An LMI-based Stable Fuzzy Control System Design with Pole Placement Constraints

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October 28, 1998

ABSTRACT

This paper proposes a systematic design methodology for the Takagi-Sugeno(TS) model based fuzzy control system with guaranteed stability and additional constraints on the closed-loop pole location. These combined two objectives are formulated as a system of LMIs (Linear Matrix Inequalities). Since LMIs intrinsically reflect constraints, they tend to offer more flexibility for combining various constraints on the closed-loop system. To demonstrate the usefulness of the proposed design methodology, it is applied to the regulation problem of a nonlinear magnetic bearing system. Simulation results show that the proposed LMI-based design methodology yields not only maximized stability boundary but also the desired transient responses.

1 Introduction

In the past two decades, Fuzzy Logic Control(FLC) has been proposed as an alternative to the traditional control techniques with many successful applications. In particular, systems which are difficult to model, because of insufficient knowledge of the dynamic characteristics, and nonlinear with significant variations in the parameter of the model are attractive candidates for the application of FLC. However it has been argued that FLC being a rule based control strategy, almost by definition, lacks an analytic and systematic methodology for the issues of stability, robustness, and other performance requirements, and therefore, it cannot be reconciled with the traditional methods of control design and analysis.

In recent years, there have been many research efforts on these issues based on the Takagi-Sugeno(TS) fuzzy model.[1] The TS fuzzy model can provide an effective representation of complex nonlinear systems with a set of linear local models, so the design of fuzzy controllers based on TS fuzzy models (Parallel Distributed Compensator(PDC), following the terminology in[3]) is attractive. The concept of PDC approach is to design a compensator using linear control design techniques for each TS linear local model. The resulting overall fuzzy controller ,which is nonlinear, behaves like a gain scheduling controller, where the gain scheduling is implemented with fuzzy logic. For this TS model based fuzzy control system, Tanaka *et al*,[23] proved the stability by finding a common symmetric positive definite matrix P for the r subsystems and suggested the idea of using Linear Matrix Inequality(LMI) for finding the common P matrix. They have been considered very important results and some refining efforts have been pursued thereafter. However the design process presented in,[2] and[3] involves a iterative process.

That is, for each rule a controller is designed based on consideration of local performance only, then LMI-based stability analysis is carried out to check the global stability condition. In the case that the stability conditions are not satisfied, the controller for each rule should be redesigned. To overcome such a defect, Wang *et al*[3] pointed out that it is more desirable to be able to directly design a controller (instead of iterative process) which guarantees global stability by recasting to LMI problems. They, however, did not consider performance issues such as transient behaviors. Generally, such a design focused on only stability issue does not directly deal with the desired dynamic characteristic of the closed-loop system which is commonly expressed in terms of transient responses. In contrast, the transient responses are more easily tuned in terms of pole location.[5] For many practical problems, exact pole assignment may not be necessary: it suffices to locate closed-loop poles in prescribed subregions in the left half plane.

In this paper, we present a systematic design methodology for the stable fuzzy control system with pole placement in a specified region in a complex plane. By imposing the additional requirment of the closed-loop pole location, we can prevent fast controller dynamics and achieve good transient behavior. More significantly, in the proposed methodology, the control design problems which considers both the global stability and the desired transient performance simutaneously are reduced to the LMI problems. Therefore solving these LMI constraints directly leads to a fuzzy state-feedback controller such that the resulting fuzzy control system meets above two objectives. As a result, this approach is superior to other approaches in[3] and[6] which achieves the desired control performances by trial-and error. To demonstrate the usefulness, the proposed design methodology is applied to the regulation problem of a nonlinear magnetic bearing system. It is well known that the design of the control system for magnetic bearings is difficult and constitutes a challenging task due to the nonlinear and open-loop unstable dynamics.[6] We used a model that has been studied previously in[7] and.[8]

2 TS Fuzzy Model and Control

2.1 TS Fuzzy Model

An nth order SISO nonlinear system can be expressed in the following form:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \\ &\cdots \\ \dot{x}_n &= f(x_1, x_1, \cdots, x_n, u) \end{aligned} \tag{1}$$

where u is the control input.

