Fuzzy Causal Knowledge-Based Expert System

Kun Chang Lee[1], Hyun Soo Kim[2], Yong Uk Song[3]
[1]*School of Management, Sung Kyun Kwan University* (leekc@yurim.skku.ac.kr)
[2]*Department of MIS, Dong-A University*
[3]*Department of MIS, Kyung Sang National University*

Abstract

Although many methods of knowledge acquisition has been developed in the expert systems field, such a need for causal knowledge acquisition has not been stressed relatively. In this respect, this paper is aimed at suggesting a causal knowledge acquisition process, and then investigate the causal knowledge-based inference process. A vehicle for causal knowledge acquisition is FCM (Fuzzy Cognitive Map), a fuzzy signed digraph with causal relationships between concept variables found in a specific application domain. Although FCM has a plenty of generic properties for causal knowledge acquisition, it needs some theoretical improvement for acquiring a more refined causal knowledge. In this sense, we refine fuzzy implications of FCM by proposing fuzzy causal relationship and fuzzy partially causal relationship. To test the validity of our proposed approach, we prototyped a causal knowledge-driven inference engine named CAKES and then experimented with some illustrative examples.

1. Introduction

Expert system consists of three major components: (1) dialogue structure, (2) inference engine, and (3) knowledge base. The dialogue structure serves as the language interface in which the user can access the expert system. The inference engine is the logic (set of procedures or program) that actually solves (matches symptoms to diagnoses) a given problem. The knowledge base is the heart of an expert system because it contains the detailed knowledge supplied by a human expert. Many ways of representing knowledge exist such as frame, semantic net, predicate logic, and IF-THEN rules, etc [4]. We will discuss a new type of knowledge-causal knowledge. Causal knowledge seems similar to IF-THEN rules at first glance, but semantically different from other knowledge.

Literature survey reveals that there exist few studies about extracting the causal knowledge from some domain, building a causal knowledge base, and making inference with it. The term causal knowledge is rarely found in the expert system literature. Rather, the term FCM (Fuzzy Cognitive Map) has been extensively used in many studies [1,3,4,5,8,9,10,11,12,14] because it has been used as a vehicle for expressing the causal knowledge type and making inference with it. We will also adopt FCM as a major vehicle of causal knowledge representation and inference. However, conventional FCM theory is not sufficient for representing more refined forms of causal knowledge. Therefore, we will enrich the conventional FCM theory with two proposed concepts: Fuzzy Causal Relationship and Fuzzy Partially Causal Relationship. Our concern is focused on the development of a new inference engine for expert systems.

Many researches about knowledge acquisition have been focused on the type of non-causal knowledge. The functioning of the inference engine depends on the knowledge type stored in knowledge base. For example, the inference engine uses backward chaining or forward chaining rules for IF-THEN type knowledge [13] which has been most popular in the arena of applying expert systems. Then what about the causal knowledge type ? Can the inference engine with backward or forward chaining inference rules deal with such causal knowledge type as well ? The answer is unfortunately No. We will discuss this important issue. Therefore, our main research objectives can be summarized as follows:

(1) To develop a theoretical background for an inference engine which is capable of handling a causal knowledge type.
(2) To implement the prototype inference engine named CAKES (CAusal Knowledge -based Expert System)
(3) To test its performance and analyze its results with some illustrative examples.

The structure of this paper is as follows. FCM is briefly reviewed in section 2 and two fuzzy concepts, fuzzy causal relationsip and fuzzy partially causal relationship, are proposed in section 3. Section 4 discusses the characteristics of a prototype CAKES and illustrates its performance. This paper is ended with some concluding remarks.

