The Third AFSS(1998), 619–624

FCM* Algorithm for Application to Fuzzy Control

Katsuari KAMEI

Computer Science
Ritsumeikan University
Kusatsu, Shiga 525-8856, Japan
Tel: +81-77-561-2699 *Fax:*+81-77-561-2669
E-mail: kamei@cs.ritsumei.ac.jp

Abstract

This paper presents a new clustering algorithm called FCM* algorithm for the design of fuzzy controller. FCM* is an extended version of FCM (Fuzzy c-Means) algorithm and can estimate the number of clusters automatically and give membership grades u^*_{ik} suitable for making fuzzy control rules. This paper also shows an example of its application to the line pursuit control of a car.

Keywords: fuzzy clustering, FCM, control design.

1 Introduction

In applications of fuzzy clustering algorithms to the fuzzy control, they have been used for an identification of Takagi and Sugeno's fuzzy model [1] rather than directly used for the design of fuzzy controller. For examples, Hirota and Sugeno used FCM algorithm for the identification of fuzzy models [2, 3, 4, 5, 6]. And other identification methods using extended version of FCM, clustering algorithms besides FCM and Neural Networks have been also proposed [7, 8, 9, 10, 11, 12]. These researches, however, just described an identification of the relationship between inputs and outputs of control objects, but never referred to controller designs.

On the other hand, there are a few researches in regard of applications of fuzzy clustering to operator's control models, in which inputs were states of the control objects and outputs were operator's behaviors (actions). For examples, Aisu and Iokibe proposed a controller design method and a generation method for fuzzy rules and membership functions, respectively [13, 14].

One of the most important problems in the fuzzy models using fuzzy clustering algorithms is how to decide the number of clusters which means the number of fuzzy rules. The solutions for this problem using proper reference indices and the repeats of union and division of clusters have been proposed [15, 16, 17, 18, 19].

In this paper, one of the extended FCM algorithms called FCM* (Fuzzy c-Means Star) algorithm, which can estimate the number of clusters automatically, are proposed and are applied to the design of fuzzy controllers.

2 FCM* Algorithm

2.1 Problems of conventional FCM

In an application of the conventional (original) FCM algorithm to making operator's control model, it has two problems as follows:

1. The algorithm needs to preset the proper number of clusters c.
2. The membership grade u_{ik} of a data point cannot compare with other data points because u_{ik} depends on the distance between the data point and cluster centers.

Especially, Problem 2 will be a fatal problem in case of making operator's control model which treats a cluster as a fuzzy subset in fuzzy control rules and a cluster center as a point standing for the rule.

Fig.1 shows an example of the above problem. In this figure, $\boldsymbol{v}_i$ is a cluster center representing a rule i, $\boldsymbol{x}_k$ is one of data points giving states of control object and u_{ik} is a membership grade of $\boldsymbol{x}_k$ to the rule i. All u_{ik} are same even though the distances between $\boldsymbol{v}_i$ and $\boldsymbol{x}_{k-1}$,$\boldsymbol{x}_k$,$\boldsymbol{x}_{k+1}$ are different. That means the rule i is used in the same weight even though the states of control object are different.

Therefore, we need a new membership grade depending on the distance between a cluster center and data points for making an operator's control model using fuzzy clustering.

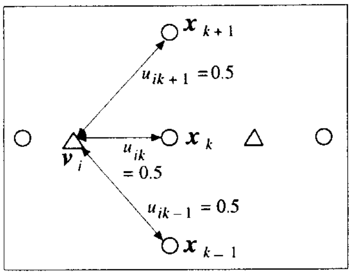


Fig. 1: Problems of FCM algorithm.

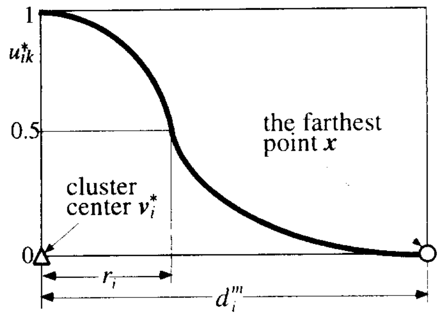


Fig. 2: Definition of u^*_{ik}

2.2 Definition of u^*_{ik}

We define a new membership function $u^*_{ik} \equiv u(v^*_i, x_k)$ instead of the original u_{ik} defined in (1) to solve the problems in 2.1. Fig.2 shows the membership function u^*_{ik} given by a bell-shaped function with three parameters: $u^*_{ik}|_{d_{ik}=0} = 1.0, u^*_{ik}|_{d_{ik}=r_i} = 0.5, u^*_{ik}|_{d_{ik}=d^m_i} = 0.0$ where $d_{ik} = \|v^*_i - x_k\|$. As the membership function u^*_{ik} does not satisfy the condition like (2) of FCM, we define a normalized membership function w_{ik} and then use it to calculate a cluster center v^*_i.