By taking the Taylor's series expansion of Eqn. (1) for r operating points (x_l^*, u^*), where $l = 1, 2, \cdot\cdot, n$, the nonlinear system can be represented by the following linearized state space form:

$$\dot{\mathbf{x}}(t) = \mathbf{A}_i \mathbf{x}(t) + \mathbf{B}_i u_i(t) + \mathbf{d}_i, \quad i = 1, 2, \cdots, r \tag{2}$$

where

$$\mathbf{A}_i = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & \cdots & 1 \\ \frac{\partial f(x_l^*, u^*)}{\partial x_1} & \frac{\partial f(x_l^*, u^*)}{\partial x_2} & \cdots & \frac{\partial f(x_l^*, u^*)}{\partial x_n} \end{bmatrix}, \mathbf{B}_i = \begin{bmatrix} 0 \\ 0 \\ \cdot \\ \cdot \\ \frac{\partial f(x_l^*, u^*)}{\partial u} \end{bmatrix}$$

$$\mathbf{d}_i = \begin{bmatrix} 0 \\ 0 \\ \cdot \\ \cdot \\ f(x_l^*, u^*) - \sum_{l=1}^{n} \frac{\partial f(x_l^*, u^*)}{\partial x_l} x_i^* - \frac{\partial f(x_l^*, u^*)}{\partial u} u^* \end{bmatrix}$$

and the variables with * denote the values at the operating points.

The continuous fuzzy dynamic model is described by fuzzy IF-THEN rules to express local linear input-output relations of nonlinear systems around each operating point by above linear local model. The ith rule of this fuzzy model is of the following form:

Plant Rule i:

$$If\ x_1(t) \text{ is } L_{i1} \cdots \text{and } x_n(t) \text{ is } L_{in}, \quad Then\ \dot{\mathbf{x}}(t) = \mathbf{A}_i\mathbf{x}(t) + \mathbf{B}_i u_i(t) + \mathbf{d}_i \tag{3}$$

$i = 1, 2, \cdots, r$ and r is the number of rules and L_{ij}s are fuzzy sets centered at the ith operating point. The inference performed via the TS model is an interpolation of all the relevant linear models. These degree of relevance becomes the weight in the interpolation process.

For any current state vector $\mathbf{x}(t)$ and input $u(t)$, the final output of the fuzzy system is given by

$$\dot{\mathbf{x}}(t) = \frac{\sum_{i=1}^{r} \lambda_i(t) \left\{ \mathbf{A}_i\mathbf{x}(t) + \mathbf{B}_{1i}u(t) + \mathbf{d}_i \right\}}{\sum_{i=1}^{r} \lambda_i(t)} \tag{4}$$

where $\lambda_i(t) = \prod_{j=1}^{n} L_{ij}(x_j(t))$ and $L_{ij}(x_j(t))$ is the grade of membership of $x_j(t)$ in L_{ij}. It should be noted that even if the rules in a TS fuzzy model (3) involve only linear combinations of the model inputs, the entire model is truly nonlinear as shown above (4).

2.2 TS Model-Based Fuzzy Control

We utilize the concept of PDC, following the terminology,[3] to design fuzzy state-feedback controllers on the basis of the TS fuzzy models (3). Linear control theory can be used to design the consequent parts of the fuzzy control rules, because the consequent parts of TS fuzzy models are described by linear state equations. If we compute the control input $u(t)$ to be

$$u_i(t) = \tilde{u}_i(t) - k_{0i} \tag{5}$$

where $k_{0i} = \frac{d_i}{B_i(1,n)}$, then the Eqn. (2) is described by

$$\dot{\mathbf{x}}(t) = \mathbf{A}_i\mathbf{x}(t) + \mathbf{B}_i\tilde{u}_i(t), \quad i = 1, 2, \cdots, r \tag{6}$$

