1}+mi+j-1} = 0\}\Big) \\ &\quad \cdot \mathrm{P}(\Phi(R_{n-1}) = k),\end{aligned}$$

for $i = 0, 1, \cdots$ and $j = 1, \cdots, m$. Note that

$$\delta_k \overset{\triangle}{=} \lim_{t\to\infty} \mathrm{P}(\tilde{X}_t^k = 0) = \frac{1 - \alpha_k \mathrm{E}(\tilde{C}^k)}{1 - \alpha_k \mathrm{E}(\tilde{C}^k) + \alpha_k \mathrm{E}(\tilde{K}^k)}, \quad (2)$$

for $k = 1, \cdots, m$. Assume that $\{\tilde{X}^k_{mi+k-1},\ i = 0, 1, \cdots\}$ is a sequence of i.i.d. random variables such that $\mathrm{P}(\tilde{X}^k_{mi+k-1} = 0) = \delta_k$ for $k = 1, \cdots, m$. Then, we have

$$\theta_k \overset{\triangle}{=} \lim_{n\to\infty} \mathrm{P}(\Phi(R_n) = k) = \frac{\delta_k}{\sum_{l=1}^{m} \delta_l}, \quad (3)$$

for $k = 1, \cdots, m$. Using Equations (2) and (3), we obtain an approximate distribution for C_n as follows:

$$\begin{aligned}&\mathrm{P}(C_n = mi + j) \\ &= \sum_{k=1}^{m} \left[\prod_{l=1}^{m} (1 - \delta_l)\right]^{i} \prod_{l=1}^{j-1} (1 - \delta_{\Phi(k+l)}) \delta_{\Phi(k+j)} \theta_k. \quad (4)\end{aligned}$$

Secondly, suppose that $A_n > R_{n-1}$, i.e., the nth CA packet initiates a busy period. Then,

$$C_n = (\lfloor A_n \rfloor - A_n) + (R_n - \lfloor A_n \rfloor).$$

Set $U_n = \lfloor A_n \rfloor - A_n$ and $V_n = R_n - \lfloor A_n \rfloor$. Since the length of an idle period associated with CA queue has an exponential distribution with parameter β, we have [4]

$$\begin{aligned}\mathrm{P}(U_n \leq x) &= 0 \cdot \mathrm{I}_{\{x<0\}} \\ &\quad + \frac{\mathrm{e}^{-\beta}}{1 - \mathrm{e}^{-\beta}} \cdot \left[\mathrm{e}^{\beta x} - 1\right] \cdot \mathrm{I}_{\{0 \leq x < 1\}} \\ &\quad + 1 \cdot \mathrm{I}_{\{x \geq 1\}}. \qquad (5)\end{aligned}$$

Note that

$$\mathrm{P}(\Phi(A_n) = k) = \frac{1}{m}, \tag{6}$$

for $k = 1, \cdots, m$. Based on the same argument for the calculation of the distribution in (4), we obtain an approximate distribution for V_n as follows:

$$\begin{aligned}&\mathrm{P}(V_n = mi + j)\\ &= \sum_{k=1}^{m}\left[\prod_{l=1}^{m}(1-\delta_l)\right]^i \prod_{l=1}^{j-1}(1-\delta_{\Phi(k+l)})\delta_{\Phi(k+j)}\frac{1}{m},\end{aligned} \tag{7}$$

for $i = 0, 1, \cdots$ and $j = 1, \cdots, m$. Using Equations (5) and (7), the distribution for C_n is calculated to be

$$\begin{aligned}&\mathrm{P}(C_n \le x)\\ &= \sum_{i=1}^{\infty}\sum_{j=1}^{m}\mathrm{P}(U_n \le x - (mi+j))\mathrm{P}(V_n = mi+j).\end{aligned} \tag{8}$$