$$u_{ik} = \frac{(\frac{1}{d_{ik}})^{\frac{2}{m-1}}}{\sum_{j=1}^{c}(\frac{1}{d_{jk}})^{\frac{2}{m-1}}} \quad (1)$$

$$\sum_{i=1}^{c} u_{ik} = 1 \quad (2)$$

$$d^m_i = \max_k d_{ik} \quad (3)$$

$$r_i = \frac{\sum_{k=1}^{N} d_{ik} u^*_{ik}}{\sum_{k=1}^{N} u^*_{ik}} \quad (4)$$

$$v^*_i = \frac{\sum_{k=1}^{N}(w_{ik})^m x_k}{\sum_{k=1}^{N}(w_{ik})^m} \quad (5)$$

$$w_{ik} = \frac{u^*_{ik}}{\sum_{j=1}^{c^*} u^*_{jk}} \quad (6)$$

where, N is the number of data and c^*, c are the number of clusters.

2.3 Initial clustering

On the initial stage of clustering, cluster centers $v^{*(0)}_i$, cluster radius $r^{(0)}_i$ and the number of clusters $c^{(0)}$ are defined as:

$$v^{*(0)}_i = \begin{cases} \frac{x_k + x_{k'}}{2} & (\ell_k = \ell_{k'}) \\ x_k & (\ell_k \neq \ell_{k'}) \end{cases} \quad (7)$$

$$r^{(0)}_i = \begin{cases} \frac{3}{2}\ell_k & (\ell_k = \ell_{k'}) \\ \ell_k & (\ell_k \neq \ell_{k'}) \end{cases} \quad (8)$$

$$c^{(0)} = N - 1 \quad (9)$$

where, $\ell_k = \max_j \|x_k + x_j\|$ $(k \neq j)$

2.4 Estimation of the number of clusters

The number of clusters are estimated by repeats of union of clusters after the initial clustering. There are four types in the union of clusters i and j as shown in Fig.3, where $r_i \geq r_j$. The union is carried out when $\alpha > \alpha^*$, where α is *distance weight* [7] defined by (10) and α^* is a preset value dependent on data sets. The estimated number of clusters c^* is calculated by (12). After the union of the clusters i and j, the cluster center $v^{*(new)}_i$ and the cluster radius $r^{(new)}_i$ of the unified clusters are given by Eqs.(14-17).

$$\alpha(v^*_i, v^*_j) = \begin{cases} 0.0 & (d^*_{jk} \geq r_i + r_j) \\ \frac{r_i + r_j}{d^*_{jk}} - 1 & (r_i - r_j < d^*_{jk} < r_i + r_j) \\ 1.0 & (d^*_{jk} \leq r_i - r_j) \end{cases} \quad (10)$$

$$d^*_{jk} = \|v^*_i - v^*_j\| \quad (11)$$

$$c^* = c - \sharp\{V^*\} \quad (12)$$

$$V^* = \{v^*_j | \alpha(v^*_i, v^*_j) > \alpha^*\} \quad (13)$$

for $i = 1, \ldots, c-1; j = i+1, \ldots, c$.

$$v^{*(new)}_i = v^*_i \quad (14)$$

$$r^{(new)}_i = r_i \quad (15)$$

in case of (a) and (b) in Fig.3,

$$v_i^{*(new)} = \frac{(d_{jk}^* + r_i - r_j)v_i^* + (d_{jk}^* + r_i - r_j)v_j^*}{d_{jk}^*} \tag{16}$$

$$r_i^{(new)} = \frac{d_{jk}^* + r_i + r_j}{2} \tag{17}$$

in case of (c) and (d) in Fig.3.

