QUASI-FUZZY EXTREMALLY DISCONNECTED SPACES

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Abstract: In this paper, we introduce the concept of quasi-fuzzy extremally disconnectedness in fuzzy bitopological space, which is a generalization of fuzzy extremally disconnectedness due to Ghosh [5] in fuzzy topological space and investigate some of its properties using the concepts of quasi-semi-closure, quasi-θ-closure and related notions in a fuzzy bitopological setting.

1. INTRODUCTION AND PRELIMINARIES

Motivated by the fact that there are some non-symmetric fuzzy topological structures, Kubiak [8] introduced the notion of fuzzy bitopological space, henceforth fbts for short, (A triple (X, τ_1, τ_2) where X is a non-empty set and τ_1 and τ_2 are fuzzy topology on X is called a fuzzy bitopological space) and initiated the bitopological aspects due to Kelly [7] in the theory of fuzzy topological spaces. Since then several authors [3,4,6,8-10,12] have contributed to the subsequent development of various fuzzy bitoplogical properties. Recently, Park et.al. [12] introduced the concept of quasi-fuzzy open (quasi-fuzzy closed) sets and studied its basic properties. They also introduced the concepts of quasi-fuzzy separation axioms, quasi-fuzzy connectedness and quasi-fuzzy continuity by using quasi Q-neighborhoods and quasi-neighborhoods.

In this paper, we introduce the concepts of quasi-fuzzy semiopen, quasi-fuzzy preopen and quasi-fuzzy pre-semiopen sets and study their basic properties. We also introduce the concept of quasi-fuzzy extremally disconnectedness and characterize its properties in terms of the concepts of quasi-semi-closure, quasi-θ-closure and related notions in fuzzy bitopological setting.

For definitions and results not explained in this paper, we refer to the papers [2,11-13] assuming them to be well known. A fuzzy point in X with support $x \in X$ and value α $(0 < \alpha \leq 1)$ is denoted by x_α. For a fuzzy set A of X, $1 - A$ will stand for the complement of A. By 0_X and 1_X we will mean respectively the constant fuzzy sets taking on the values 0 and 1 on X. A fuzzy set A of a fbts (X, τ_1, τ_2) is called quasi-fuzzy-open (briefly, qfo) if for each fuzzy point $x_\alpha \in A$ there exists either a τ_1-fo set U such that $x_\alpha \in U \leq A$, or a τ_2-fo set V such that $x_\alpha \in V \leq A$. A fuzzy set is quasi-fuzzy-closed (briefly, qfc) set if the complement is a qfo set. A fuzzy set A is said to be quasi-Q-nbd (resp. quasi-nbd) of a fuzzy point x_α if there exists a qfo set U such that $x_\alpha q U \leq A$ (resp. $x_\alpha \in U \leq A$). A fuzzy point x_α belongs to qcl(A) (the quasi closure of a fuzzy set A with respect to a fuzzy bitopology) if every qfo quasi-Q-nbd of x_α is q-coincident with A [12].

Definition 1.1. Let A be a fuzzy set of a fbts (X, τ_1, τ_2). Then A is called

(a) quasi-fuzzy semiopen (briefly, qfso) if there exists a qfo set B such that $B \leq A \leq \mathrm{qcl}(B)$,
(b) quasi-fuzzy semiclosed (briefly, qfsc) if there exists a qfc set B such that $\mathrm{qint}(B) \leq A \leq B$,
(c) quasi-fuzzy preopen (briefly, qfpo) if $A \leq \mathrm{qint}(\mathrm{qcl}(A))$,

(d) quasi-fuzzy preclosed (briefly, qfpc) if $A \geq \mathrm{qcl}(\mathrm{qint}(A))$,
(e) quasi-fuzzy semi-preopen (briefly, qfspo) if $A \leq \mathrm{qcl}(\mathrm{qint}(\mathrm{qcl}(A)))$,
(f) quasi-fuzzy semi-preclosed (briefly, qfspc) if $A \geq \mathrm{qint}(\mathrm{qcl}(\mathrm{qint}(A)))$.