Based on the piecewise linear model (6) we determine state feedback controller described by

$$\tilde{u}_i(t) = \mathbf{K}_i\mathbf{x}(t) \tag{7}$$

It should be noted that, however, the value of the control input actually used in the fuzzy rules would be derived from Eqn. (5). Hence a set of r control rules takes the following form:

Control Rule i:

$$If\quad x_1(t) \text{ is } L_{i1} \cdots \text{ and } x_n(t) \text{ is } L_{in}, \quad Then\quad u_i(t) = \mathbf{K}_i\mathbf{x}(t) - k_{0i} \tag{8}$$

Each of the rules can be viewed as describing a "local" state-feedback controller associated with the corresponding "local" submodel of the system to be controlled. The resulting total control action is

$$u(t) = \frac{\sum_{i=1}^{r} \lambda_i(t)\,(\mathbf{K}_i\mathbf{x}(t) - k_{0i})}{\sum_{i=1}^{r} \lambda_i(t)} \tag{9}$$

where $\lambda_i(t)'s$ are the fuzzy weights obtained in the fuzzy model of the controlled system. Note that the resulting fuzzy controller (9) is nonlinear in general since the coefficient of the controller depend nonlinearly on the system input and output via the fuzzy weights. Substituting (9) into (4), the fuzzy control system (closed-loop) can be represented by

$$\dot{\mathbf{x}}(t) = \frac{\sum_{i=1}^{r}\sum_{j=1}^{r} \lambda_i(t)\lambda_j(t)\,\{\mathbf{A}_i + \mathbf{B}_i\mathbf{K}_j\}\,\mathbf{x}(t)}{\sum_{i=1}^{r}\sum_{j=1}^{r} \lambda_i(t)\lambda_j(t)} \tag{10}$$

3 An LMI-based Fuzzy Control System Design

We consider the synthesis of fuzzy state feedback control system that guarantees stability and satisfies additional constraints on the closed-loop pole location. Wang and Tanaka[3] presented a design methodology for the stable fuzzy control system by an LMI approach. In this Section, we extend these earlier results by incorporating pole placement requirements in LMI region. By solving these two kinds of LMI constraints directly leads to a fuzzy state-feedback controller such that the resulting fuzzy control system meets both the global stability and the desired transient performance simutaneously. In particular, an attractive interior point algorithm is available to solve such LMIs.[10]

3.1 Stability Analysis via Lyapunov Approach

A sufficient quadratic stability condition derived by Tanaka and Sugeno,[12] for ensuring stability of (10) is given as follows:

THEOREM 1. *The fuzzy control system (10) is asymptotically stable in the large if there exists a common positive definite matrix P such that*

$$\{\mathbf{A}_i + \mathbf{B}_i\mathbf{K}_j\}^T\mathbf{P} + \mathbf{P}\{\mathbf{A}_i + \mathbf{B}_i\mathbf{K}_j\} < 0 \quad i,j = 1,2,\cdots,r \tag{11}$$

Note that system(10) can be also rewritten as

$$\dot{\mathbf{x}}(t) = \frac{1}{W}[\sum_{i=1}^{r} \lambda_i(t)\lambda_i(t)\mathbf{G}_{ii}\mathbf{x}(t) + 2\sum_{i<j} \lambda_i(t)\lambda_j(t)\mathbf{G}_{ij}\mathbf{x}(t \tag{12}$$

where $\mathbf{G}_{ii} = \{\mathbf{A}_i + \mathbf{B}_i\mathbf{K}_i\}, \mathbf{G}_{ij} = \frac{\{\mathbf{A}_i+\mathbf{B}_i\mathbf{K}_j\}+\{\mathbf{A}_j+\mathbf{B}_j\mathbf{K}_i\}}{2}$ $\quad i<j$, and $W = \sum_{i=1}^{r}\sum_{j=1}^{r} \lambda_i(t)\lambda_j(t)$

Applying Theorm 1, we have the following revised sufficient condition for the fuzzy control system (12).