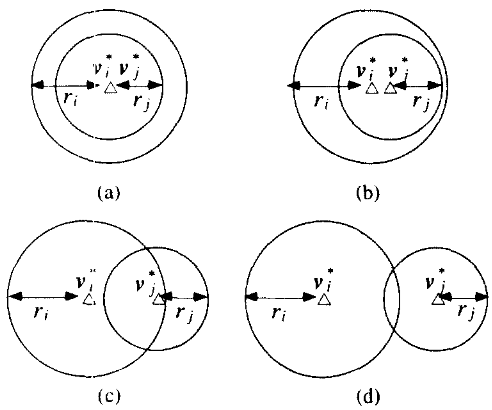


Fig. 3: Types of union.

2.5 Procedure of FCM*

1. Calculate $v_i^{*(0)}, r_i^{(0)}$ and $c^{(0)}$.
2. Calculate a matrix $U^{*(0)}$ of u_{ik}^*, then unify clusters and calculate c^*, then set $b = 1$.
3. Calculate $v_i^{*(b)}, r_i^{(b)}$.
4. Calculate $U^{*(b)}$.
5. Calculate $v_i^{*(b+1)}, r_i^{(b+1)}$.
6. Calculate $U^{*(b+1)}$.
7. If $\|U^{*(b+1)} - U^{*(b)}\| < \epsilon$ then set $c = c^*$ and go to next step else set $b = b + 1$ and go back to 4.
8. Unify the clusters and calculate c^*.
9. If $c^* = c$ then stop else set $c^* = c$, then go back to 3.

2.6 Test clustering

Fig.4 shows test data and cluster centers calculated by FCM and FCM*. Table 1 shows the parameters used in the calculation. Fig.5 - 7 show u_{ik}^*, w_{ik} and u_{ik} for test data shown in Fig.4, respectively. From the figures, we can confirm that FCM* could solve the two problems of FCM described in 2.1. For examples, compare u_{ik}^* of #1 and #13 of data for the cluster center v_1^* and see the results of #14 - #18. We could also show that w_{ik} indicated almost same as u_{ik}.

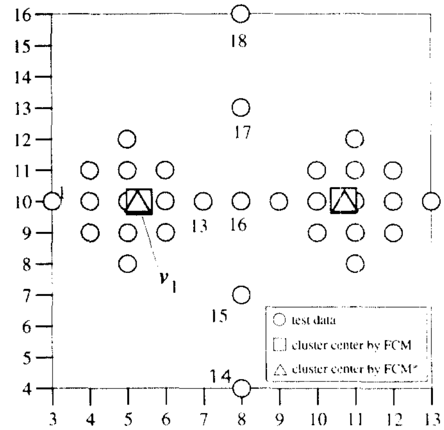


Fig. 4: Test data and cluster centers.

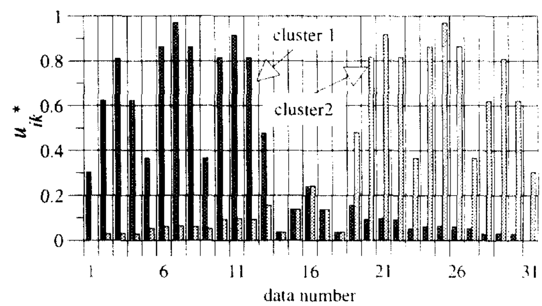


Fig. 5: Membership grade u_{ik}^*.

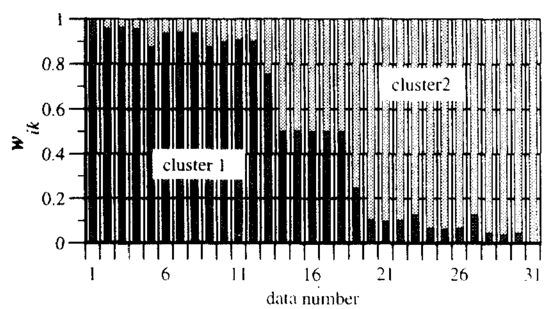


Fig. 6: Normalized membership grade w_{ik}.

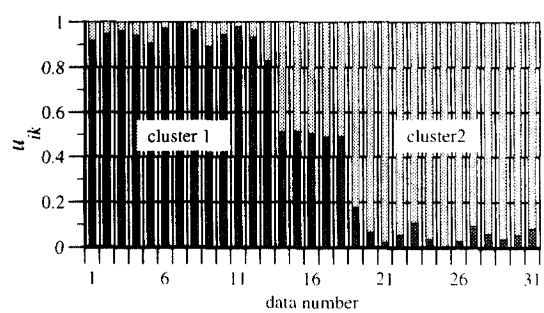


Fig. 7: Membership grade u_{ik}.

Table 1: Parameters used in FCM and FCM*.

