Noise Reduction using Fuzzy Mathematical Morphology

Takuo Kikuchi*1, Mikio Nakatsuyama*2, Shuta Murakami*3

*1 Department of Electronic Technology, Kochi Polytechnic College
1595-1 Nishino, Noichi ,Kami-gun ,Kochi 781-5232, Japan
Tel:+81-8875-6-4107 Fax:+81-8875-6-4130 E-mail:takuo@kochi-pc.ac.jp
*2 Department of Industrial Design, Akita Municipal Junior College of Art and Craft
12-3 Araya-Okawamachi, Akita 010-16, Japan
Tel: +81-188-88-8139 Fax: +81-188-88-8109 E-mail: nakatsu@eos.amcac.ac.jp
*3 Department of Computer Science, Faculty of Engineering, Kyushu Institute of technology
1-1 Sensuicho, Tobata, Kitakyushu, Fukuoka 804, Japan
Tel/Fax: +81-93-884-3244 E-mail: murakami@comp.kyutech.ac.jp

Abstract

Mathematical morphology (MM) has been introduced as a powerful tool for studying the geometrical properties of images. MM is a good approach to digital image processing, which is based on the shape features. The MM operators such as dilation, erosion, closing and opening have been applied successfully to image noise reduction. The MM filters can easily filter the noise when the noise factors are known. However it is very difficult to reduce the noise when images are ambiguous, because the boundary between the noise and object is vague. In this paper, we propose a new method to reduce noise from ambiguous images by using Fuzzy Mathematical Morphology (FMM) operators. Performance evaluation via simulations show that the FMM filters efficiently reduce the image noise. Furthermore, the FMM filters show a good performance compared with the conventional filters.

Keywords: Fuzzy mathematical morphology, structuring elements, noise reduction, image processing

1. Introduction

MM has been introduced as a powerful tool for studying the geometrical properties of images[1][2]. The basic MM operators are dilation, erosion, closing and opening. These transformations use the structuring element to interact with the set and to extract the information. A structuring element is a set which has a simple size and shape. The structuring element is chosen for its geometric properties.

The MM filters can filter the signals based on MM theory operators. The MM filters can be used for filtering the noise when the noise factors are known. However, they have a bad performance for noise reduction of ambiguous images. This is because the boundary between the noise and object is vague and a cross-over point can't be found easily. To deal with this problem, FMM has been proposed as a good approach [3][4][5][6][7]. Minjin uses FMM for image analysis[3]. Shinha and Dougetry have given the FMM definition and features[4]. They use the brightness of the pixel as the membership functions. Images are modeled as fuzzy subsets of the Euclidean plane or Cartesian grid, and the morphological operations are defined in terms of a fuzzy index function. Gesu has applied FMM to analyze real images[5]. Nakatsuyama has given some definitions of a new FMM[6].

In this paper, we propose a method to reduce the noise in the noisy and ambiguous images, by using FMM operators definition in ref.[6]. We explain the characteristics of FMM operators and how to use them for noise reduction. Furthermore, we show that the establishment of the cross-over point is very important element to extract the object in ambiguous images. For this reason, the S-function is extended and the cross-over point is decided using the histogram method. Using the proposed method , the object characteristic can be extracted from the noisy images. Performance evaluation via simulations show that the FMM is a good approach for noise reduction.

The organization of this paper is as follows. The definitions of FMM operators are introduced in Section 2. In Section 3, the noise reduction algorithm will be presented. Simulation results are discussed in Section 4.

Conclusions are given in Section 5.

2. Definitions of FMM Operators

In this section, we define FMM operations: fuzzy translation, fuzzy erosion, fuzzy dilation, fuzzy opening and fuzzy closing.

2.1 Fuzzy Translation

U_x is denoted as fuzzy translation.

$$U = (a_1/u_1 + a_2/u_2 + \ldots\ldots a_n/u_n). \qquad (1)$$

Let μ_x be b. Then, U_x is,

$$U_x = \{ (a_1 \vee b) / (u_1 + x) + (a_2 \vee b) / (u_2 + x) + \ldots\ldots + (a_n \vee b) / (u_n + x) \}. \qquad (2)$$

where u_n and x are vectors.