THEOREM 2. *The fuzzy control system (12) is asymptotically stable in the large if there exists a common positive definite matrix P such that*

$$\mathbf{G}_{ii}^T\mathbf{P} + \mathbf{P}\mathbf{G}_{ii} < 0 \quad i, = 1,2,\cdots,r \quad \mathbf{G}_{ij}^T\mathbf{P} + \mathbf{P}\mathbf{G}_{ij} < 0 \quad i < j \le r \tag{13}$$

Conditions (13) are not jointly convex in $\mathbf{K}_i's$ and $\mathbf{P}$. To cast these conditions into LMIs, we define $\mathbf{Q} = \mathbf{P}^{-1}$. Then we can rewrite (13) as:

$$\mathbf{Q}\mathbf{G}_{ii}^T + \mathbf{G}_{ii}\mathbf{Q} < 0 \quad i = 1, 2, \cdots, r \quad \mathbf{Q}\mathbf{G}_{ij}^T + \mathbf{G}_{ij}\mathbf{Q} < 0 \quad i < j \le r \tag{14}$$

3.2 LMI Formulation for Pole-Placement Requirement

In the synthesis of control system, meeting some desired performances should be considered in addition to stability. Generally, stability condition(Theorem 2) does not directly deal with the transient responses of the closed-loop system. In contrast, a satisfactory transient response of a system can be guaranteed by confining its poles in a prescribed region. This section discusses a Lyapunov characterization of pole clustering regions in terms of LMIs. To this purpose, we introduce the following LMI-based representation of stability regions.

DEFINITION 1. *(LMI stability region)*[5] *A subset of D of the complex plane is called an LMI region if there exist a symmetric matrix $\alpha = [\alpha_{kl}] \in R^{m \times m}$ and a matrix $\beta = [\beta_{kl}] \in R^{m \times m}$ such that*

$$D = \{z \in C : \quad f_D(z) < 0\} \tag{15}$$

where the characteristic function $f_D(z)$ is given by $f_D(z) = [\alpha_{kl} + \beta_{kl}\ z + \beta_{kl}\ \bar{z}]_{1 \le k,l \le m}$ (f_D is valued in the space of $m \times m$ Hermitian matrices)

It is easily seen that LMI regions are convex and symmetric with respect to the real axis. Specifically, we consider circle LMI region D

$$D = \left\{x + jy \in C : \ (x+q)^2 + y^2 < r^2\right\} \tag{16}$$

centered at $(-q, 0)$ and has radius $r>0$, where the charateristic function is given by

$$f_D(z) = \begin{pmatrix} -r & \bar{z}+q \\ z+q & -r \end{pmatrix} \tag{17}$$

This circle region puts a lower bound on both exponential decay rate and the damping ratio of the closed-loop response, and thus is very common in practical control design. Motivated by Chilali and Gahinet's Theorem,[5] an extended Lyapunov Theorem for the fuzzy control system (10) is developed with above definition of an LMI-based circular pole region as belows.

THEOREM 3. *The fuzzy control system (10) is D-stable if and only if there exists a positive symmetric matrix $\mathbf{Q}$ such that*

$$\begin{pmatrix} -r\mathbf{Q} & q\mathbf{Q} + \mathbf{Q}\{\mathbf{A}_i + \mathbf{B}_i\mathbf{K}_j\}^T \\ q\mathbf{Q} + \{\mathbf{A}_i + \mathbf{B}_i\mathbf{K}_j\}\mathbf{Q} & -r\mathbf{Q} \end{pmatrix} < 0 \tag{18}$$

The proof and more details of this theorem can be found in.[5]

It should be noted that since Theorem 3 will be used for the supplimentary constraints in our problem, constraints of the LMI region to both case of $i = j$ and $i < j$ may not be necessary: it suffices to locate the poles of only dominant term(in the case of $i = j$) in the prescribed LMI regions.