Remark 1.2. Every qfo (resp. qfc) set is both qfso (resp. qfsc) and qfpo (resp. qfpc) but the converses are not true. Every qfso (resp. qfsc) set, or qfpo (resp. qfpc) set is qfspo (resp. qfspc) set but the converses are not true.

Example 1.3. Let $X = \{a, b, c\}$ and A_i $(i = 1, 2, 3, 4)$ be a fuzzy set of X defined as follows:

$$A_1(a) = 0.5, A_1(b) = 0.8, A_1(c) = 0.5; \quad A_2(a) = 0.5, A_2(b) = 0.3, A_2(c) = 0.5;$$
$$A_3(a) = 0.6, A_3(b) = 0.2, A_3(c) = 0.6; \quad A_4(a) = 0, \quad A_4(b) = 0.3, A_4(c) = 0.$$

(a) Let $\tau_1 = \{1_X, 0_X, A_1\}$ and $\tau_2 = \{1_X, 0_X, A_2\}$ be fuzzy topologies on X. Consider a fuzzy set B of X defined $B(a) = 0.5, B(b) = 0.6, B(c) = 0.5$. Then B is a qfso set but neither a qfo set nor a qfpo set.

(b) Let $\tau_1 = \{1_X, 0_X, 1 - A_1\}$ and $\tau_2 = \{1_X, 0_X, 1 - A_2\}$ be fuzzy topologies on X. Consider a fuzzy set C of X defined $C(a) = 0.5, C(b) = 0.4, C(c) = 0.5$. Then C is a qfpo set but neither a qfo set nor a qfso set.

(c) Let $\tau_1 = \{1_X, 0_X, A_3\}$ and $\tau_2 = \{1_X, 0_X, A_4\}$ be fuzzy topologies on X. Consider a fuzzy set D of X defined $D(a) = 0.7, D(b) = 0.5, D(c) = 0.2$. Then D is a qfspo set but neither a qfso set nor a qfpo set.

Theorem 1.4. *For a fuzzy set A in a fbts (X, τ_1, τ_2), the following are equivalent:*
(a) A is qfsc (resp. qfso) set,
(b) $\mathrm{qint}(\mathrm{qcl}(A)) \leq A$ (resp. $A \leq \mathrm{qcl}(\mathrm{qint}(A))$),
(c) $\mathrm{qint}(A) = \mathrm{qint}(\mathrm{qcl}(A))$ (resp. $\mathrm{qcl}(A) = \mathrm{qcl}(\mathrm{qint}(A))$).

Theorem 1.5. *(a) Any union of qfso (resp. qfpo, qfspo) sets is a qfso (resp. qfpo, qfspo) set, and*
(b) any intersection of qfsc (resp. qfpc, qfspc) sets is a qfsc (resp. qfpc, qfspc) set.

Definition 1.6. Let A be a fuzzy set of fbts X. Then
(a) $\mathrm{qscl}(A) = \cup\{B \mid B$ is qfsc set and $A \leq B\}$ is called quasi semi-closure of A,
(b) $\mathrm{qsint}(A) = \cup\{B \mid B$ is qfso set and $A \geq B\}$ is called quasi semi-interior of A.

Remark 1.7. For a fuzzy set of fbts X, $\mathrm{qint}(A) \leq \mathrm{qsint}(A) \leq A \leq \mathrm{qscl}(A) \leq \mathrm{qcl}(A)$, $\mathrm{qsint}(1 - A) = 1 - \mathrm{qscl}(A)$ and $\mathrm{qscl}(A)$ (resp. $\mathrm{qsint}(A)$) is qfsc (resp. qfso) set.

Theorem 1.8. *Let A be a fuzzy set of a fbts X. Then $x_\alpha \in \mathrm{qscl}(A)$ if and only if for each qfso set U with $x_\alpha qU$, UqA.*

2. QUASI-FUZZY EXTREMALLY DISCONNECTED SPACES

Definition 2.1. A fbts X is said to be quasi-fuzzy extremally disconnected (briefly, QFED) if quasi closure of every qfo set is qfo in X.