N	m	ϵ	c(FCM)	α
31	2.0	0.01	2	0.1

3 Design of Fuzzy Controller

There are a lot of books and papers which describe the design of fuzzy controller. This section describes how to directly calculate control values (outputs of controller) based on states of control object (inputs of controller).

3.1 Fuzzy control rules

Let $\boldsymbol{p}$ be states of a control object and $\boldsymbol{q}$ be control values. Fuzzy control rules using FCM* are presented as:

$$R_i : \textit{ if } \boldsymbol{x} \textit{ is near } \boldsymbol{v}_i \textit{ then } \boldsymbol{y} = \boldsymbol{q} \textit{ with } u(\boldsymbol{v}_i^*, \boldsymbol{x}) \tag{18}$$

$$\boldsymbol{x} = (\boldsymbol{p}, \boldsymbol{q}) \in X \subset R^{n+m} \tag{19}$$

$$\boldsymbol{p} \in P \subset R^n, \boldsymbol{q} \in Q \subset R^m \tag{20}$$

$$\boldsymbol{v}_i^* \in V \subset R^{n+m} \tag{21}$$

3.2 Reasoning of output $\boldsymbol{y}^*$

When inputs $\boldsymbol{p}^*$ are given, outputs $\boldsymbol{y}^*$ are calculated by (22) using the fuzzy rules represented in (18). Fig.8 will help you understand the reasoning method.

$$\boldsymbol{y}^* = \frac{\sum_{i=1}^{c^*} \omega_i \boldsymbol{q}^*}{\sum_{i=1}^{c^*} \omega_i} \tag{22}$$

$$\begin{aligned} \omega_i &= u(\boldsymbol{v}_i^*, \boldsymbol{x}) \quad (23) \\ &= u(\boldsymbol{v}_i^*, (\boldsymbol{p}^*, \boldsymbol{q}^*)) \\ &= \max_{q \in Q} u(\boldsymbol{v}_i^*, (\boldsymbol{p}^*, \boldsymbol{q})) \quad (24) \end{aligned}$$

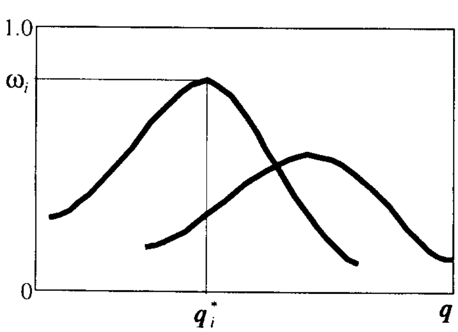


Fig. 8: Reasoning of output y^* ($m = 1$).

4 Application to Line Pursuit Control

4.1 Acquisition of operator data

Human operators input a steering angle $\delta(|\delta| \leq 30$[deg]) to the line pursuit control simulator based on η[m] and θ[deg] which are a distance and an angle between the goal line and the car, respectively. After the enough training of control, the operators started the experiments from initial points $\eta_0 = \{4, 5, 6, 7, 8, 9, 10\}$[m], $\theta_0 = 0$[deg], then finished it when $|\eta| < 0.1$ and $|\theta| < 1.72$. We obtained 144 data points which show the relationship between inputs (η, θ) and output δ. Fig.9 shows an example of experiment results in case of $\eta_0 = 10$[m].

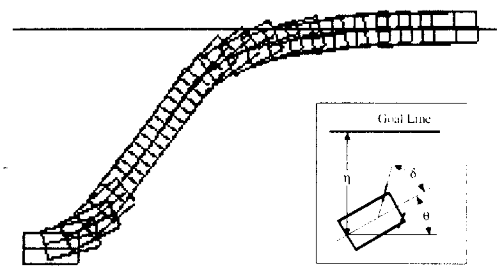


Fig. 9: An example of operator control results ($\eta_0 = 10$[m]).