3.3 Formulation for the Synthesis

In this section, we formulate a problem for the design of fuzzy state feedback control system that guarantees stability and satisfies desired transient responses by using above LMI constraints (14) and

(18). With change of variable $\mathbf{Y}_i = \mathbf{K}_i\mathbf{Q}$,$i = 1, 2, \cdots, r$ and substituting into (14) and (18), this leads to the following LMI formulation of our fuzzy state-feedback synthesis problem.

THEOREM 4. *The fuzzy control (10) is stabilizable in the specified region D via PDC if there exists a common* $\mathbf{Q}$ *and* $\mathbf{Y}_i$ *such that the following LMI conditions hold:*

$$\mathbf{A}_i\mathbf{Q} + \mathbf{Q}\mathbf{A}_i^T + \mathbf{B}_i\mathbf{Y}_i + \mathbf{Y}_i^T\mathbf{B}_i^T < 0 \quad (19)$$

$$\frac{\mathbf{A}_i\mathbf{Q} + \mathbf{Q}\mathbf{A}_i^T + \mathbf{B}_i\mathbf{Y}_j + \mathbf{Y}_j^T\mathbf{B}_i^T}{2} + \frac{\mathbf{A}_j\mathbf{Q} + \mathbf{Q}\mathbf{A}_j^T + \mathbf{B}_j\mathbf{Y}_i + \mathbf{Y}_i^T\mathbf{B}_j^T}{2} < 0 \quad (20)$$

$$\begin{pmatrix} -r\mathbf{Q} & q\mathbf{Q} + \mathbf{Q}\mathbf{A}_i^T + \mathbf{Y}_i^T\mathbf{B}_{1i}^T \\ q\mathbf{Q} + \mathbf{A}_i\mathbf{Q} + \mathbf{B}_{1i}\mathbf{Y}_i & -r\mathbf{Q} \end{pmatrix} < 0 \quad (21)$$

$$\mathbf{Q} > 0 \quad (22)$$

Given a solution $(\mathbf{Q},\ \mathbf{Y}_i)$, *the fuzzy state feedback gain is obtained as*

$$\mathbf{K}_i = \mathbf{Y}_i\mathbf{Q}^{-1} \quad (23)$$

As a result, the obtained gain guarantees global stability while provides desired transient behavior by constaint the closed-loop poles in region D.

4 Application to an Active Magnetic Bearing System

The objective of the control system of active magnetic bearing(AMB) is to maximize the stable boundary of operation with desired transient performance through overcoming the gap nonlinearity. To achieve such an objective, we design a fuzzy state-feedback controller based on Theorem 3 for a nonlinear AMB system. Then we demonstrate the validity and practicality of the obtained controller by some simulations. The model that has been studied previously in[7] and[8] will be used.

4.1 Active Magnetic Bearing(AMB) System

The AMB system which will be used is a two-axis controlled vertical shaft magnetic bearing with a symmetric structure. Due to the small gyroscopic effect of this setup,[8] the system can be devided into two identical subsystems(x-z and y-z planes), which means that each gap displacement for the x-direction and y-direction can be controlled individually. Thus, without loss of generality, we will focus our analysis strictly on the x- direction motion only.

The equations of motion for the AMB can be represented as[8]:

$$\begin{aligned} \dot{x}_1(t) &= x_2(t) \\ \dot{x}_2(t) &= \left(\frac{l^2k}{J_T}\right)\left(\frac{(i_b + i_p(t))^2}{(G - \beta x_1(t))^2} - \frac{(i_b - i_p(t))^2}{(G + \beta x_1(t))^2}\right) \end{aligned} \quad (24)$$

where x_1 denotes the displacement of the rotor from the center position, x_2 is the velocity, and i_p is the control input current applied to the electromagnets. The physical parameters of this experimental setup are given as follows:

$k(force\ constant):\ 0.00186\ lb\cdot in^2/A^2$	$\beta(sensitivity\ of\ air\ gap\ to\ shaft\ disp.):\ 0.974$
$i_b(bias\ current):\ 0.3\ A$	$G(nominal\ air\ gap):\ 0.02\ in$
$l(length\ of\ the\ rotor):\ 4.8\ in$	$J_T(transverse\ MOI\ of\ the\ rotor):0.134 lb\cdot in\cdot sec^2$