Theorem 2.2. *A fbts X is QFED if and only if any two non-q-coincident qfo sets of fbts X have non-q-coincident quasi-closures.*

Proof. Let X be a QFED and A, B be qfo sets of X such that $A\bar{q}B$. Then $\mathrm{qcl}(A)$ is a qfo set and hence $\mathrm{qcl}(A)\bar{q}\,\mathrm{qcl}(B)$. Conversely, let U be a qfo set of X. Then $1-\mathrm{qcl}(U)$ is qfo in X. Now, we have

$$\begin{aligned} U\bar{q}(1-\mathrm{qcl}(U)) &\Rightarrow \mathrm{qcl}(U)\bar{q}\,\mathrm{qcl}(1-\mathrm{qcl}(U)) \\ &\Rightarrow \mathrm{qcl}(1-\mathrm{qcl}(U)) \leq 1-\mathrm{qcl}(U) \\ &\Rightarrow \mathrm{qcl}(1-\mathrm{qcl}(U)) = 1-\mathrm{qcl}(U) \\ &\Rightarrow (1-\mathrm{qcl}(U)) \text{ is qfc} \\ &\Rightarrow \mathrm{qcl}(U) \text{ is qfo.} \end{aligned}$$

Hence X is QFED.

Theorem 2.3. *The following are equivalent for a fbts X:*
(a) X is QFED.
(b) For each qfso set A of X, $qcl(A)$ is a qfo set.
(c) For each qfso set A of X, $qscl(A)$ is a qfo set.
(d) For each qfso set A and each qfso set B with $A\bar{q}B$, $qcl(A)\bar{q}\,qcl(B)$.
(e) For each qfso set A of X, $qcl(A) = qscl(A)$.
(f) For each qfso set A of X, $qscl(A)$ is a qfc set.
(g) For each qfsc set A of X, $qint(A) = qsint(A)$.
(h) For each qfsc set A of X, $qsint(A)$ is a qfo set.

Proof. (a)⇔(b): It follows from Theorem 1.4.

(a)⇒(e): Since $\mathrm{qscl}(A) \leq \mathrm{qcl}(A)$ for any fuzzy set A of X, it sufficient to show that $\mathrm{qscl}(A) \geq \mathrm{qcl}(A)$ for any qfso set A of X. Let $x_\alpha \notin \mathrm{qscl}(A)$. Then there exist a qfso set W with $x_\alpha qW$ such that $W\bar{q}A$. Thus $\mathrm{qint}(W)$ and $\mathrm{qint}(A)$ are qfo sets such that $\mathrm{qint}(W)\bar{q}\,\mathrm{qint}(A)$. By Theorem 2.2, $\mathrm{qcl}(\mathrm{qint}(W))\bar{q}\,\mathrm{qcl}(\mathrm{qint}(A))$ and then by Theorem 1.4, $x_\alpha \notin \mathrm{qcl}(\mathrm{qint}(A)) = \mathrm{qcl}(A)$. Hence $\mathrm{qcl}(A) \leq \mathrm{qscl}(A)$.

(e)⇒(f): Obvious.

(f)⇒(e): For any fuzzy set A of X, $A \leq \mathrm{qscl}(A) \leq \mathrm{qcl}(A)$ from Remark 1.7. Then $\mathrm{qcl}(A) = \mathrm{qcl}(\mathrm{qscl}(A))$. Since A is a qfso set, by (f), $\mathrm{qscl}(A)$ is a qfc set. Hence $\mathrm{qcl}(A) = \mathrm{qscl}(A)$.

(f)⇔(h): Follows from Remark 1.7.

(g)⇒(h): Obvious.

(h)⇒(g): For any fuzzy set A of X, $\mathrm{qint}(A) \leq \mathrm{qsint}(A) \leq A$ and hence $\mathrm{qint}(A) = \mathrm{qint}(\mathrm{qsint}(A))$. Since A is a qfsc set, by (h), $\mathrm{qsint}(A)$ is qfo set. Hence $\mathrm{qint}(A) = \mathrm{qsint}(A)$.