2. Fuzzy Cognitive Maps

FCMs are fuzzy signed directed graphs with feedback, and they model the world as a collection of concepts (or factors) and causal relations between concepts [6,7]. Usually, a concept is depicted as a node in FCM, and a causal relationship between two concepts is represented as an edge. Therefore an edge value (or causality value) between concept i and concept j, e_{ij}, indicates a causality value between the two concepts. To clearly understand the FCM's logic, let us define a concept and a causality. The causality value e_{ij} take values in the interval [-1,1]. $e_{ij}=0$ indicates no causality. $e_{ij} > 0$ indicates causal increase or positive causality: a concept C_j increases as C_i increases, and C_j decreases as C_i decreases. $e_{ij} < 0$ indicates causal decrease or negative causality: C_j decreases as C_i increases, and C_j increases as C_i decreases. Simple FCMs have edge values in {-1, 0, 1}. Then, if causality occurs, it occurs to a maximal positive or negative degree. Simple FCMs provide a quick approximation to an expert's stated or printed causal knowledge. For instance, consider Figure 1 in which the causal knowledge on the Middle East peace policy is depicted, based on the article by Henry Kissinger, printed in the *Los Angeles Times* (1982) [6,7]. There exist various issues related to how to use FCM, but we will deal with issues about the causal knowledge-based FCM representation and inference.

* Insert Figure 1 *

3. Enriched Fuzzy Cognitive Map

For more clear understanding of the fuzzy relationships in FCM, we discuss the characteristics of fuzzy relation [2]. Fuzzy relation R from set A to set B, or (A, B) represents its degree of membership in the unit interval [0,1]. The corresponding membership function is

$$R : A \times B \rightarrow [0,1].$$

R(x,y) is interpreted as the "strength" of membership of the relation (x,y), where $x \in A$ and $y \in B$. Then the causality value e_{ij} is interpreted as the degree of relationship between two concept nodes C_i and C_j. So, e_{ij} can be denoted by the membership function value $R(C_i, C_j)$. So we will call $R(C_i, C_j)$ used in representing a causal relationship of FCM as a fuzzy causal relationship (FCR) in the sequel.

3.1 Fuzzy Causal Relationship

The fuzzy relation in FCM is more general than the fuzzy relation concept [2] originally defined in fuzzy literature. The reason is that it can include negative (-) fuzzy relations. This is because FCM's fuzzy relations mean fuzzy causality. Causality can have a negative sign. In FCM, the negative fuzzy relation (or causality) between two concept nodes is the degree of a relation with "negation" of a concept node. For example, if the negation of a concept node C_i is noted as $\tilde{}C_i$, then $R(C_i,C_j) = -0.6$ means that $R(C_i,\tilde{}C_j) = 0.6$. Conversely, $R(C_i,C_j) = 0.6$ means that $R(C_i, \tilde{}C_j) = -0.6$. Now, let us define more formally FCR (Fuzzy Causal Relationship) in FCM. What A causally increases B means that if A increases then B increases, and if A decreases then B also decreases. On the other hand, what A causally decreases B means that if A increases then B decreases and if A decreases then B increases. So, in the concepts that constitute causal relationships, there must exist quantitative elements that can increase or decrease. Kosko [6,7] defined the concept C_i that constitutes FCR as follows:

$$C_i = (Q_i \cup \tilde{}Q_i) \cap M_i$$

where Q_i is a quantity fuzzy set and $\tilde{}Q_i$ is a dis-quantity fuzzy set. $\tilde{}Q_i$ is the negation of Q_i. M_i is a modifier fuzzy set that modifies Q_i or $\tilde{}Q_i$. Each Q_i and $\tilde{}Q_i$ partitions the whole set C_i. Double negation $\tilde{}\,\tilde{}Q_i$ is equal to Q_i, implying that $\tilde{}Q_i$ is corresponding to Q_i^c, the complement of Q_i. However, negation does not mean antonym. For example, assume that Q_i is "tall" and $\tilde{}Q_i$ is "short" in height. The complement of fuzzy set "tall" does not correspond to the fuzzy set "short". That is, in verbal representation, "not tall" does not necessarily mean "short". Therefore, if a dis-quantity fuzzy set $\tilde{}Q_i$ does not correspond to the complement of Q_i, we will call it as the anti-quantity fuzzy set to clarify the subtle meaning in the dis-quantity fuzzy set. From the discussion so far, the following two theorems hold.

