The Third AFSS(1998). 691=695

Stabilized Control of Inverted Pendulum System by ANFIS

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Abstract - Most of systems has nonlinearity. And also accurate modelings of these uncertain nonlinear systems are very difficult. In this paper, a fuzzy modeling technique for the stabilization control of an IP(inverted pendulum) system with nonlinearity was proposed. The fuzzy modeling was acquired on the basis of ANFIS(Adaptive Neuro Fuzzy Infernce System) which could learn using a series of input-output data pairs. Simulation results showed its superiority to the PID controller. We believe that its applicability can be extended to the other nonlinear systems.

Keyword : nonlinear system, ANFIS(Adaptive Neuro-Fuzzy Inference System), IP(inverted pendulum), fuzzy modeling

1. Introduction

Generally speaking, most of system has nonlinearity and also modeling of this uncertainty nonlinear system is very complicated structure. Thus, for the control of this nonlinear system is proposed fuuzzy control technique by L.A. Zadeh in 1995. It is often applies to the difficult of mathematical modeling and strong of its nonlinearity.

But, design of stable fuzzy control system is very hard task because of all for human expert knoeledges are difficult to create automatically membership functions, and optium of each rule.

Therefore, in this paper discuss to the no accurate mathematical models of system under control and human expert knowledges are available to provied linguistic control rules descriptions about the IP(inverted pendulum)systems. The classical methods such as statespace analysis are difficult to tune their parameters.

Therefore, in this paper comparing with the ordinary fuzzy control technique, the proposed ANFIS can make efficiently the IP system through its simulations. Furthermore simulation results showes its effectiveness and usefulness other nonlinear system for the side of dynamic characteristic and adapt to the disturbance.

2. Mathematical Modeling

The mathematical model of IP system was derived from Fig.1.

Cart

$$M\ddot{z}(t)+\mu\dot{z}(t)=\alpha\cdot\mu(t)-H(t) \qquad (1)$$

Pendulum in the horizontal direction

$$H(t) = m\frac{d^2}{dt^2}\{ z(t) + L \sin \phi(t)\}$$
$$= m\,\ddot{z}(t) + mL$$
$$\{ \ddot{\phi}(t) \cos \phi(t) - \dot{\phi}^2(t) \sin \phi(t)\} \quad (2)$$

Pendulum in the vertical direction

$$V(t) - mg = m\frac{d^2}{dt^2}\{ L \cos \phi(t)\}$$
$$= -mL\{ \ddot{\phi}(t) \sin \phi(t) + \dot{\phi}^2(t) \cos \phi(t)\} \quad (3)$$

Turning direction

$$J_p \ddot{\phi}(t) = LV(t) \sin\phi(t) - LH(t) \cos\phi(t) - \eta\dot{\phi} \quad (4)$$

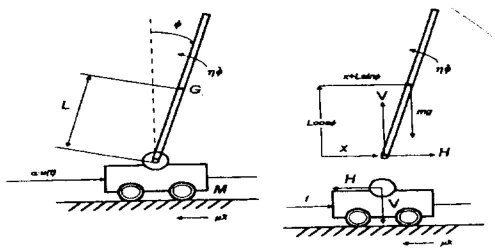


Fig. 1 IP System

And its parameters are Table 1.

Table 1 System Parameters

parameter	DESCRIPTION	VALUE	UNIT
m	Mass of the pendulum	0.112	kg
L	Length from the pivot to the mass center of pendulum	0.255	m
V	Vertical reactive force at the pivot	———	$kg \cdot m/s^2$
H	Horizontal reactive force at the pivot	———	$kg \cdot m/s^2$
J_p	Inertia moment of the pendulum	0.00292	$kg \cdot m^2$
M	Mass of the cart	0.886	kg
z	Horizontal position of the cart	-0.655~ 0.655	m
ϕ	Angle of the pendulum from the vertical line	$-\pi \sim \pi$	rad
r	Wire belt pulley radius	0.0235	m
a	Driving force coefficient	0.738	$kg \cdot m/s^2/V$
u	Input voltage to push the cart	-10 ~ 10	V
G	Mass center of the pendulum	———	———
g	Gravitational acceleration constant	9.8	m/s^2
μ	Friction coefficient	2.215	kg/s
η	Coefficient related to rotation moment of the pendulum	0.00199	$kg \cdot m^2/s$

As shown in Fig.1, the process is represented by four variables: the angle between the pendulum and the vertical axis ($x_3 = \phi$), its deriative as the angular vemocity ($x_4 = \dot{\phi}$), the horizontal position ($x_1 = x$) and its deriative as the horizontal velocity ($x_2 = \dot{x}$). By neglecting the infinitesimal H, the model was rewritten as follow.

$$\ddot{\phi} = \frac{1}{(J - p + mL^2)}\{ mgL\phi - mL\ddot{x} - \eta\dot{\phi}\} \quad (6)$$