2.2 Fuzzy Erosion and Fuzzy Dilation definitions

Given two fuzzy sets A and B, fuzzy erosion($\ominus$)and fuzzy dilation($\oplus$) of image A by structuring element B can be founded to be [6],

$$A \oplus B = \{ \mu a/a: Bx \subset X \ \ \mu x \geqq \mu b, \ y \in B \ \ \mu a \leftarrow \mu a \wedge \mu y, \ a = x + y \}, \qquad (3)$$

where x or y denotes the coordinate of the pixel and means as a vector, too.

$$A \ominus B = \{ \mu a/a: Bx' \subset X' \ \ \mu x' \geqq \mu b, \ y' \in B' \ \ \mu a \leftarrow \mu a \wedge \mu y, \ a = x' + y \}, \qquad (4)$$

where X' is the complement of X.

2.3 Fuzzy Opening and Fuzzy Closing definitions

The FMM operations such as fuzzy opening(X_B) and fuzzy closing(X^B) can be expressed in terms of fuzzy erosion and fuzzy dilation as follows:

$$X_B = (X \ominus B) \oplus B, \qquad (5)$$

$$X^B = (X \oplus B) \ominus B. \qquad (6)$$

3. Noise Reduction

The noise reduction algorithm flow is shown in Fig.1. First, the input images are transformed into fuzzy sets (Fuzzification). The extended S function is used in order to get the optimal membership values which are suitable for removing the noise. Next, the combination of fuzzy operators to ambiguous images is applied. Finally, defuzzification is performed by applying the inverse of the membership function previously used in the input original image (Defuzzification).

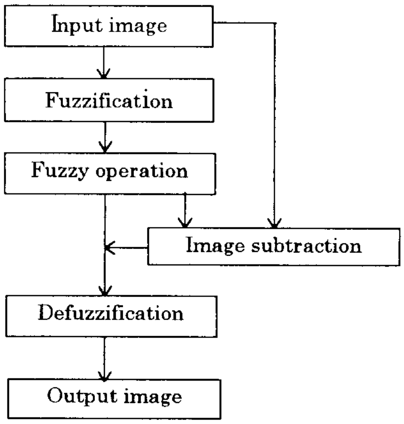

Fig.1 The flow of noise reduction

3.1 Fuzzification

Each pixel of the picture has its own characteristics such as brightness, color, etc. The brightness or color is essentially a fuzzy quantity which has some quantity of grades. A membership function is used to assign the fuzzy grade to the object. To extend MM to real images, we transform the original data in a fuzzy set. In this way, each fuzzified pixel value can be interpreted as the grade of membership of that pixel into the original data set. In the fuzzification step, the extended S-function is used to fuzzify the input image.

The S-function is defined as:

$$S(X_{mn}: a, b, c) = \begin{cases} 0 & X_{mn} \leqq a \\ 2\{(X_{mn} - a) / (c - a)\}^2 & a \leqq X_{mn} \leqq b \\ 1 - 2\{(X_{mn} - a) / (c - a)\}^2 & b \leqq X_{mn} \leqq c \\ 1 & X_{mn} \geqq c, \end{cases} \qquad (7)$$

where b = (a + c)/2 is the cross-over point, c is the point at which the height of S-function is equal to 1, and a is the initial point. The X_{mn} parameter is the intensity value of the input image at the pixel(m,n).

The establishment of cross-over point is a difficult task in ambiguous images. The degree of image characteristics decreases toward the background. Therefore, it is difficult to assign a suitable membership function value using the conventional methods. In our method, we use the histogram to define the cross-over point. There are two histogram patterns in images with shadow.

3.1.1 One peak value in histogram

(a)Rapid changes of gray level.

In this case, the cross-over point value can be decided as $(x_{max} + x_{min})/2$.

(b)Slow changes of gray level.

In this case, the cross-over point which has a high characteristic in the boundary between the object and background is moved behind the middle point toward the decrease direction of gray level. Based on this a high membership value is assigned to the boundary between the shadow and object. This procedure is illustrated in Fig.2.

3.1.2 Several peak values in histogram

In this case, the lowest peak is absorbed into the neighboring peak. Therefore, the cross-over point is assigned optionally at the peak of the object. This operation is shown in Fig.3.

3.2 Structuring element

The structuring element is a fuzzy matrix of n * m elements with a given central pixel. A structuring element is shown in Fig.4. A high membership values is assigned to an input image when the structuring element is applied to a definite image. The structuring element used in our work has the following characteristics:

1. Disk type,
2. 3*3 size,
3. All membership values in the structuring element are fixed,
4. The membership value of the structuring element is high.