Theorem 1. When a concept C_i is ($Q_i \cap M_i$) and the negative concept $\tilde{}C_i$ is ($\tilde{}Q_i \cap M_i$), the following FCRs are all equivalent.

$$C_i \xrightarrow[+]{} C_j,\ \tilde{}C_i \xrightarrow[+]{} \tilde{}C_j,\ C_i \xrightarrow[-]{} \tilde{}C_j,\ \tilde{}C_i \xrightarrow[-]{} C_j$$

Theorem 2. When a concept C_i is ($Q_i \cap M_i$) and the negative concept $\tilde{}C_i$ is ($\tilde{}Q_i \cap M_i$), the following FCRs are all equivalent.

$$C_i \xrightarrow[-]{} C_j,\ \tilde{}C_i \xrightarrow[-]{} \tilde{}C_j,\ C_i \xrightarrow[+]{} \tilde{}C_j,\ \tilde{}C_i \xrightarrow[+]{} C_j$$

The following four theorems hold in case of a real valued causality. Comaprison with theorems 1 and 2 will be useful.

Theorem 3. When fuzzy causal concepts C_i, and C_j are given, the following FCRs are all equivalent.

$$C_i \xrightarrow[r]{} C_j,\ \tilde{}C_i \xrightarrow[r]{} \tilde{}C_j,\ C_i \xrightarrow[-r]{} \tilde{}C_j,\ \tilde{}C_i \xrightarrow[-r]{} C_j$$

where $-1 \le r \le 1$.

Theorem 4. When $\tilde{C}_i$ is a negative concept of C_i and the dis-quantity fuzzy set of $\tilde{C}_i$ is equal to the complement of C_i's quantity fuzzy set, then the following FCRs are all equivalent.

$$C_i \underset{r}{\rightarrow} C_j,\quad C_i \underset{1-r}{\rightarrow} \tilde{C}_j,\quad C_i \underset{r-1}{\rightarrow} C_j$$

where $0 < r < 1$.

Theorem 5. When $\tilde{C}_i$ is a negative concept of C_i and the dis-quantity fuzzy set of $\tilde{C}_i$ is equal to the complement of C_i's quantity fuzzy set, then the following FCRs are all equivalent.

$$C_i \underset{r}{\rightarrow} C_j,\quad C_i \underset{-1-r}{\rightarrow} \tilde{C}_j,\quad C_i \underset{r+1}{\rightarrow} C_j$$

where $-1 < r < 0$.

Theorem 6. When $\tilde{C}_i$ is a negative concept of C_i and the dis-quantity fuzzy set of $\tilde{C}_i$ is equal to the complement of C_i's quantity fuzzy set, then the followings hold.

$$C_i \underset{1}{\rightarrow} C_j \text{ implies } C_i \underset{+0}{\rightarrow} \tilde{C}_j,\quad C_i \underset{-0}{\rightarrow} C_j$$

$$C_i \underset{-1}{\rightarrow} C_j \text{ implies } C_i \underset{-0}{\rightarrow} \tilde{C}_j,\quad C_i \underset{+0}{\rightarrow} C_j$$

However, Theorem 3 cannot be applied to Fuzzy Partially Causal Relationships which will be discussed in the next section. In case that r is in {-1, 0, 1}, Theorems 4 and 5 do not hold. Theorem 6 can apply to the case that we can distinguish +0 from -0, where +0 implies that the degree of a positive causality is 0 and also -0 indicates that the degree of a negative causality is 0. Theorem 6 results from the fact that "not C_i" means "$\tilde{C}_i$", and "C_i" means "not $\tilde{C}_i$". For example, if there is a full positive causality on $C_i \rightarrow C_j$ (i.e., edge value is +1), this means that the FCR has no positive causality on $C_i \rightarrow \tilde{C}_j$ (edge value is +0) and its equivalent expression, $C_i \rightarrow C_j$ (edge value is -0). However, the reverse is not true. For example, no negative causality (-0) on $C_i \rightarrow C_j$ does not necessarily mean full positive causality (+1) because there may be also a positive causality (+0) on $C_i \rightarrow C_j$.