4.2 TS Fuzzy Model

We represent the nonlinear system (24) by a TS fuzzy model (3) via linearization (using Taylor's series expansion) around several operating points.[8] With considerations of the nonlinear dynamic characteristics of AMB[8] shown in Fig. 1(a), the membership functions of the fuzzy sets for x_1 and u ($= i_p$) are defined as Fig. 1(b)

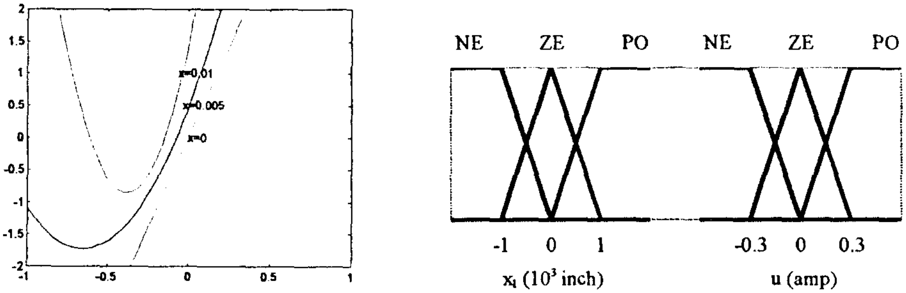


Fig.1 (a) 2-dimensional view of the characteristic of magnetic force (b) Membership function

With this definition we have totally $3^2 = 9$ rules. However, to reduce the number of fuzzy rules, the rules with similar antecedents and same consequent were grouped together and described by a single approximate rule. As a result, three rules are used to describe nonlinear dynamics (24). Denoting $\mathbf{x} = \begin{bmatrix} x_1 & x_2 \end{bmatrix}$, the piecewised linear TS fuzzy model can be written as:

Plant Rule 1 : $IF x_1 = ZE\ THEN\ \dot{\mathbf{x}}(t) = \mathbf{A}_1\mathbf{x}(t) + \mathbf{B}_1 u(t) \pm \mathbf{d}_1$
Plant Rule 2 : $IF x_1 = PO(or NE)\ and\ u = ZE,\ THEN\ \dot{\mathbf{x}}(t) = \mathbf{A}_2\mathbf{x}(t) + \mathbf{B}_2 u(t) \pm \mathbf{d}_2$
Plant Rule 3 : $IF x_1 = PO(or NE)\ and\ u = NE(or PO),\ THEN\ \dot{\mathbf{x}}(t) = \mathbf{A}_3\mathbf{x}(t) + \mathbf{B}_3 u(t) \pm \mathbf{d}_3$

where

$$\mathbf{A}_1 = \begin{bmatrix} 0 & 1 \\ 16506 & 0 \end{bmatrix}, \mathbf{A}_2 = \begin{bmatrix} 0 & 1 \\ 63640 & 0 \end{bmatrix}, \mathbf{A}_3 = \begin{bmatrix} 0 & 1 \\ 10040 & 0 \end{bmatrix}$$

$$\mathbf{B}_1 = \begin{bmatrix} 0 \\ 1130 \end{bmatrix}, \mathbf{B}_2 = \begin{bmatrix} 0 \\ 2402 \end{bmatrix}, \mathbf{B}_3 = \begin{bmatrix} 0 \\ 511 \end{bmatrix}$$

$$\mathbf{d}_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \mathbf{d}_2 = \begin{bmatrix} 0 \\ 352.7 \end{bmatrix}, \mathbf{d}_3 = \begin{bmatrix} 0 \\ 100.4 \end{bmatrix}$$

4.3 Synthesis of Fuzzy Control System

Using Theorem 4, we can design fuzzy state feedback controller that guarantees global stability while provides desired transient behavior by constraint the closed-loop poles in D. The stability region D is a circle of center (-q, 0) and radius r and the LMI synthesis is performed for a set of values:

$$\begin{pmatrix} q & r \end{pmatrix} = \begin{pmatrix} 450 & 250 \end{pmatrix}$$

then the LMI region has the following charateristic function as

$$f_D(z) = \begin{bmatrix} -250 & 450 + \bar{z} \\ 450 + z & -250 \end{bmatrix}$$