(a)⇒(d): Let A be a qfso set and B be a qfso set such that $A\bar{q}B$. Then we have $\mathrm{qint}(A)\bar{q}\,\mathrm{qint}(B)$ and thus by Theorem 2.2, $\mathrm{qcl}(\mathrm{qint}(A))\bar{q}\,\mathrm{qcl}(\mathrm{qint}(B))$. Hence by Theorem 1.4, $\mathrm{qcl}(A)\bar{q}\,\mathrm{qcl}(B)$.

(d)⇒(b): Let A be a qfso set of X. Then $1-\mathrm{qcl}(A)$ is a qfso set and $A\bar{q}(1-\mathrm{qcl}(A))$. Thus by (d), $\mathrm{qcl}(A)\bar{q}\,\mathrm{qcl}(1-\mathrm{qcl}(A))$ which implies $\mathrm{qcl}(A) \leq \mathrm{qint}(\mathrm{qcl}(A))$. Hence $\mathrm{qcl}(A) = \mathrm{qint}(\mathrm{qcl}(A))$ and consequently $\mathrm{qcl}(A)$ is qfo set of X.

(e)⇒(d): Let A be a qfso set and B be a qfso set such that $A\bar{q}B$. Then $\mathrm{qscl}(A)$ is qfso and $\mathrm{qscl}(B)$ is qfso in X and hence $\mathrm{qscl}(A)\bar{q}\,\mathrm{qscl}(B)$. By (e), $\mathrm{qcl}(A)\bar{q}\,\mathrm{qcl}(B)$.

(a)⇒(c): Follows from Theorem 1.4 using same method as (a)⇒(e).

(c)⇒(a): Let A be a qfo set of a fbts X. It is sufficient to prove $\mathrm{qcl}(A) = \mathrm{qscl}(A)$. Obviously, $\mathrm{qscl}(A) \leq \mathrm{qcl}(A)$. Let $x_\alpha \notin \mathrm{qscl}(A)$. Then there exists a qfso set U with $x_\alpha qU$ such that $A\bar{q}U$. Hence $\mathrm{qscl}(U) \leq \mathrm{qscl}(1-A) = 1-A$ and thus $\mathrm{qscl}(U)\bar{q}A$. Since $\mathrm{qscl}(U)$ is qfo set with $x_\alpha q\,\mathrm{qscl}(U)$, $x_\alpha \notin \mathrm{qscl}(A)$. Hence $\mathrm{qcl}(A) \leq \mathrm{qscl}(A)$.

Definition 2.4 **[Park et.al].** A fuzzy point x_α in a fbts X is said to be a quasi-θ-cluster point of a fuzzy set A if for every qfo quasi-Q-nbd U of x_α, $\mathrm{qcl}(U)\mathrm{q}A$. The set of all fuzzy quasi-θ-cluster points of A is called the quasi-θ-closure of A and will be denoted by $\mathrm{qcl}_\theta(A)$. A fuzzy set A is called quasi-θ-closed if $A = \mathrm{qcl}_\theta(A)$.

It is easy to see that $\mathrm{qcl}(A) \leq \mathrm{qcl}_\theta(A)$ for any fuzzy set A of a fbts X.

Lemma 2.5. *For any qfpo set A of a fbts X, $qcl(A) = qcl_\theta(A)$.*

Theorem 2.6. *The following are equivalent for a fbts X:*
(a) X is QFED.
(b) The quasi-closure of every qfspo set of X is qfo set.
(c) The quasi-θ-closure of every qfpo set of X is qfo set.
(d) The quasi-closure of every qfpo set of X is qfo set.

Proof. (a)$\Rightarrow$(b): Let A be a qfspo set. Then $\mathrm{qcl}(A) = \mathrm{qcl}(\mathrm{qint}(\mathrm{qcl}(A)))$. Since X is QFED, $\mathrm{qcl}(A)$ is qfo set.
(b)$\Rightarrow$(d): Follows from the fact that every qfpo set is qfspo set.
(d)$\Rightarrow$(a): Clear.
(c)$\Leftrightarrow$(d): Follows from Lemma 2.5.