3.2 Fuzzy Partially Causal Relationship

In the previous section, we have explored the properties and characteristics of FCR. However, in reality, there exist many cases in which the definition of causality is not met. There may be a case that even though $(Q_i \cap M_i) \subset (Q_j \cap M_j)$ is true, but $(\tilde{Q}_i \cap M_i) \subset (\tilde{Q}_j \cap M_j)$ is not true. Also such a case would happen that $(Q_i \cap M_i) \subset (\tilde{Q}_j \cap M_j)$ is true, but $(\tilde{Q}_i \cap M_i) \subset (Q_j \cap M_j)$ is not true. For example, there may be a stock market situation that institute investors' buying causes the increase of composite stock price but their selling cannot cause the decrease of composite stock price. This kind of market situation can be observed when individual investors rush in the stock market because of their prospect of bull and/or optimistic market. In that case, institute investors' selling may not cause the decrease of composite stock price. This phenomonon shows another types of FCR, which will be termed as "Fuzzy Partially Causal Relation (FPCR)". We define FPCR as follows.

Definition 1. C_i *partially causes* C_j iff $(Q_i \cap M_i) \subset (Q_j \cap M_j)$.
Definition 2. C_i *partially* causally decreases C_j iff $(Q_i \cap M_i) \subset (\tilde{Q}_j \cap M_j)$.

Figure 2 is an exemplar FCM without adopting FPCR. However, if FPCR exists between the concept nodes, the FCM shown in Figure 2 should be transformed into Figure 3 to represent the FPCRs. As shown in Figure 3, in case that FPCR exists in a FCM, it is necessary to explicitly name both the quantity and dis-quantity fuzzy sets such as support ↔ regulation, improvement ↔ degeneration, buying ↔ selling, increase ↔ decrease to make clear the information which FCM represents.

* Insert Figures 2 and 3 *

4. Design and Implementation of CAKES

4.1 Design

CAKES was coded in Delphi running on Windows95 environment. Main menus of CAKES are composed of File, Concept Node, Relationship, Inference, Window, Help, of which core menus are (1) Concept Node, (2) Relationship, and (3) Inference. Concept node menu helps user build a causal knowledge base. Relationship menu enables user to define a causal knowledge base as a matrix form and input an appropriate causality value on each edge between two concept nodes of interest. Inference menu enables two types of inference: (1) Matrix Multiplication Method and (2) Advanced Inference Method. We will discuss the Advanced Inference Method in detail.

Let us illustrate how to build a causal knowledge base with a simple example For example, suppose that we want to build a causal knowledge base for an economic situation affecting market rate of interest and that there are seven concept nodes such as Inflation Expectation, Desire for Allowing a Loan, Desire for Taking a Loan, Business Condition, Government Expenditure, Private Demand for Credit, Governmental Demand for Money, and Market Rate of Interest. Figure 4 depicts an illustrative FCM representing economic situation affecting market rate of interest.

Figure 5 is a screen of CAKES illustrating Figure 4, where Inflation stands for Inflation

Expectation, Borrowing Desire for Taking a Loan, Lending Desire for Allowing a Loan, B. Condition Business Condition, P. Demand Private Demand for Credit, G. Expend Government Expenditure, Fund, G. Fund Governmental Demand for Fund.

* Insert Figures 4 and 5 *

4.2 Inference Mechanism

Traditional inference mechanism is based on matrix multiplication in which 0 threshold [12] or 1/2 threshold [7] is usually adopted to ensure convergence after finite multiplications. However, the traditional inference mechanism suffers from illogical conclusion due to its absurd inference logic [3]. So CAKES adopts more advanced version of causal knowledge-based inference mechansim which is refined by using FCR and FPCR concepts. CAKES's Advanced Inference Mechanism is based on the following five inference principles.

