비선형 퍼지 미분 시스템에 대한 α-수준 완전 제어가능성

The α-level controllability for the nonlinear fuzzy differential systems

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1. Introduction and Preliminaries

Let E_N be the set of all upper semicontinuous convex fuzzy numbers with bounded α-level intervals.

The purpose of this note is to investigate the α-level controllability of nonlinear fuzzy control system.

(F.D.E)
$$\begin{cases} \dot{x}(t) = a(t)x(t) + f(t, x(t)) + u(t), \\ x(0) = x_0 \end{cases}$$

where $a : [0, T] \to E_N$ is fuzzy coefficient, $x_0 \in E_N$ is initial value and nonlinear function $f : [0, T] \times E_N \to E_N$ satisfies a global Lipschitz condition and control function $u(t)$ is fuzzy number.

We find the α-level exact controllability conditions of the (F.C.S) on the assumption that the following linear fuzzy control system (F.C.S 1) is α-level exact controllable :

(F.C.S.1)
$$\begin{cases} \dot{x}(t) = a(t)x(t) + u(t), \\ x(0) = x_0 \in E_N. \end{cases}$$

We consider properties of the fuzzy number and metrics.

A fuzzy subset of R^n is defined in terms of a membership function which assigns to each point $x \in R^n$ a grade of membership in the fuzzy set. Such a membership function

$$u : R^n \rightarrow [0,1]$$

is used synonomously to denote the corresponding fuzzy set.

Assumption 1. u maps R^n onto $[0,1]$.

Assumption 2. $[u]^0$ is a bounded subset of R^n.

Assumption 3. u is upper semicontinuous.

Assumption 4. u is fuzzy convex.

We denote by E^n the space of all fuzzy subsets u of R^n which satisfy Assumptions 1-4; that is, normal, fuzzy convex, upper semicontinuous fuzzy sets with bounded supports.

In particular $n=1$, denote by E^1 the space of all fuzzy subsets u of R which satisfy Assumptions 1-4.

A fuzzy number a in real line R is a fuzzy set characterized by a membership function μ_a as $\mu_a : R \rightarrow [0,1]$.

A fuzzy number a is expressed as $a = \int_{x \in R} \mu_a(x)/x$ with the understanding that $\mu_a(x) \in [0,1]$ represent the grade of membership of x in a and $\int$ denotes the union of $\mu_a(x)/x$'s.

A fuzzy number a in R is said to be convex if for any real numbers $x, y, z \in R$ with $x \leq y \leq z$,

$$\mu_a(y) \geq \min\{\mu_a(x), \mu_a(z)\}.$$

A fuzzy number a in R is called normal if the following holds

$$\max_x \mu_a(x) = 1.$$

Let E_N be the set of all upper semicontinuous convex fuzzy numbers with bounded α-level intervals. This means that if $a \in E_N$ then the α-level set

$$[a]^\alpha = \{x \in R \mid a(x) \geq \alpha, 0 < \alpha \leq 1\}$$

is a closed bounded interval which we denote by

$$[a]^\alpha = [a_l^\alpha, a_r^\alpha]$$

and there exists a $t_0 \in R$ such that $a(t_0) = 1$.

Two fuzzy numbers a and b are called equal $a = b$, if $a(x) = b(x)$ for all $x \in R$. It follows that

$$a = b \Leftrightarrow [a]^\alpha = [b]^\alpha \text{ for all } \alpha \in (0,1].$$

A fuzzy number a may be decomposed into its level sets through the resolution identity

$$a = \int_0^1 \alpha[a]^\alpha ,$$

where $\alpha[a]^\alpha$ is the product of a scalar α with the set $[a]^\alpha$ and $\int$ is the union of $[a]^\alpha$'s with α ranging from 0 to 1.

