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On-line Identification of a Fuzzy System

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Abstract

This paper presents an explanation regarding on-line identification of a fuzzy system. The fuzzy system to be identified is assumed to be in the type of singleton consequent parts and be represented by a linear combination of fuzzy basis functions (FBF's). For on-line identification, squared-cosine (SCOS) fuzzy basis function is introduced to reduce the number of parameters to be identified and make the system consistent and differentiable. Then the parameters of the fuzzy system are identified on-line by the gradient search method. Finally, a computer simulation is performed to illustrate the validity of the suggested algorithms.

I. INTRODUCTION

In most applications of the fuzzy theory (*e.g.*, the application in control systems or prediction systems), the main design objective is to construct a fuzzy system to approximate a desired control system or process. Until now, however, only a few studies on the automatic identification of the fuzzy system have been conducted.

Pedrycz[1] suggested the identification algorithms of fuzzy relational model. Sugeno and his colleagues proposed the identification of so-called TSK(Takagi-Sugeno-Kang) fuzzy system [2][3]. Recently, other researchers also participate in the identification of the TSK fuzzy system [4][5]. Sugeno and Yasukawa reported qualitative modeling of a fuzzy system in [6] and some researchers attempted to identify the fuzzy system via the neural-network-based approaches [7][8].

However, most of these are the off-line algorithms and cannot be applied to the situations where real-time processing is required such as adaptive control and signal processing. Even though on-line successive fuzzy modeling was suggested in [9], it cannot be viewed as an on-line algorithm since it requires an initial fuzzy model which is fully constructed in advance by other algorithms.

To solve this problem, this paper proposes on-line identification algorithms of a fuzzy system. For on-line identification, s*quared-cosine* (SCOS) fuzzy basis functions (FBF's) are introduced for the following purposes:

(1) to reduce the number of parameters to be identified,

(2) to make the inference of the fuzzy system simple by equating the products of the membership functions to FBF's,

(3) to make the system consistent and differentiable.

The parameters of the fuzzy system are identified on-line by the gradient search.

II. FUZZY SYSTEMS AND SOME PROPERTIES

Since Mamdani applied fuzzy logic to a practical system, many different fuzzy systems have been used with different structures, membership functions, etc. In this paper, the following fuzzy system is considered:

$$R^{n_1 n_2 \cdots n_m}: \text{If } x_1 \text{ is } A_1^{n_1} \text{ and } \cdots, x_m \text{ is } A_m^{n_m} \text{ then } y \text{ is } \theta^{n_1, n_2, \cdots, n_m} \quad (n_1 = 1, \cdots, M_1, \cdots, n_m = 1, \cdots, M_m) \tag{1}$$

where $\boldsymbol{x} \equiv [x_1, x_2, \cdots x_m]$ and y are the input and the output variables of the fuzzy system $\mathfrak{I}$, respectively. As noted in Eq. (1), the fuzzy system $\mathfrak{I}$ is assumed to be

$$\mathfrak{I}: U \subset R^m \rightarrow V \subset R$$

where $U = U_1 \times U_2 \times \cdots U_m \subset R^m$ and the number of fuzzy rules is $\prod_{i=1}^{m} M_i$. If the fuzzifier is a singleton, the T-norm in fuzzy implication and inference is a product inference and the defuzzifier is the center average, then the fuzzy system of Eq. (1) can be formulated by Eq. (2).

$$y_m = \mathfrak{I}(\boldsymbol{x}) = \mathfrak{I}(x_1, x_2, \cdots, x_m)$$

$$= \sum_{n_1=1}^{M_1} \cdots \sum_{n_m=1}^{M_m} \left\{ \frac{\prod_{i=1}^{m} A_i^{n_i}(x_i)}{\sum_{n_1=1}^{M_1} \cdots \sum_{n_m=1}^{M_m} \prod_{i=1}^{m} A_i^{n_i}(x_i)} \right\} \theta^{n_1, n_2, \cdots, n_m}$$

$$= \sum_{\mathbf{n}=1}^{\mathbf{M}} \left\{ \frac{\Omega^{\mathbf{n}}(\mathbf{x})}{\sum_{\mathbf{n}=1}^{\mathbf{M}} \Omega^{\mathbf{n}}(\mathbf{x})} \right\} \theta^{\mathbf{n}} \tag{2}$$

$$= \sum_{\mathbf{n}=1}^{\mathbf{M}} \{\xi^{\mathbf{n}}(\mathbf{x})\} \theta^{\mathbf{n}}$$

where the following notations are introduced for simplification:

$$\mathbf{n} \equiv (n_1, n_2, \cdots, n_m), \qquad \mathbf{M} \equiv (M_1, M_2, \cdots, M_m)$$

$$\sum_{\mathbf{n}=1}^{\mathbf{M}} \equiv \sum_{n_1=1}^{M_1} \sum_{n_2=1}^{M_2} \cdots \sum_{n_m=1}^{M_m}, \qquad \Omega^{\mathbf{n}}(\mathbf{x}) \equiv \prod_{i=1}^{m} A_i^{n_i}(x_i)$$

$$\xi^{\mathbf{n}}(\mathbf{x}) \equiv \frac{\Omega^{\mathbf{n}}(\mathbf{x})}{\sum_{\mathbf{n}=1}^{\mathbf{M}} \Omega^{\mathbf{n}}(\mathbf{x})}$$