Inference Principle 1

If two causal relationships support the same conclusion, then the addition of those two causality value is greater than each causality value.

Inference Principle 2

If a causal relationship is connected consecutively to a causal relationship, then the absolute value of its additive value of the two causality values is less than or equal to the least of absolute value of the two causality values.

Inference Principle 3

The final additive value remains same irrespective of the order of addition of causality values of interest.

Inference Principle 4

Both a positive cauality value and a negative causality value have the same amount of strength although they have the opposite direction with each other.

Inference Principle 5

The final causality value lies between +1 and -1.

To support the above principles, CAKES adopts Mini-Max operation in the multiplication between a concept node vector and an FCM matrix; the minimum operation is applied for the muliplication between an element of the vector and an element of the matrix and the maximum operation is applied for the summation at the vector multiplication. This is formulated as follows:

$$\underline{C}^t = \underline{C}^{t-1} \cdot \underline{E} + \underline{C}^{t-1} = \{C_i^t \mid C_i^t = \Psi(X_j(N(C_j^{t-1}, E_{ij})), C_i^{t-1}), i=1,\dots,n, j=1,\dots,n\}$$

where

$\underline{C}^t$ = concept node vector at iteration t

$\underline{E}$ = FCM matrix

n = number of concepts

$$N(a,b) = \begin{cases} \min(\mathrm{abs}(a), \mathrm{abs}(b)) & \text{if } a \cdot b \ge 0 \\ -\min(\mathrm{abs}(a), \mathrm{abs}(b)) & \text{if } a \cdot b < 0 \end{cases}$$

with respect to FCR (1)

$$N(a,b) = \begin{cases} \min(\mathrm{abs}(a), \mathrm{abs}(b)) & \text{if } a \cdot b \ge 0 \\ 0 & \text{if } a \cdot b < 0 \end{cases}$$

with respect to FPCR

$$X_j(a_j) = \begin{cases} \max_j(\mathrm{abs}(a_j)) & \text{if } a_j \ge 0 \quad \forall j \\ -\max_j(\mathrm{abs}(a_j)) & \text{if } a_j \le 0 \quad \forall j \\ \gamma(\sum_j a_j) & \text{otherwise} \end{cases} \quad (2)$$

$$\gamma(a) = \begin{cases} +1 & \text{if } a > 1 \\ a & \text{if } -1 \le a \le 1 \\ -1 & \text{if } a < -1 \end{cases}$$

with respect to FCR (3)

$$\gamma(a) = \begin{cases} +1 & \text{if } a > 1 \\ a & \text{if } 0 \le a \le 1 \\ 0 & \text{if } a < 0 \end{cases}$$

with respect to FPCR

$$\Psi(a,b) = \begin{cases} a & \text{if } b = 0 \\ b & \text{if } b \ne 0 \end{cases} \quad (4)$$

While the values of the concept node vector decays to zero in traditional matrix multiplication mechanism, the concept node vector in Mini-Max mechanism retains the values. It is because the Mini-Max mechanism does not conduct the multiplication between real numbers of which values lie between -1 and 1 while the matrix multiplication mechanism conducts the multiplication and tends to converge to zero.

The operators N, X, and Ψ in the above formula should be explained more in detail to validate the formula and to support the different cases of FCR and FPCR respectively. Formula (1) represents the minimum operation. When the signs of C_j^t and E_{ij} are different to each other in FPCR, the cause does not support the effect. So, the value of $N(a,b)$ is zero with respect to FPCR. Formula (2) and (3) represents maximum operation. Since the causality value must lies between 0 and 1 in FPCR, formula (3) ensures that with respect to FPCR. The purpose of formula (4) is to keep the known values of concept node vector.