4.2 Fuzzy clustering of operator data

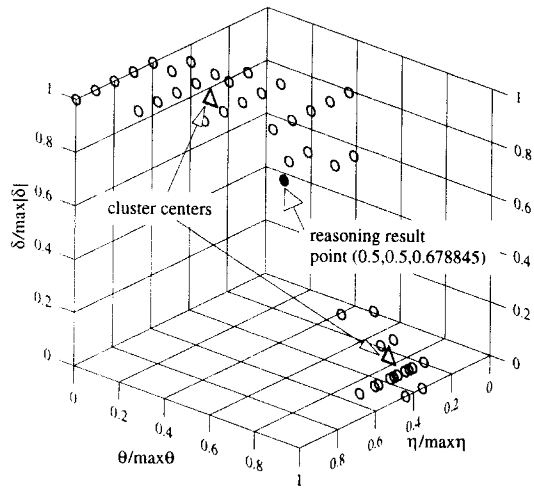
First, the operator data were divided into *Case1* ($\delta \geq 0$) and *Case2* ($\delta < 0$), then FCM* was applied to the two *Cases*. Consequently, FCM* gave the number of clusters $c^* = 2$ and two cluster centers $\boldsymbol{v}_i^* (i = 1, 2)$ in each *Case* as shown in Fig.10. Therefore, we got four fuzzy control rules.

4.3 Simulation results

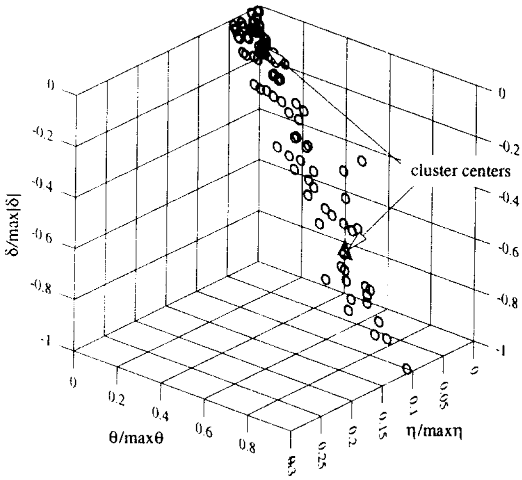
Fig.11 shows ω_1 and ω_2 for $\delta / \max ||\delta||$ when $\eta^* / \max \eta = 0.5$, $\theta^* / \max \theta = 0.5$ as an example of the calculation y^*. We got $y^* = 0.678845$ from (22). The dot in Fig.10(a) shows the result of reasoning. Fig.12 shows a control result when $\eta_0 = 10$[m]. Fig.13 also shows a control result when $\eta_0 = 12$[m] which we did not obtain data from the operators in the experimetns.

5 Conclusions

We could show the useful results of the line pursuit control using FCM* algorithm and the reasoning method which directly calculate the control



(a) $Case1 (\delta \geq 0)$



(b) $Case2 (\delta < 0)$

Fig. 10: Data points and cluster centers obtained from operators.

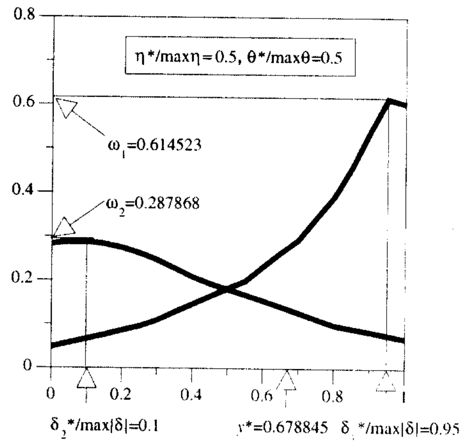


Fig. 11: Reasoning of y^*

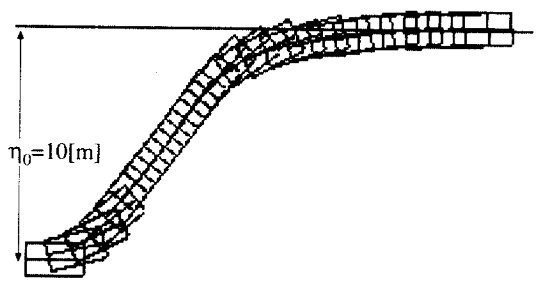


Fig. 12: Control result ($\eta_0 = 10$[m])

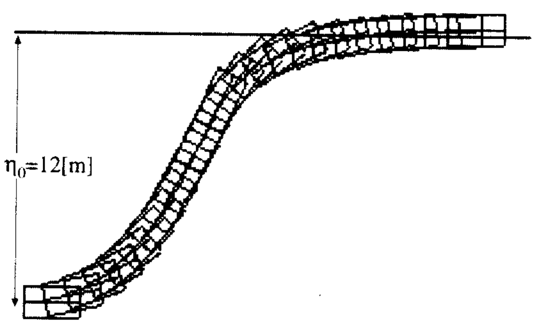


Fig. 13: Control result ($\eta_0 = 12$[m])

value y^*. The design of fuzzy controller described here is very simple method that the better and the more operator data we obtain, the better control results we can get.