The support Γ_a of a fuzzy number a is defined, as a special case of level set, by the following

$$\Gamma_a = \{x \mid \mu_a(x) > 0\}.$$

A fuzzy number a in R is said to be positive if $0 < a_1 < a_2$ holds for the support $\Gamma_a = [a_1, a_2]$ of a, that is, Γ_a is in the positive real line. Similarly, a is called negative if $a_1 \le a_2 < 0$ and zero if $a_1 \le 0 \le a_2$.

Lemma 1.1. ([12]) If $a, b \in E_N$, then for $\alpha \in (0, 1]$,

$$[a+b]^\alpha = [a_l^\alpha + b_l^\alpha, a_r^\alpha + b_r^\alpha],$$

$$[a \cdot b]^\alpha = [\min\{a_i^\alpha b_j^\alpha\}, \max\{a_i^\alpha b_j^\alpha\}] \quad (i, j = l, r),$$

$$[a-b]^\alpha = [a_l^\alpha - b_r^\alpha, a_r^\alpha - b_l^\alpha].$$

Lemma 1.2 ([12]) Let $[a_l^\alpha, a_r^\alpha]$, $0 < \alpha \le 1$, be a given family of nonempty intervals. If (1) $[a_l^\beta, a_r^\beta] \subset [a_l^\alpha, a_r^\alpha]$ for $0 < \alpha \le \beta$ and (2) $[\lim_{k\to\infty} a_l^{\alpha_k}, \lim_{k\to\infty} a_r^{\alpha_k}] = [a_l^\alpha, a_r^\alpha]$ whenever (α_k) is nondecreasing sequence converging to $\alpha \in (0, 1]$, then the family $[a_l^\alpha, a_r^\alpha], 0 < \alpha \le 1$, represents the α-level sets of a fuzzy number $a \in E_N$.

Conversely, if $[a_l^\alpha, a_r^\alpha], 0 < \alpha \le 1$, are the α-level sets of a fuzzy number $a \in E_N$, then the conditions (1) and (2) holds true.

Let x be a point in R^n and A be a nonempty subset of R^n. We define the distance $d(x, A)$ from x to A by

(1.1) $$d(x, A) = \inf\{\|x - a\| : a \in A\}.$$

Now let A and B be nonempty subsets of R^n. We define the Hausdorff separation of B from A by

(1.2) $$d_H^*(B, A) = \sup\{d(b, A) : b \in B\},$$

in general $d_H^*(A, B) \neq d_H^*(B, A)$.

We define the Hausdorff distance between nonempty subsets of A and B of R^n by

(1.3) $$d_H(A, B) = \max\{d_H^*(A, B), d_H^*(B, A)\}.$$

This is now symmetric in A and B. Consequently

(1) $d_H(A,B) \geq 0$ with $d_H(A,B)=0$ if and only if $\overline{A}=\overline{B}$.
(2) $d_H(A,B) = d_H(B,A)$
(3) $d_H(A,B) \leq d_H(A,C)+d_H(C,B)$

for any nonempty subsets of A, B and C of R^n. The Hausdorff distance (1.3) is a metric, the Hausdorff metric.

The supremum metric d_∞ on E^n is defined by

(1.4) $$d_\infty(u, v) = \sup\{d_H([u]^\alpha, [v]^\alpha) : \alpha \in (0,1]\} \text{ for all } u, v \in E^n$$

and is obviously metric on E^n.

The supremum metric H_1 on $C([0,T]:E^n)$ is defined by

(1.5) $$H_1(x,y) = \sup\{d_\infty(x(t), y(t)) : t \in [0,T]\} \text{ for all } x, y \in C([0,T]:E^n).$$

2. The α-level controllability of nonlinear fuzzy differential system

We consider the α-level controllability of nonlinear fuzzy control system.