4.3 Representation of Causal Knowledge

The causal knowledge base of CAKES is constructed from fuzzy causal rules. The syntax of a fuzzy causal rule is as follows:

```
(RULE rule-name
    [RELATION-VALUE          value]
```

```
[RELATION-TYPE            COMPLETE / PARTIAL]
  IF
                  cause
     THEN
                  effect)
```

The words with capital characters like RULE, RELATION-VALUE, RELATION-TYPE, *etc.* are reserved words for CAKES. The *rule-name* is the unique name for the causal relation. The *cause* and *effect* are the names of the concepts. The *value* is the value of causal relation between the concepts *cause* and *effect*. The RELATION-TYPE is COMPLETE for FCR and PARTIAL for FPCR.

4.4 Experiments with CAKES

CAKES is able to perform more intelligent inference with a given causal knowledge base. Let us consider an example depicted in Figure 4. If we use a Matrix Multiplication Inference Method (traditional version of inference) starting with Business Condition = 0.6 and Government Expenditure = 0.6, then the final Market Rate of Interest = 0.43. The inference history is as follows:

[Inference History]

	Inflation	Lending	Borrowing	B.Condition	P.Demand	G.Expend	G.Fund	I.Rate
1.	0.0	0.0	0.0	0.6	0.0	0.6	0.6	0.0
2.	0.0	0.0	0.0	0.6	.36	0.6	.36	0.0
3.	0.0	0.0	0.0	0.6	.36	0.6	.36	.43*

* 0.43 = 0.36 * 0.6 + 0.36 * 0.6

Does 0.43 for Market Rate of Interest make sense ? The answer is No because starting causality values (0.6) are greater than 0.5, but the final value for Market Rate of Interest is just 0.43 which is less than 0.6. The reason is that multiplication of less-than-1.0 values yields smaller value as inference processes become long. Figure 6 shows the final conclusion screen for Matrix Multiplication Inference Method. However, Advanced Inference Method proposed in this paper yields more sensible result with the same starting value 0.6 Inference history is as follows:

[Inference History]

	Inflation	Lending	Borrowing	B.Condition	P.Demand	G.Expend	G.Fund	I.Rate
1.	0.0	0.0	0.0	0.6	0.0	0.6	0.6	0.0
2.	0.0	0.0	0.0	0.6	0.6	0.6	0.6	0.0
3.	0.0	0.0	0.0	0.6	0.6	0.6	0.6	0.7*

* 0.7 = min(0.6, 0.6) + min(0.6, 0.6) - 0.5

With Advanced Inference Method, the final causality value for Market Rate of Interest is 0.70 which can be interpreted more naturally than Matrix Multiplication Method that strong starting value for Business Condition and Government Expenditure yields certainly strong causality value for Market Rate of Interest. Figure 7 shows the final conclusion screen for Advanced Inference Method.

* Insert Figures 6 and 7 *

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5. Concluding Remarks

Causal knowledge is a knowledge type usually found in a wide variety of ill-structured problem domains such as politics, OR/MS, economics, and strategic planning decision making, etc. However, few studies exist dealing with topics of causal knowledge representation and inference. Although FCMs have been extensively used in literature so far to represent the causal knowledge representation and inference, need to develop more improved FCM theory was required to build more refined form of causal knowledge. When the improved FCM is used for extracting causal knowledge from a certain problem domain, the resulting causal knowledge base can be built more precisely for a given problem. To prove our proposed idea, a causal knowledge-based expert systems shell prototype, named CAKES, was implemented in Delphi language. We proved with an illustrative examples how a robust causal knowledge base can be extracted and used for more intelligent inference. The major contributions of this paper are as follows:

(1) A more robust causal knowledge base can be constructed with our proposed improved FCM.
(2) The proposed Advanced Inference Method yields more natural conclusions for a given problem.

We hope that this paper draws more attention from researchers on the topic of causal knowledge representation and inference. But the limitations still remain as follows:

(1) More refined form of causal knowledge representation is needed.
(2) Unification with other AI techniques such as neural networks and fuzzy logic is required to solve more complicated problems.