By solving LMI feasibility problem of Theorm 4, we can obtain a positive symmetric matrix $\mathbf{Q}$ as

$$\mathbf{Q} = \begin{bmatrix} 0.0001 & -0.0158 \\ -0.0158 & 4.8053 \end{bmatrix}$$

and Y_1, Y_2 and Y_3 as

$$\mathbf{Y}_1 = \begin{bmatrix} 0.0021 & -1.0435 \end{bmatrix}, \mathbf{Y}_2 = \begin{bmatrix} -0.0002 & -0.2052 \end{bmatrix}, \mathbf{Y}_3 = \begin{bmatrix} 0.0049 & -2.2716 \end{bmatrix}$$

Finally, the state feedback gain can be obtaind by (23).

$$\mathbf{K}_1 = \begin{bmatrix} -108.49 & -0.57 \end{bmatrix}, \mathbf{K}_2 = \begin{bmatrix} -71.81 & -0.28 \end{bmatrix}, \mathbf{K}_2 = \begin{bmatrix} -212.34 & -1.17 \end{bmatrix}$$

For comparison, we also calculate the state feedback gains when the constraint for the pole-placement is ommited(i.e. considering only stability condition). At that time a positive symmetric matrix $\mathbf{Q}$, matix $\mathbf{Y}_i$, and gain matrix $\mathbf{K}_i$ are as belows:

$$\mathbf{Q} = \begin{bmatrix} 0.0007 & -0.0414 \\ -0.0414 & 7.3873 \end{bmatrix}$$

$$\mathbf{Y}_1 = \begin{bmatrix} -0.0161 & 0.5540 \end{bmatrix}, \mathbf{Y}_2 = \begin{bmatrix} -0.0203 & 1.0659 \end{bmatrix}, \mathbf{Y}_3 = \begin{bmatrix} -0.0229 & 0.6460 \end{bmatrix}$$

$$\mathbf{K}_1 = \begin{bmatrix} -31.00 & -0.10 \end{bmatrix}, \mathbf{K}_2 = \begin{bmatrix} -34.35 & -0.05 \end{bmatrix}, \mathbf{K}_2 = \begin{bmatrix} -46.12 & -0.17 \end{bmatrix}$$

The resulting fuzzy control law for each piecewise linear segment of the fuzzy model can be written as follows:

Control Rule 1 : $IFx_1 = ZE,\ THEN\ u(t) = \mathbf{K}_1\mathbf{x}(t) - k_{01}$
Control Rule 2 : $IFx_1 = PO(orNE)\ and\ u = ZE(orPO),\ THEN\ u(t) = \mathbf{K}_2\mathbf{x}(t) - k_{02}$
Control Rule 3 : $IFx_1 = PO(orNE)\ and\ u = NE(orPO),\ THEN\ u(t) = \mathbf{K}_2\mathbf{x}(t) - k_{03}$

where $k_{01} = 0,\ k_{02} = 0.147$, and $k_{03} = 0.197$.

The overall fuzzy state feedback controller is

$$u(t) = \frac{(\lambda_1(t)\mathbf{K}_1 + \lambda_2(t)\mathbf{K}_2 + \lambda_3(t)\mathbf{K}_3)\,\mathbf{x}(t)}{\lambda_1(t) + \lambda_2(t) + \lambda_3(t)} \tag{25}$$

which is nonlinear.

4.4 Simulations

To investigate the effectiveness of the proposed controller, some simulations were performed. For comparison another fuzzy state feedback controller which was obtained by stability constraints only (without pole-placement constraints) was employed. In addition, a linear local controller (control rule 1) which was designed for a single equilibrium point is employed. We can see that the results of fuzzy controller which was obtained by both stability and pole placement constraints(Fig 2(b)) indicate better transient performance than those of another fuzzy controller which was obtained by stability constraints only(Fig 2(a)), while both fuzzy controller give stable response regardless of any initial displacement. Therefore, it is desiarable to tune the stability and trasient response simultaneously by combining these two objectives. The performance of these two controllers were measured by the following quadratic error index.