(F.D.E) $$\begin{cases} \dot{x}(t) = a(t)x(t) + f(t,x(t)) + u(t), \\ x(0) = x_0 \end{cases}$$

where $a:[0,T] \to E_N$ is fuzzy coefficient, initial value $x_0 \in E_N$ and control function $u:[0,T] \to E_N$ and nonlinear function $f:[0,T] \times E_N \to E_N$ satisfies a global Lipschitz condition. i.e., there exists a finite constant $k > 0$ such that

$$d_H([f(s,\xi_1(s))]^\alpha, [f(s,\xi_2(s))]^\alpha) \leq k\, d_H([\xi_1(s)]^\alpha, [\xi_2(s)]^\alpha)$$

for all $\xi_1(s)$, $\xi_2(s) \in E_N$.

The (F.C.S) is related to the following fuzzy integral system :

(F.I.S) $$\begin{cases} x(t) = S(t)x_0 + \int_0^t S(t-s)f(s,x(s))\,ds + \int_0^t S(t-s)u(s)\,ds, \\ x(0) = x_0 \in E_N. \end{cases}$$

Definition 2.1. The (F.I.S) is α-level exact controllable if, there exists $u(t)$ such that the fuzzy solution $x(t)$ of (F.I.S) satisfies $[x(T)]^\alpha = [x^1]^\alpha$ where x^1 is target set.

We assume that the following linear fuzzy control system with respect to nonlinear fuzzy control system (F.C.S) :

(F.C.S 1)
$$\begin{cases} \dot{x}(t) = a(t)x(t) + u(t) , \\ x(0) = x_0 \in E_N \end{cases}$$

is α-level exact controllable. Then

$$[x(T)]^\alpha = [S(T)x_0 + \int_0^T S(T-s)u(s)ds]^\alpha$$
$$= [S_l^\alpha(T)x_{0l}^\alpha + \int_0^T S_l^\alpha(T-s)\,u_l^\alpha(s)\,ds\ ,\ S_r^\alpha(T)x_{0r}^\alpha + \int_0^T S_r^\alpha(T-s)\,u_r^\alpha(s)\,ds]$$
Defined
$$= [(x^1)_l^\alpha,\ (x^1)_r^\alpha].$$

the fuzzy mapping $\tilde{g}: \tilde{P}(R) \to E_N$ by

$$\tilde{g}^\alpha(v) = \begin{cases} \int_0^T S^\alpha(T-s)\,v(s)ds\,, & v \subset \overline{\Gamma_u}, \\ 0\,, & \text{otherwise}. \end{cases}$$

Then there exist $\tilde{g}_i^\alpha$ $(i = l, r)$ such that

$$\tilde{g}_l^\alpha(v) = \int_0^T S_l^\alpha(T-s)\,v_l(s)ds\,,\quad v_l(s) \in [\,u_l^\alpha(s)\,,\,u^1(s)\,],$$
$$\tilde{g}_r^\alpha(v) = \int_0^T S_r^\alpha(T-s)\,v_r(s)ds\,,\quad v_r(s) \in [\,u^1(s)\,,\ u_r^\alpha(s)\,].$$

We assume that $\tilde{g}_l^\alpha$, $\tilde{g}_r^\alpha$ are bijective mappings. Hence α-level of $u(s)$ are

$$[u(s)]^\alpha = [u_l^\alpha(s)\,,\,u_r^\alpha(s)]$$
$$= [(\tilde{g}_l^\alpha)^{-1}((x^1)_l^\alpha - S_l^\alpha(T)x_{0l}^\alpha)(s),\ (\tilde{g}_r^\alpha)^{-1}((x^1)_r^\alpha - S_r^\alpha(T)x_{0r}^\alpha)(s)].$$