III. THE STRUCTURE OF THE PROPOSED FUZZY SYSTEM ADOPTING SCOS FBF'S

A. Gaussian or Triangular Membership Functions

The system using Gaussian membership functions shown in Fig 2(a) are inconsistent and the denominator $\sum_{\mathbf{n}=1}^{\mathbf{M}} \Omega^{\mathbf{n}}(\mathbf{x})$ of the FBF of Eq. (2) makes it difficult to identify the system on-line. It also needs more parameters than the systems using triangular functions as shown in Fig. 2(a) and Fig. 2(b).

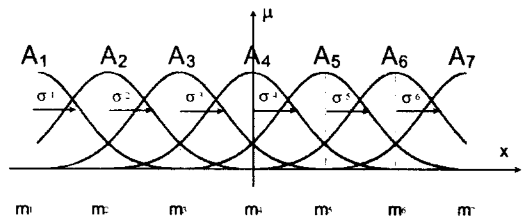

Fig. 2. (a) Gaussian membership functions

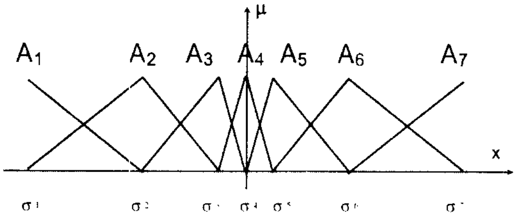

Fig. 2. (b) Triangular membership functions

On the contrary, the system using triangular membership functions shown in Fig 2(b) are consistent and the denominator $\sum_{n=1}^{M} \Omega^{n}(\mathbf{x})$ of the FBF (2-6) turns to be *one*. However, the triangular membership function may not be differentiable at some points. In the next subsection, SCOS membership function-based fuzzy system is proposed. The proposed system is (1) consistent and (2) differentiable at all points and (3) has the FBF's identical to the products of membership functions as in the case of triangular membership functions.

B. The Proposed Fuzzy System Using Squared-Cosine Membership Functions

The proposed fuzzy system is of the type of Eq. (2) and adopts the squared-cosine (SCOS) membership functions as shown in Fig. 3.

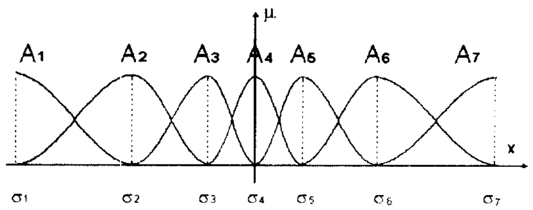

Fig 3. The universe of discourse and squared-cosine membership functions of x_i

The SCOS membership functions and the output of the fuzzy system are given in Eq. (3) and Eq. (4), respectively.

$$A_i^n(x_i) = \begin{cases} \cos^2 \dfrac{\pi(x_i - \sigma_i^n)}{2d_i^{n-1}} & \text{if } \sigma_i^{n-1} \le x_i < \sigma_i^n \\ \cos^2 \dfrac{\pi(x_i - \sigma_i^n)}{2d_i^{n}} & \text{if } \sigma_i^{n} \le x_i \le \sigma_i^{n+1} \\ 0 & \text{elsewhere} \end{cases} \quad (3)$$

where $d_i^n \equiv \sigma_i^{n+1} - \sigma_i^n \quad (n = 1, \cdots, M_i)$

$$y_m = \sum_{n_1=1}^{M_1} \cdots \sum_{n_m=1}^{M_m} \left\{ \frac{\prod_{i=1}^{m} A_i^{n_i}(x_i)}{\sum_{n_1=1}^{M_1} \cdots \sum_{n_m=1}^{M_m} \prod_{i=1}^{m} A_i^{n_i}(x_i)} \right\} \theta^{n_1, n_2, \cdots, n_m}$$

(4)

Remark 1.

(1) From Fig. 3, it can be noted that the SCOS membership functions are normal, consistent and complete.