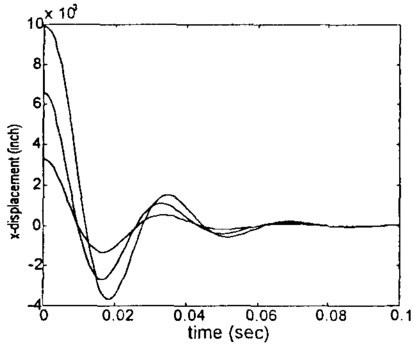
$$I_{q.e} = \int_0^t e(t)^2 dt \tag{26}$$

The above performance index was calculated for each response of three different initial dispalcements. They are summarized in Table 1. Through these results we can verify the effectiveness of the proposed multi-objective(stability+closed-loop pole location) design approach.

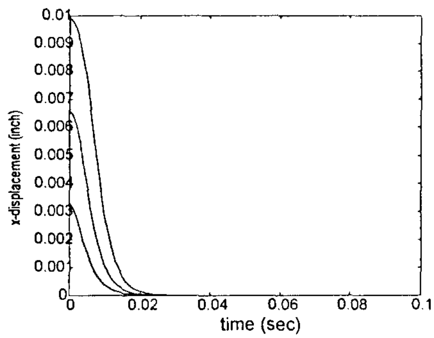
	Quadratic Error		
Initial disp.	$x_0 = 0.0033$	$x_0 = 0.0066$	$x_0 = 0.0099$
Fig. 2(a)	$7.058e-5$	$2.939e-4$	$7.498e-4$
Fig. 2(b)	$4.292e-5$	$2.120e-4$	$6.338e-4$

Table 1: Comparisons of two LMI approaches

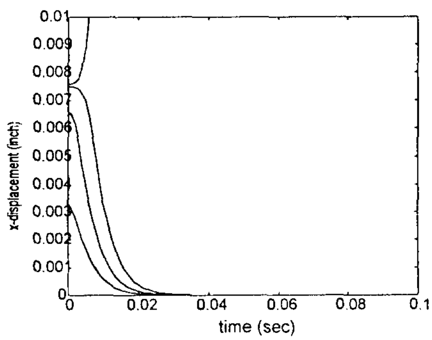
We also tested the performance of the linear local controller (control rule 1) which was designed for a single equilibrium point. As can be seen in Fig 2(c), it performs well near the equilibrium point, but its effectiveness deteriorates outside of the limited operating region and fails to regulate the rotor for the initial displacement of $|x_0 = 0.0075\ inch|$. This small boundary of stability is due largely to the nonlinearity of AMB.



(a)



(b)



(c)

Fig. 2 2 Responses of (a) fuzzy control with constraints for stability only (b) fuzzy control with constraints for both stability and pole-placement (c) linear local control with constraints for both stability and pole-placement

5 Conclusion

In this paper, we presented a systematic design methodology for the stable fuzzy control of nonlinear dynamic syatems with pole placement in a specified region in a complex plane . By imposing the additional requirment of the closed-loop pole location, we can prevent fast controller dynamics and achieve good transient behavior with guaranteed stability. More significantly, in the proposed methodology, the control design problems which considers both the global stability and the desired transient performance are reduced to the LMI problems. Therefore solving these LMI constraints directly leads to a fuzzy state-feedback controller such that the resulting fuzzy control system meets above two objectives. As a result, this approach is superior to other existing approaches which achieves the desired control performances by trial-and error. To demonstrate the usefulness of the proposed design methodology, it is applied to the regulation problem of a nonlinear magnetic bearing system. Simulation results show that the proposed LMI-based design methodology yields not only maximized stability boundary but also the desired transient responses.

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