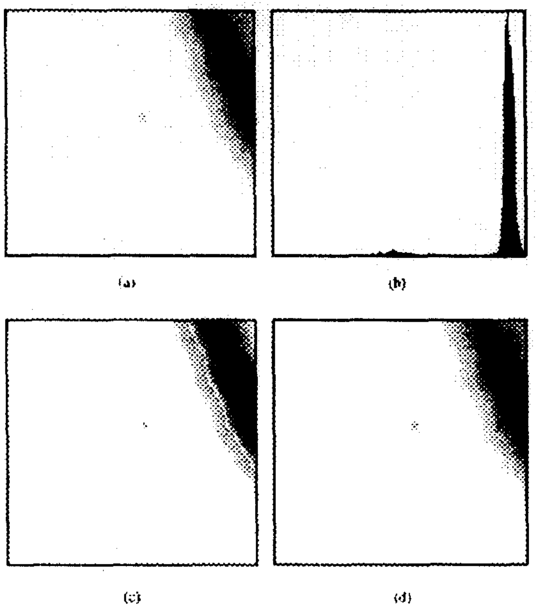

Fig.2 An aurora image (the example of gray level decreasing monotonously) : (a) Input image, (b) Histogram of the image, (c) Use of S-function, (d) Use of optional cross-over points .

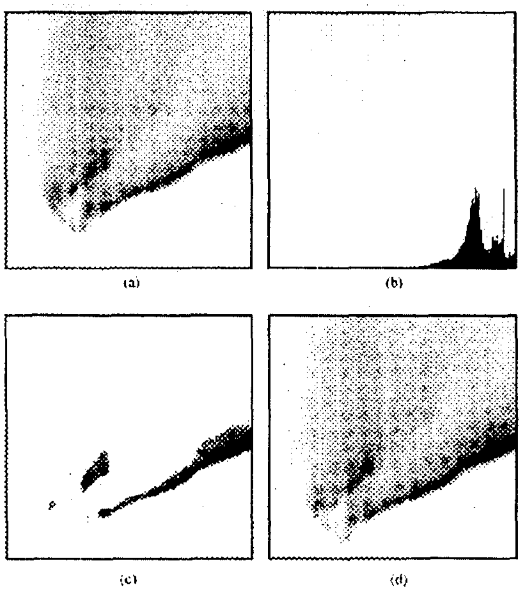

Fig.3 An aurora image (the example of gray level decrease rapidly) : (a) Input image, (b) Histogram of the image, (c) Using of S-function, (d) Use of optional cross-over points .

3.3 FMM Operations

An example of FMM operations is shown in Fig.5. In Fig.5(a) and (b) are shown the input image membership values and input image, respectively. Fig.5 (c) shows the membership values after the 'fuzzy erosion' operation is applied using structuring element of Fig.4. The output image is shown in Fig.5(d).

	0.9	
0.9	0.9	0.9
	0.9	

Fig.4 A structuring element.

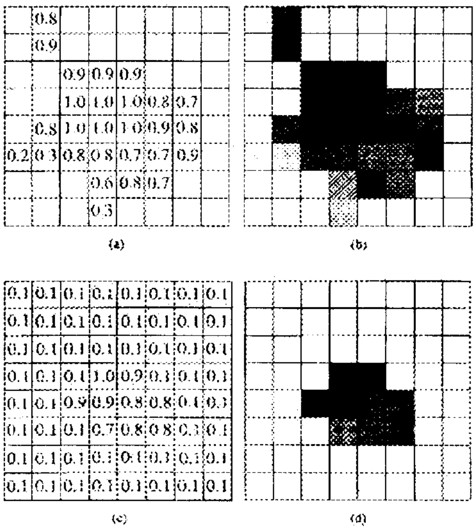

Fig.5 Example of fuzzy erosion: (a) Membership function of input image , (b) Input image, (c) Membership function after fuzzy erosion, (d) Output image.

4. Simulation

In Fig.6, we present an example of FMM filtering performed on a shaded image. The original images are shown in Fig.6(a). After fuzzification, the FMM operator 'fuzzy erosion' have been applied to the original image (see Fig.6(b)) with a disk flat structuring element of size 3*3. Fig.(c) is shown the use of the FMM filter 'fuzzy opening'. The simulation show that the FMM operators are able to remove the noise and to preserve the inner finer structures of the object.