(2) The SCOS membership function $A_i^n(x_i)$ as defined in Eq. (3) is differentiable at every point including even σ_i^{n-1}, σ_i^n and σ_i^{n+1}. The differentiability at σ_i^{n-1}, σ_i^n and σ_i^{n+1} can be clearly demonstrated by showing that

$$\lim_{x_i \to \sigma_i^n -} \frac{dA_i^n(x_i)}{dx_i} = \lim_{x_i \to \sigma_i^n +} \frac{dA_i^n(x_i)}{dx_i}$$

$$\lim_{x_i \to \sigma_i^{n-1} -} \frac{dA_i^n(x_i)}{dx_i} = \lim_{x_i \to \sigma_i^{n-1} +} \frac{dA_i^n(x_i)}{dx_i}$$

$$\lim_{x_i \to \sigma_i^{n+1} -} \frac{dA_i^n(x_i)}{dx_i} = \lim_{x_i \to \sigma_i^{n+1} +} \frac{dA_i^n(x_i)}{dx_i}$$

(3) The SCOS FBF turns to be the product of membership functions, that is,

$$\sum_{n=1}^{M} \Omega^{n}(\mathbf{x}) = \sum_{n_1=1}^{M_1} \sum_{n_2=1}^{M_2} \cdots \sum_{n_m=1}^{M_m} \prod_{i=1}^{m} A_i^{n_i}(x_i) = 1$$

and

$$\xi^{n}(\mathbf{x}) = \frac{\Omega^{n}(\mathbf{x})}{\sum_{n=1}^{M} \Omega^{n}(\mathbf{x})} = \Omega^{n}(\mathbf{x}) = \prod_{i=1}^{m} A_i^{n}(x_i)$$

Then the output is reduced to the following expression as in the case of the triangular membership functions.

$$y_m = \mathfrak{I}(\mathbf{x}) = \mathfrak{I}(x_1, x_2, \cdots, x_m) = \sum_{\mathrm{n}=1}^{\mathrm{M}} \{\xi^{\mathrm{n}}(\mathbf{x})\}\theta^{\mathrm{n}}$$

$$= \sum_{n_1=1}^{M_1} \cdots \sum_{n_m=1}^{M_m} \left\{ \frac{\prod_{i=1}^{m} A_i^{n_i}(x_i)}{\sum_{n_1=1}^{M_1} \cdots \sum_{n_m=1}^{M_m} \prod_{i=1}^{m} A_i^{n_i}(x_i)} \right\} \theta^{n_1, n_2, \cdots, n_m}$$

$$= \sum_{\mathrm{n}=1}^{\mathrm{M}} \left\{ \frac{\Omega^{\mathrm{n}}(\mathbf{x})}{\sum_{\mathrm{n}=1}^{\mathrm{M}} \Omega^{\mathrm{n}}(\mathbf{x})} \right\} \theta^{\mathrm{n}} = \sum_{\mathrm{n}=1}^{\mathrm{M}} \{ \Omega^{\mathrm{n}}(\mathbf{x})\} \theta^{\mathrm{n}}$$

IV. ON-LINE IDENTIFICATION

A. Initial Fuzzy System Construction

The numbers of fuzzy sets for each coordinate (*i.e.*, M_i for the ith coordinate) are assumed to be given in advance. Then, M_i SCOS fuzzy membership functions $A_i^{n_i}$'s are defined, which uniformly cover U_i, the projection of U onto the ith coordinate. In other words, the premise parameters ($\sigma_i^1, \cdots, \sigma_i^{M_i}$) of the fuzzy system adopting SCOS membership functions are positioned in this order with the identical intervals between the neighboring points to cover U_i. During the on-line identification, the order should be kept and cannot be changed.

After the aforementioned initial construction is completed, the given initial fuzzy system with initialized premise parameters and arbitrary consequent parameters is represented as in Eq. (5):

$$R^{n_1 n_2 \cdots n_m}: \text{ If } x_1 \text{ is } A_1^{n_1} \text{ and}, \cdots, \; x_m \text{ is } A_m^{n_m} \text{ then } y \text{ is } \theta^{n_1, n_2, \cdots, n_m} \quad (5)$$

$$(n_1 = 1, \cdots, M_1, \; \cdots, \; n_m = 1, \cdots, M_m)$$

or in input-output representation as in Eq. (6):

$$y_m = \mathfrak{I}(\mathbf{x}, \boldsymbol{\theta}) = \mathfrak{I}(x_1, x_2, \cdots, x_m, \boldsymbol{\theta}) \quad (6)$$

$$= \sum_{n_1=1}^{M_1} \cdots \sum_{n_m=1}^{M_m} \theta^{n_1, \cdots, n_m} \prod_{i=1}^{m} A_i^{n_i}(x_i, \sigma_i^{n_i-1}, \sigma_i^{n_i}, \sigma_i^{n_i-1})$$