Lemma 2.7. *For a fuzzy set A of a fbts X,*
(a) $qint(qcl(A)) \leq qscl(A)$,
(b) $qint(qscl(A)) = qint(qcl(A))$.

Definition 2.8. Let A be a fuzzy set of a fbts X. Then A is called
(a) quasi-fuzzy regularly open (briefly, qfro) if $A = \mathrm{qint}(\mathrm{qcl}(A))$,
(b) quasi-fuzzy regularly closed (briefly, qfrc) if $A = \mathrm{qcl}(\mathrm{qint}(A))$.

Every qfro (resp. qfrc) set is qfo (resp. qfc) set and qfsc (resp. qfso) set but the converse need not be true as following example shows.

Example 2.9. Let $X = \{a, b, c\}$, $\tau_1 = \{1_X, 0_X, A\}$ and $\tau_2 = \{1_X, 0_X, B\}$, where A and B are fuzzy sets in X defined as follows: $A(a) = 0.7, A(b) = 0.5, A(c) = 0.3$; $B(a) = 0.4, B(b) = 0.6, B(c) = 0.5$. Then A is a qfo set but not a qfro set. Also a fuzzy set C of X, defined by $C(a) = 0.4, C(b) = 0.5, C(c) = 0.3$, is a qfsc set but not a qfro set.

Lemma 2.10. *Let A be a fuzzy set of a fbts X. Then we have*
(a) A is qfpo if and only if $qscl(A) = qint(qcl(A))$.
(b) A is qfpo if and only if $qscl(A)$ is qfro set.
(c) A is qfro set if and only if A is qfpo and qfsc.

Theorem 2.11. *In a fbts X, the following are equivalent:*
(a) X is QFED.
(b) $qscl(A) = qcl_\theta(A)$ for every qfpo (or qfso) set A of X.
(c) $qscl(A) = qcl(A)$ for every qfspo set A of X.

Proof. (a)$\Rightarrow$(b): Since $\mathrm{qscl}(A) \leq \mathrm{qcl}_\theta(A)$ for any fuzzy set A of X, it sufficient to show that $\mathrm{qcl}_\theta(A) \leq \mathrm{qscl}(A)$ for any qfpo or qfso set A of X. Let $x_\alpha \notin \mathrm{qscl}(A)$. Then there exists a qfso set with $x_\alpha \mathrm{q} U$ such that $U \bar{\mathrm{q}} A$ and thus there exists a qfo set V such that $V \leq U \leq \mathrm{qcl}(V)$ with $V \bar{\mathrm{q}} A$ which implies $V \bar{\mathrm{q}}\ \mathrm{qcl}(A)$. This means $V \bar{\mathrm{q}}\ \mathrm{qint}(\mathrm{qcl}(A))$ and hence $\mathrm{qcl}(V) \bar{\mathrm{q}}\ \mathrm{qint}(\mathrm{qcl}(A))$. Now, if A is qfpo set, then $A \leq \mathrm{qint}(\mathrm{qcl}(A))$ and hence $\mathrm{qcl}(V) \bar{\mathrm{q}} A$. If A is qfso set, since X is QFED, $\mathrm{qcl}(V)$ is qfo set, and thus $\mathrm{qcl}(V) \bar{\mathrm{q}}\ \mathrm{qcl}(\mathrm{qint}(\mathrm{qcl}(A)))$ which implies $\mathrm{qcl}(V) \bar{\mathrm{q}} A$. Hence in any case, $x_\alpha \notin \mathrm{qcl}_\theta(A)$.

(b)$\Rightarrow$(a): First let A be a qfpo set of X. By Lemmas 2.10 and 2.5, we have qint(qcl(A)) = qscl(A) = qcl$_\theta$(A) = qcl(A). Then qcl(A) is qfo in X and hence by Theorem 2.7, X is QFED. Next, let A be a qfso set of X. Then qscl(A) $\leq$ qcl(A) $\leq$ qcl$_\theta$(A) = qscl(A) and thus qscl(A) = qcl(A). Hence X is QFED from Theorem 2.6.

(a)$\Rightarrow$(c): Let A be a qfspo set of X. Since X is QFED, by Theorem 2.6, qcl(A) is qfo in X. Hence by Lemma 2.7, qscl(A) = qcl(A).