Acknowledgment

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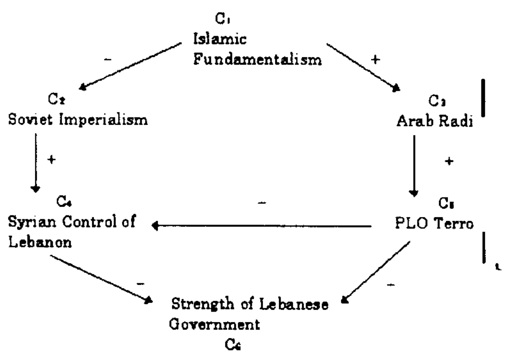

Figure 1. Illustrative Fuzzy Cognitive Map

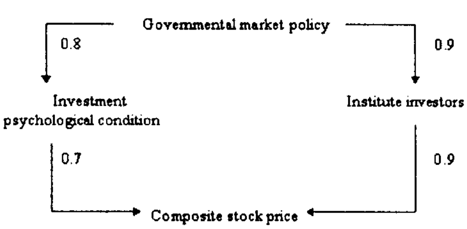

Figure 2. Exemplar FCM

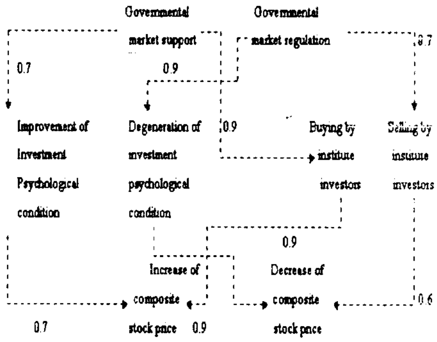

Figure 3. An illustrative FCM representing with FPCR

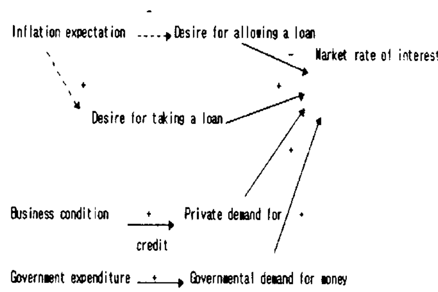

Figure 4. Economic Situation Affecting Market Rate of Interest

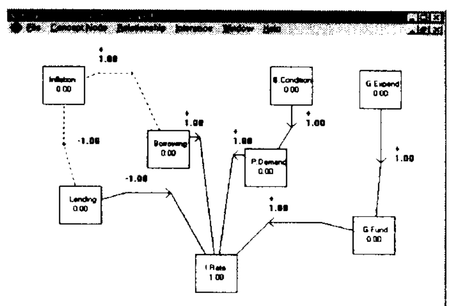

Figure 5. Causal Knowledge Representation of Figure 4

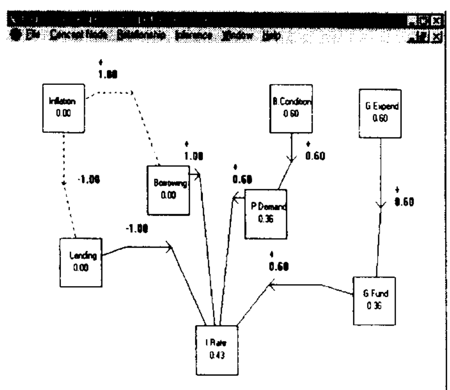

Figure 6. Inference Result with Matrix Multiplication Inference Method

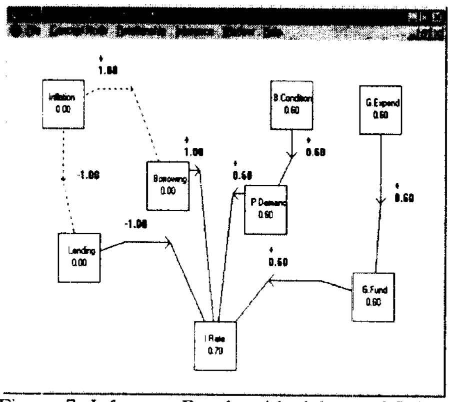

Figure 7. Inference Result with Advanced Inference Method