Thus we can be introduced $u(s)$ of nonlinear system Then substitutin

$$[u(s)]^\alpha = [u_l^\alpha(s), u_r^\alpha(s)]$$
$$= [(\tilde{g}_l^\alpha)^{-1}((x^1)_l^\alpha - S_l^\alpha(T)x_{0l}^\alpha - \int_0^T S_l^\alpha(T-s)f_l^\alpha(s,x(s))ds),$$
$$(\tilde{g}_r^\alpha)^{-1}((x^1)_r^\alpha - S_r^\alpha(T)x_{0r}^\alpha - \int_0^T S_r^\alpha(T-s)f_r^\alpha(s,x(s))ds)].$$

$$[x(T)]^\alpha$$
$$= [S_l^\alpha(T)x_{0l}^\alpha + \int_0^T S_l^\alpha(T-s)f_l^\alpha(s,x(s))ds$$
$$+ \int_0^T S_l^\alpha(T-s)(\tilde{g}_l^\alpha)^{-1}((x^1)_l^\alpha - S_l^\alpha(T)x_{0l}^\alpha - \int_0^T S_l^\alpha(T-s)f_l^\alpha(s,x(s))ds)ds\ ,$$
$$S_r^\alpha(T)x_{0r}^\alpha + \int_0^T S_r^\alpha(T-s)f_r^\alpha(s,x(s))ds$$
$$+ \int_0^T S_r^\alpha(T-s)(\tilde{g}_r^\alpha)^{-1}((x^1)_r^\alpha - S_r^\alpha(T)x_{0r}^\alpha - \int_0^T S_r^\alpha(T-s)f_r^\alpha(s,x(s))ds)ds]$$

this expression into the (F.I.S) yields α-level of $x(T)$

$$= [S_l^\alpha(T)x_{0l}^\alpha + \int_0^T S_l^\alpha(T-s)f_l^\alpha(s,x(s))\,ds$$

$$+ \widetilde{g}_l^\alpha \cdot (\widetilde{g}_l^\alpha)^{-1}((x^1)_l^\alpha - S_l^\alpha(T)x_{0l}^\alpha - \int_0^T S_l^\alpha(T-s)f_l^\alpha(s,x(s))\,ds),$$

$$S_r^\alpha(T)x_{0r}^\alpha + \int_0^T S_r^\alpha(T-s)f_r^\alpha(s,x(s))\,ds$$

$$+ \widetilde{g}_r^\alpha \cdot (\widetilde{g}_r^\alpha)^{-1}((x^1)_r^\alpha - S_r^\alpha(T)x_{0r}^\alpha - \int_0^T S_r^\alpha(T-s)f_r^\alpha(s,x(s))\,ds)\,]$$

$$= [(x^1)_l^\alpha, (x^1)_r^\alpha] = [x^1]^\alpha.$$

We now set

$$\Phi x(t) =_\alpha S(t)x_0 + \int_0^t S(t-s)f(s,x(s))\,ds$$

$$+ \int_0^t S(t-s)\ \widetilde{g}^{-1}(x^1 - S(T)x_0 - \int_0^T S(T-s)f(s,x(s))\,ds)\,ds,$$

where the fuzzy mappings $\widetilde{g}^{-1}$ is satisfied above statements.

Notice that $\Phi x(T) =_\alpha x^1$, which means that the control $u(t)$ steers the (F.C.S) from the origine to x^1 in time T provided we can obtain a fixed point of the nonlinear operator Φ.

Assume that the following hypotheses:

(H1) (F.C.S 1) is α-level exact controllable.

(H2) Inhomogeneous term $f: [0,T] \times E_N \rightarrow E_N$ satisfies a global Lipschitz condition. i.e., there exists a finite constant constant $k > 0$ such that

$$d_H([f(s,\xi_1(s))]^\alpha, [f(s,\xi_2(s))]^\alpha) \le k\, d_H([\xi_1(s)]^\alpha, [\xi_2(s)]^\alpha)$$

for all $\xi_1(s)$, $\xi_2(s) \in E_N$.

Theorem 2.1. Suppose that hypotheses (H1), (H2) are satisfied. Then the state of the (F.I.S) can be steered from the initial value x_0 to any final state x^1 in time T.

Proof. Omitted.

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