Assume that the CA packet completion times are independent. Let C and K denote random variables governed by the distributions given in Equations (4) and (8), respectively. Then, the process $\{X_t,\ t \ge 0\}$ is stochastically equivalent to the system-size process of an M/G/1 queueing system with exceptional service time for a packet initiating a busy period, where the arrival rate is β and the nominal and exceptional packet service times are C and K. Let D_n denote the delay time for the nth CA packet. Suppose that $\sum_{k=1}^{m}\alpha_k + \beta < 1$. Then, from Lindley's theorem [1], there exists a random variable D such that $D_n \xrightarrow{d} D$ as $n \to \infty$. Let $G_D(s)$ denote the Laplace-Stieltjes transform of the distribution for D. Then, we have

$$\begin{aligned}&G_D(s)\\ &= \frac{1-\beta\mathrm{E}(C)}{1-\beta\mathrm{E}(C)+\beta\mathrm{E}(K)}\cdot\frac{[\beta-s]G_K(s)-\beta G_C(s)}{[\beta-s]-\beta G_C(s)},\end{aligned} \tag{9}$$

where $G_C(s)$ and $G_C(s)$ are the Laplace-Stieltjes transforms of the distribution functions for C and K, respectively. By differentiating both sides of Equation (9), we can calculate the moments of the CA packet delay time at steady-state. For example, the mean of the CA packet delay time is given by

$$\mathrm{E}(D) = \frac{2\mathrm{E}(K) - \beta[\mathrm{E}(C^2) - \mathrm{E}(K^2)]}{2[1-\beta[\mathrm{E}(C)-\mathrm{E}(K)]]} + \frac{\beta\mathrm{E}(C^2)}{2[1-\beta\mathrm{E}(C)]}.$$

4 Numerical Examples

In Section 3, we have presented an analytical method for calculating the moments of the PG and CA packet delay times under the dynamic multiplexing scheme. The delay levels experienced by PG and CA packets depend on the network parameters: packet arrival rates α_k and β, the number of groups m and the threshold value c. In this section, we demonstrate the impact of these parameters on the packet delay performance. Hereafter, we set $m = 4$ and $\alpha_k = \frac{1}{2}\alpha_1$ for all $k = 2, 3, 4$. Define the peak PG and CA packet delay levels to be $\mathrm{E}(D^k)+3\sqrt{\mathrm{Var}(D^k)}$ and $\mathrm{E}(D)+3\sqrt{\mathrm{Var}(D)}$, respectively, where D^k denote a random variable governed by the steady-state distribution for the delay time sequence of PG packets destined to a MS in group k. Set $\alpha = \sum_{k=1}^{m}\alpha_k$ and $\eta = \frac{\beta}{\alpha+\beta}$.

Figure 1 shows the peak delay levels of the PG and CA packets with respect to the aggregate packet arrival rate $(\alpha+\beta)$, where $\eta = 0.5$ and the threshold $c = 5$. In this figure, we observe that as the aggregate packet arrival rate is increased, PG and CA packets experience higher peak delay levels. In Figure 2, the peak PG and CA packet delay levels are illustrated with respect to the fraction η, where $\alpha+\beta = 0.8$ and $c = 5$. As the fraction η is increased, the amount of CA packet load increases, while the amount of PG packet load decreases. Thus, the peak CA packet delay level is higher and the peak PG packet delay level is lower for higher fraction value, as shown in this figure. In Figure 3, we demonstrate the effect of the threshold on the peak packet delay levels, where $\alpha+\beta = 0.8$ and $\eta = 0.5$. As the threshold level is increased, the peak delay level of the PG packet destined to a MS in group 1 (group 2) increases (decreases). We also observe that the peak CA packet delay level decreases as the threshold level is increases. Note that a PG packet destined to a MS in group 1 is more likely to join the PG queue 1 as the threshold level is increased. Then, in the second, third and fourth slots of each frame, the probability to carry a PG packet is lower while the probability is higher in the first slot of each frame. Thus, a CA packet at the top of the CA queue may have a chance to be transmitted earlier for a higher threshold level. In Figure 4, we exhibit the power consumption level with respect to the threshold level by using Equation (1). In this figure, we observe that the power consumption level decreases as the threshold level is increased. The delay levels experienced by PG and CA packets must be limited. On the other hand, a reduced power consumption level is preferred. Since the power consumption level is reduced as the threshold level is increased, we may formulate an optimization problem as follows:

Given	α_k and β for $k = 1, \cdots, m$
maximize	threshold level
subject to	peak CA packet delay level $\le \epsilon_{CA}$
	peak PG packet delay level $\le \epsilon_{PG}$

where ϵ_{CA} and ϵ_{PG} the prescribed delay limits for CA and PG packets. From Figure 3, we can obtain an optimal threshold level for the problem given above.

5 Conclusions

In this paper, focusing on the forward signaling channel used by connection-oriented wireless cellular networks, we investigated a dynamic scheme for multiplexing paging and channel allocation packets which are transmitted across the forward signaling channel. For this dynamic multiplexing scheme, an approximation method was presented to obtain the delay characteristics for the channel allocation packet. Using the performance analysis method derived here, we presented numerical examples which illustrate the underlying trade-off in packet delay and power consumption performances. We derive the following conclusions.

(1) Under the condition of non-uniform arrival rates of paging packets for each group, the peak delay level experienced by a channel allocation packet monotonically increases as the channel allocation packet arrival rate is increased.
(2) As the threshold level is decreased, paging packet transmissions take place more uniformly over a frame. As a result, the peak paging packet delay level decreases. As the threshold level is decreased, the power consumption level also decreases. However, the peak channel allocation packet delay level is higher for a lower threshold level. Thus, a proper threshold level is limited by the delay constraint for the channel allocation packet.

References

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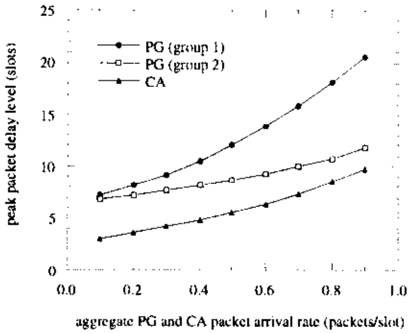

Figure 1: Peak PG and CA packet delay levels with respect to aggregate PG and CA packet arrival rate

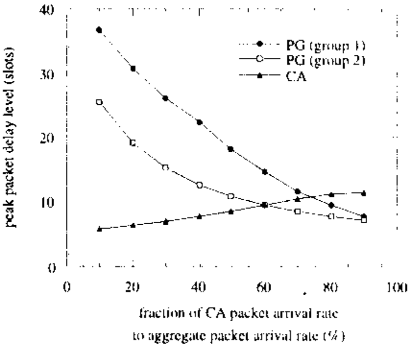

Figure 2: Peak PG and CA packet deley levels with resepct to fraction of CA packet arrival rate to aggregate PG and CA packet arrival rate

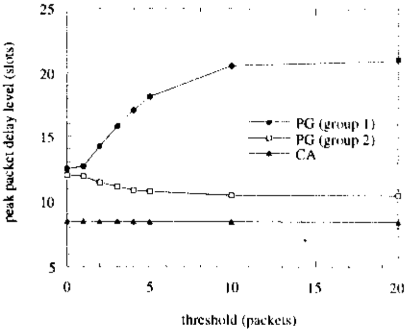

Figure 3: Peak PG and CA packet delay level with respect to threshold

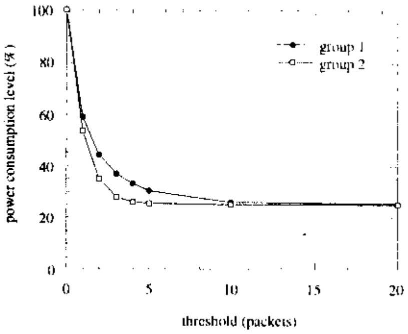

Figure 4: Power consumption level with respect to threshold