The resultant dynamic equation is given :

$$\dot{x} = Ax + Bu \quad (7)$$

$$y = Cx \quad (8)$$

Where,

$$A = \begin{bmatrix} -\frac{\mu}{M} & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ \frac{\mu mL}{M(J_p + mL^2)} & 0 & -\frac{\eta}{J_p + mL^2} & \frac{mgL}{J_p + mL^2} \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} \frac{a}{M} \\ 0 \\ -\frac{amL}{M(J_p + mL^2)} \\ 0 \end{bmatrix} \qquad C = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3. Design of Fuzzy Controller

An architecture of the fuzzy logic controller used in this paper is illustrated in figure 2 and is consisted of 4 parts; the *Fuzzification interface,* the *Knowledge base,* the *Fuzzy inference engine and* the *Defuzzification interface.*

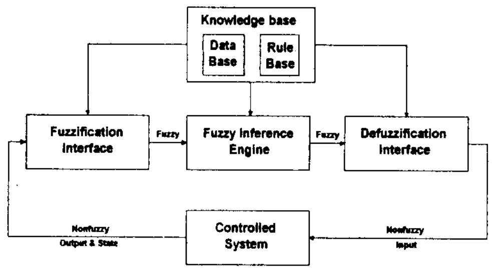
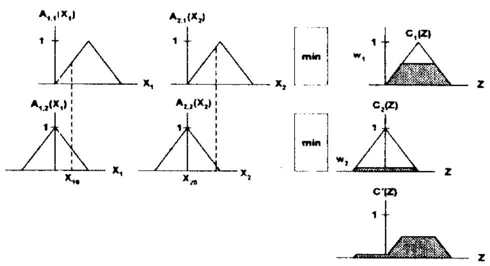


Fig. 2 Structure of FLC

The method inferring the resultant defuzzified value is as Fig. 3.



$$z = \frac{w_1 z_1 + w_2 z_2}{w_1 + w_2}$$

Fig. 3 Fuzzy Inference Diagram (Sugeno-Takagi Method)

4. Design of ANFIS

4.1 Adaptive Neuro Fuzzy Control

The architecture and learning procedure of the ANFIS(fig.4) was shown. This has multilayer feedforward networks with supervised learning capability. An adaptive network is consisted of nodes and directional links for interconnections of the nodes. Moreover, using outputs of all the nodes, their parameters were learned by the steepest gradient method and the chain rule which was proposed by Werbros in the 1970's, in order to minimize the output errors calculated from the learning data.

1) Basic Learning Rule

ANFIS in Fig.4 has a adaptive square node and a fixed circle node. For adaptive network of L layers, the output of the ith node on the kth layer, O_i^k is given.

$$O_i^k = O_i^k(O_i^{k-1}, \ldots, O_{*(k-1)}^{(k-1)}, a, b, c, \ldots) \quad (10)$$

Where, $a, b, c, \ldots$ are the parameters of these nodes. If the given training data have P entires, the error measure for the pth($1 \leq p \leq P$) entry is defined as the sum of squred errors of eq.(11).

$$E_p = \sum_{m=1}^{*(L)} (T_{m,p} - O_{m,p}^L)^2 \quad (11)$$

Therefore, $\frac{\partial E_p}{\partial O_{i,p}^L}$, the error rate at the i th node of layer L can be calculated readily.

$$\frac{\partial E_p}{\partial O_{i,p}^L} = -2(T_{i,p} - O_{i,p}^L) \quad (12)$$

The error rate at (i, k) can be derived by the chain rule.

$$\frac{\partial E_p}{\partial O_{i,p}^k} = \sum_{m=1}^{*(k+1)} \frac{\partial E_p}{\partial O_{m,p}^{k+1}} \cdot \frac{\partial O_{m,p}^{k+1}}{\partial O_{i,p}^k} \quad (13)$$

If a is a parameter of the given adaptive network, we can get eq.(14).

$$\frac{\partial E_p}{\partial a} = \sum_{O* \in S} \frac{\partial E_p}{\partial O*} \cdot \frac{\partial O*}{\partial a} \quad (14)$$

Then the derivative of the overall error measure E with respect to a is given.

$$\frac{\partial E}{\partial a} = \sum_{p=1}^{p} \frac{\partial E_p}{\partial a} \quad (15)$$

Accordingly, the updated formula for the generic parameter a is acquired.

$$\Delta a = -\eta \frac{\partial E}{\partial a} \quad (16)$$

Where, η is a learning rate defined as eq.(17) for the step size k_s which means the length of each gradient transition in the parameter space.

$$\eta = \frac{k_s}{\sqrt{\sum_a \frac{\partial E}{\partial a}^2}} \quad (17)$$

The speed of convergence is dependent on the magnitude of k_s.