where the parameters to be identified on-line are consequent parameters $\theta^{n_1, \cdots, n_m}$ ($n_1 = 1, \cdots, M_1$, $\cdots$, $n_m = 1, \cdots, M_m$) and premise parameters $\sigma_i^{k_i}$ ($i = 1, \cdots, m$, $k_i = 1, \cdots, M_i$). For simplicity, the parameters are collected to form a vector as follows:

$$\boldsymbol{\theta} = (\boldsymbol{\theta}_c^T, \; \boldsymbol{\theta}_p^T)^T \quad (L \times 1 \text{ matrix})$$

where $\boldsymbol{\theta}_c^T = (\theta^{1,1,\cdots,1}, \cdots, \theta^{M_1, M_2, \cdots, M_m})$: consequent parameters ($L_c \equiv \prod_{i=1}^{m} M_i$ elements)

$\boldsymbol{\theta}_p^T = (\sigma_1^1, \cdots, \sigma_1^{M_1}, \; , \; \cdots \; , \sigma_m^1, \cdots, \sigma_m^{M_m})$: premise parameters ($L_p \equiv \sum_{i=1}^{m} (M_i - 2)$ elements)

$$L \equiv L_c + L_p = \prod_{i=1}^{m} M_i + \sum_{i=1}^{m} (M_i - 2)$$

(the size of $\boldsymbol{\theta}$).

B. Gradient Search Algorithm

The gradient search algorithm is widely-used parameter tuning algorithm [10]. In this paper, gradient search algorithm is used to adapt the parameters on-line in the direction of the negative gradient as shown in Eq. (7)

$$\hat{\boldsymbol{\theta}}(t+1) = \hat{\boldsymbol{\theta}}(t) - \eta \frac{\partial E}{\partial \boldsymbol{\theta}(t)} + \alpha(\hat{\boldsymbol{\theta}}(t) - \hat{\boldsymbol{\theta}}(t-1)) \quad (7)$$

where t refers to the number of iterations (time), η is the learning rate and α is the momentum rate. For $\hat{\boldsymbol{\theta}}_p(t)$ and $\hat{\boldsymbol{\theta}}_c(t)$, different learning rates are used and they are denoted by η_p and η_c, respectively. The cost function is given by

$$E = \frac{1}{2}(y_d(t) - y_m(t))^2$$

where y_d is the desired value and y_m is the output from the fuzzy model. In Eq. (7), the gradient of the cost function with respect to each parameter is expressed as

$$\frac{\partial E}{\partial \theta(t)} = -(y_d(t) - y_m(t))\frac{\partial y_m(t)}{\partial \theta(t)}$$

and $\frac{\partial y_m(t)}{\partial \theta(t)} = \frac{\partial \mathfrak{I}}{\partial \theta(t)}$ can be obtained on either analytical or numerical way.

V. COMPUTER SIMULATION

In what follows, an illustrative example is provided to illustrate the validity of the suggested methods. For comparison, the performance measure used in the paper of [5] and [6] is adopted. The target system to be modeled is a nonlinear static function expressed by Eq.(8) taken from [5] and [6].

$$y = (1 + x_1^{-2} + x_2^{-1.5})^2, \quad 1 \le x_1,\ x_2 \le 5 \qquad (8)$$

One hundred points are taken randomly from $1 \le x_1,\ x_2 \le 5$ to form input-output data and the fuzzy models are built on-line by the suggested three algorithms one by one. In this example and the following examples, each universe of discourse is assumed to be covered by five fuzzy membership functions ($M_1 = \cdots = M_m = 5$). η_p, η_c and α are set to 0.2, 0.6, and 0.2, respectively, because they provide the fastest convergence in several trials. In this example, the MSE (Mean Squared Error) is used as a performance measure as follows:

$$\text{MSE} = \overline{e}^2 = \frac{1}{n}\sum_{i=1}^{n}(y_d(i) - y_m(i))^2$$

The performance measures of the suggested three on-line algorithms are given in Table 1 with those of other conventional methods. While fifty sample data are trained off-line for thousands of epoches in [5] and [6], one hundred data are used on-line (one epoch) in the suggested algorithms.

Table 1. COMPARISON OF PERFORMANCE (EXAMPLE 1)

Model	PM
Kim et al [5]	0.0197
Sugeno and Yasukawa [6]	0.0790
Gradient search	0.0247
Extended Kalman filter	0.0193
Hybrid	0.0255

VI. CONCLUSION

In this paper, on-line identification methodologies for a fuzzy system are proposed and their validity is verified through computer simulations. First squared-cosine fuzzy basis functions are introduced to reduce the number of parameters and to make on-line identification tractable. Then the on-line identification of the fuzzy system adopting SCOS FBF's is carried out by the gradient search method. However, as the number of input increases, the suggested algorithms may lead to the combinatorial explosion of the fuzzy rules and the further studies regarding the problem are needed.

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