(c)$\Rightarrow$(a): Let U and V be qfo sets such that $U\bar{q}V$. Then $U \leq 1 - V$ which implies qscl(U) $\leq$ qscl($1 - V$) $= 1 - V$ and hence qscl(U)$\bar{q}V$. Since qscl(U) is qfso in X, qscl(U)$\bar{q}$ qscl(V). Then by (c) qcl(U)$\bar{q}$ qcl(V) and hence by Theorem 2.2, X is QFED.

Theorem 2.12. *In a fbts X, the following are equivalent:*

(a) X is QFED.

(b) For each qfspo set A and each qfso set B such that $A\bar{q}B$, qcl(A)$\bar{q}$ qcl(B).

(c) For each qfpo set A and each qfso set B such that $A\bar{q}B$, qcl(A)$\bar{q}$ qcl(B).

Proof. (a)$\Rightarrow$(b): Let A be a qfspo set and B be a qfso set such that $A\bar{q}B$. Then $A\bar{q}$ qint(B) and hence qcl(A)$\bar{q}$ qint(B). By Theorem 2.6, qcl(A) is a qfo set of X and hence qcl(A)$\bar{q}$ qcl(qint(B)). Since B is qfso in X, by Theorem 1.4, qcl(B) = qcl(qint(B)). Thus qcl(A)$\bar{q}$ qcl(B).

(b)$\Rightarrow$(c): Straightforward.

(c)$\Rightarrow$(a): Let A and B be qfo sets such that $A\bar{q}B$. Since every qfo set is both qfso and qfpo, qcl(A)$\bar{q}$ qcl(B). Hence by Theorem 2.2, X is QFED.

Theorem 2.13. *A fbts X is QFED if and only if every qfso set is qfpo set.*

Proof. Let A be a qfso set. Then $A \leq$ qcl(qint(A)). Since X is QFED, qcl(qint(A)) is qfo set and then $A \leq$ qcl(qint(A)) = qint(qcl(qint(A))) $\leq$ qint(qcl(A)). Hence A is qfpo set. Conversely, let A be a qfro set. Then A is qfso set. By hypothesis, A is qfpo set so that qcl(A) = qint(qcl(A)). Then qcl(A) is qfo in X and hence X is QFED.

Definition 2.14. Let $f : X \to Y$ be a mapping from a fbts X to another fbts Y. f is called a

(a) quasi-fuzzy semi-continuous (briefly, qfsc) if $f^{-1}(V)$ is a qfso set of X for each qfo set V of Y, equivalently, $f^{-1}(V)$ is a qfsc set of X for each qfc set V of Y.

(b) quasi-fuzzy almost-open (briefly, qfao) if $f(U)$ is a qfo set of Y for each qfro set U of X.

Lemma 2.15. *Let $f : X \to Y$ be a mapping from a fbts X to another fbts Y. Then*

(a) f is qfsc if and only if f(qscl(A)) $\leq$ qcl($f(A)$) for each fuzzy set A of X.

(b) f is qfao if and only if f(qint(A)) $\leq$ qint($f(A)$) for each qfsc set A of X.

Lemma 2.16. *If $f : X \to Y$ is qfsc and qfao mapping, then $f(A)$ is a qfpo set of Y for each qfpo set A of X.*

Lemma 2.17. *If $f : X \to Y$ is qfsc and qfao mapping, then we have*

(a) $f^{-1}(B)$ is a qfsc set of X for each qfsc set B of Y.

(b) $f^{-1}(B)$ is a qfso set of X for each qfso set B of Y.

Theorem 2.18. *Let $f : X \to Y$ be a qfsc and qfao surjection. If X is QFED, then Y is also QFED.*

Proof. Let B be a qfso set of Y. By Lemma 2.17, $f^{-1}(B)$ is qfso in X. Since X is QFED, by Theorem 2.13, $f^{-1}(B)$ is qfpo in X. By Lemma 2.16, B is qfpo in Y and hence by Theorem 2.13, Y is QFED.

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