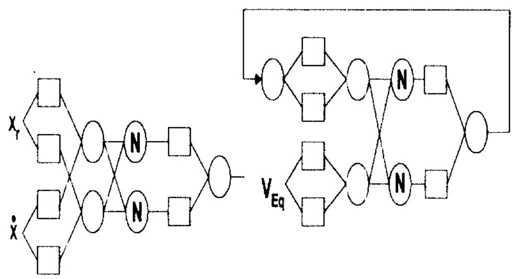


Fig.4 Structure of ANFIS

4.2 Fuzzy Conrtrol Rules

The block diagram of fuzzy control system to stabilize the structurally unstable IP system is shown in Fig.5.

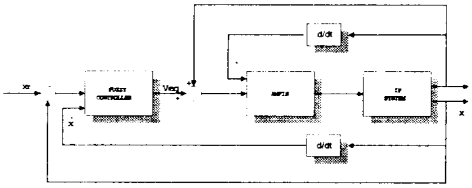


Fig.5 Block diagram of fuzzy control system

It is important to decide the fuzzy rules written in If-Then format. For example, if a pendulum slide to the right direction, then apply the positive control input u to move cart toward the right direction.

Table 2 shows the whole 25 fuzzy control rules for ϕ and $\dot{\phi}$ on the basis of above principles.

Table 2 Fuzzy rules for Stabilization

$\dot{\phi}$ \ ϕ	NB	NM	ZO	PM	PB
NB	NG	NB	NM	NS	ZO
NM	NB	NM	NS	ZO	PS
ZO	NM	NS	ZO	PS	PM
PM	NS	ZO	PS	PM	PB
PB	ZO	PS	PM	PB	PG

Fig.6 shows the membership functions of ϕ, $\dot{\phi}$ and u. Specially, u was given as single tone.

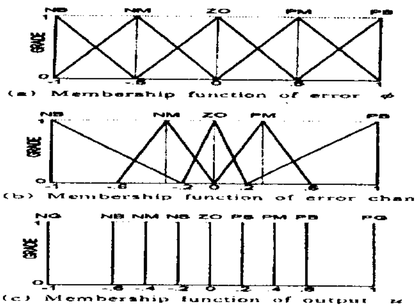


Fig. 6 Membership function for the inverted pendulum

Fig. 7 shows the membership function of n , x $\dot{x}$ and the virtual equilibrium point ϕ_{VEq}. The virtual equilibrium point ϕ_{VEq} is necessary to move the cart with the stabilized pendulum toward the desired position.

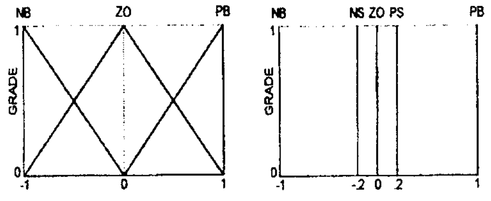


Fig. 7 Membership functions of x, $\dot{x}$ and ϕ_{VEq}

Table 3 shows the rules for the inference of the ϕ_{VEq}.

Table 3. Fuzzy rules for ϕ_{VEq}

x \ $\dot{x}$	NB	ZO	PB
NB	NS	NB	ZO
ZO	NS	ZO	PS
PB	ZO	PB	PS

5. Simulation Results

In this paper, two simulation results are shown for the initial position of cart, -0.4[m] and the initial angle of pendulum, -0.2[rad]. The simulation results of a general fuzzy controller is shown in Fig.8 and the one of ANFIS controller is shown Fig.9 . The maximum overshoot of cart was 0.1[m] and the reaching time was shortened by 1.5[sec] than one of the general fuzzy controller. The maximum overshoot of pendulum was 0.58[rad]. This data was reduced by 0.22[rad] than one of the general fuzzy controller.

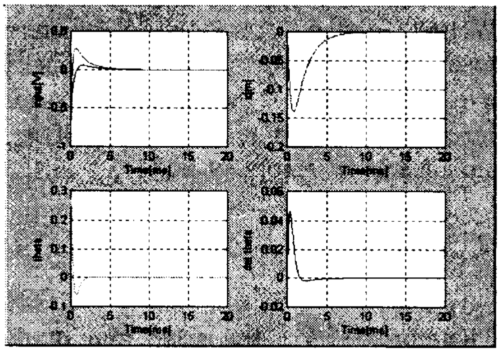


Fig.8 General fuzzy controller

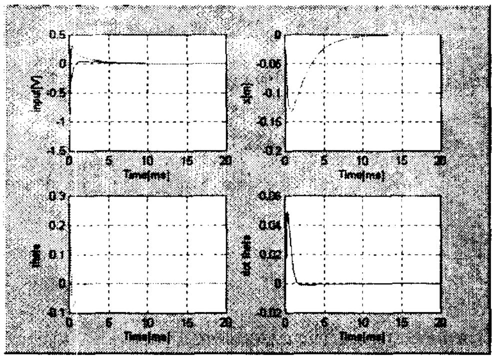


Fig.9 ANFIS Controller

6. Conclusions

In this paper, the ANFIS controller was designed to stabilize the unstable IP system. The performances of this proposed controller was superior to the conventional fuzzy one in indices such as reaching time, residual steady-state deviation and overshoot.

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