References

[1] T.Takagi and M.Sugeno, "Fuzzy Identification of Systems and Its Application to Modeling and Control," IEEE Trans. on Systems, Man, and Cybernetics, Vol.15, pp.116-132, 1985

[2] J.C.Bezdek, "Pattern Recognition with Fuzzy Objective Function Algorithms," Plenum Press, N.Y., 1981

[3] K.Hirota and Y.Yoshinari,"Identification of Fuzzy Control Rule Based on Fuzzy Clustering Method," 5th Fuzzy System Symposium, pp.253-258, 1989 (in Japanese)

[4] Y.Yoshinari, W.Pedrycz and K.Hirota "Construction of fuzzy models through clustering techniques, Fuzzy Sets and Systems, Vol.54, pp.157-165, 1993

[5] M.Sugeno and T.Yasukawa "A fuzzy-logic-based approach to qualitative modeling," IEEE Trans. on Fuzzy Systems, Vol.1, No.1, pp.7-31, 1993

[6] T.Yasukawa and M.Sugeno,"Qualitaive System Description Based on Knowledge and Numerical Data and Its Application to the Design of a Fuzzy Controller," J. of Japan Society for Fuzzy Theory and Systems,Vol.6, No.4, pp.720-735, 1994 (in Japanese)

[7] K.Kamei, D.M.Auslander and K.Inoue, "A Fuzzy Clustering Method for Multidimensional Parameter Selection in System with Uncertain Parameters," Proceedings of IEEE International Conference on Fuzzy System, pp.355-362, March 1992

[8] S.Kundu and J.Chen, "FLIC: Fuzzy Linear Invariant Clustering for Applications in Fuzzy Control," Proceedings of NAFIPS/IFIS/ NASA 94, pp.196-200, 1994

[9] R.R.Yager and D.P.Filev, "Generation of Fuzzy Rules by Mountain Clustering," Journal of Intelligent and Fuzzy Systems, Vol.2, pp.209-219, 1994

[10] J.Zhao, V.Wertz and R.Gorez, "A Fuzzy Clustering Method for the Identification of Fuzzy Models for Dynamic Systems," Proceedings of IEEE International Symposium on Intelligent Control, pp.172-177, 1994

[11] D.Nauck and R.Kruse, "NEFCLASS A Neuro-Fuzzy Approach for the Classification of Data," Proceedings of the 1995 ACM Symposium on Applied Computing, http://www.cs.tu-bs.de/ibr/projects/nefcon/nefclass.html

[12] C-T.Sun and J.-S.Jang, "Fuzzy Modeling Based on Generalized Neural Networks and Fuzzy Clustering Objective Functions," Proceedings of the 30th Conference on Decision and Control, pp.2924-2928,1991

[13] H.Aisu, T.Endo,"Fuzzy Control Method Using Clustering Algorithm with Measured Data," Proceedings of SICE'91, JS30-4, 1991 (in Japanese)

[14] T. Iokibe,"Fuzzy Clustering Method by Discretionary Fuzzy Performance Function -Automatic Rule and Membership Function Generation -, J. of Japan Society for Fuzzy Theory and Systems, Vol.4, No.2, pp.334-343, 1992 (in Japanese)

[15] R.C.Dubes, "How Many Clusters Are Best? - An Experiments*," Pattern Recognition, Vol.20, No.6, pp.645-663, 1987

[16] Y.Fukuyama and MiSugeno,"A New Method of Choosing the Number of Clusters for Fuzzy c-Means Method," Proceedings of Fuzzy System Symposium '89, pp.247-252, 1989 (in Japanese)

[17] R.Krishnapuram and C.-P.Freg,"Fitting an Unknown Number of Lines and Planes to Image Data Through Compatible Cluster Merging," Pattern Recognition, Vol.25, No.4, pp.385-400, 1992

[18] Y.K.Harada, M.Miyakoshi and M.Shimbo,"A Fuzzy Clustering Method for Automatic Segmentation of Color Scene," J. of Japan Society for Fuzzy Theory and Systems, Vol.6, No.5, pp.1021-2036, 1994 (in Japanese)

[19] S.Inui, K.Kamei and K.Inoue,"An Adovanced Fuzzy C-Means Algorithm with Clustering Number Estimation," T.IEE Japan, Vol. 114-C, No.11, pp. 1166-1171, 1994 (in Japanese)