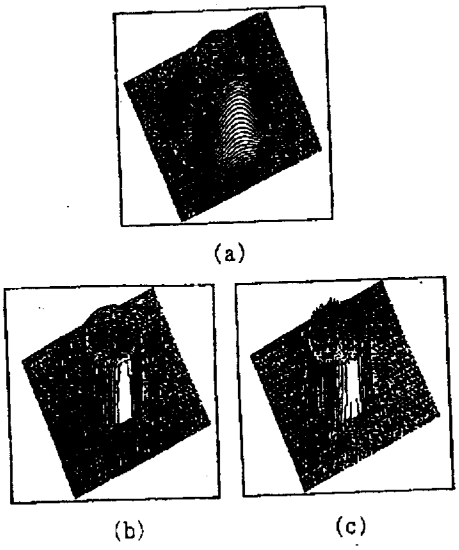

Fig.6 Use of fuzzy erosion and fuzzy opening operators: (a) Original image, (b) Fuzzy erosion, (c) Fuzzy opening.

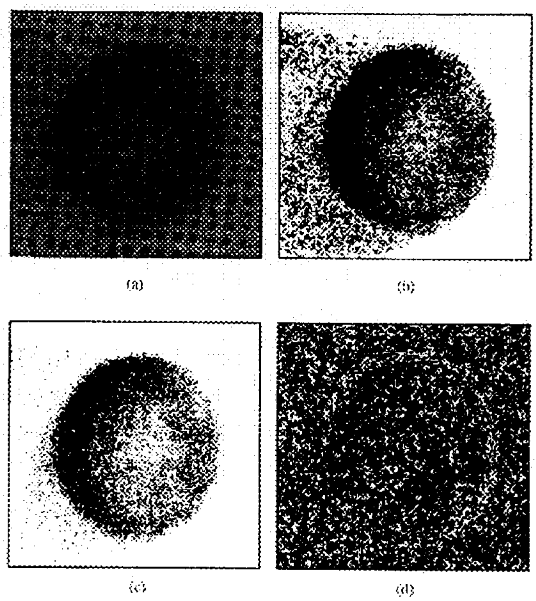

Fig7. Noise reduction results: (a) Input image, (b) F uzzy erosion, (c) Fuzzy opening, (d) Traditional filte r.

Fig.7 is shown an example when the picture is taken during an insufficient light. The input image is shown in Fig.7(a). Fig.7(b) shows the result when the 'fuzzy erosion' operator is applied. Fig.7(c) shows the difference between the images of Fig.7(b) and the original image was taken after 'fuzzy opening' was applied. Fig7(d) shows the result when a traditional filter is applied. The simulation shows that the FMM filter gives a good performance compared with the traditional filter.

Fig.8 is shown an example when a shadow exists a character. The input image is shown in Fig.8(a). Fig.8(b) is shown the result when the 'fuzzy opening' was applied. Fig.8(c) shows the result when the threshold method was applied. The results show the FMM filters have a better extraction characteristic compared with threshold method.

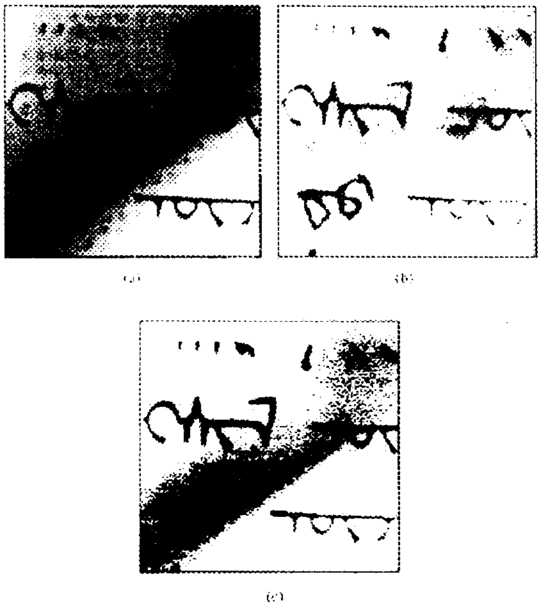

Fig.8 Character extraction: (a) Input image, (b) FM M operation, (c) Threshold method.

5. Conclusion

In this paper, we propose a new method to remove noise in images by using FMM filters. We explained the characteristics of FMM operators and their use for noise reduction. We showed that the establishment of the cross-over point is a very important for extracting the object from ambiguous images. For this reason, the S-function was extended and the cross-over point was decided using the histogram method. The proposed method was investigated by simulations. From the simulation, we conclude:

1. The combination of FMM operators give good results for noise reduction.
2. The FMM is an efficient method for noise reduction, especially when image have a bad contrast.
3. The FMM filters show a better performance than the traditional filters.

In the future, the authors would like to extend the simulations of the FMM for recognition of knots.

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