The Third AFSS(1998), 579—584

An Interval Valued Bidirectional Approximate Reasoning Method Based on Similarity Measure

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Abstract

In this work, we present a method to deal with the interval valued decision making systems. First, we propose a new type of equality mesure based on the Ordered Weighted Averaging(OWA) operator. The proposed equality mesure has a structure to render the extreme values of the mesure by choosing a suitable weighting vector of the OWA operator. From this property, we derive a bidirectional fuzzy inference network which can be applied for the decision making systems requiring the inverval valued decisions.

1 Introduction

A lot of human knowledges may be viewed as a collection of facts and rules, each of which may be represented as a fuzzy relation having some possibility value[5]. From this fact, several forms of fuzzy relational equations and their analytical solution methods have been presented[5][6]. However, most of these analytical solution methods are based on the impractical assumption that there exists a fuzzy relation for all pairs of input and output fuzzy data simultaneously.

To overcome this difficulty, there were approaches to adopt learning capability of the neural network for finding an approximate solution of the max-min fuzzy relational equations [7][2] . These neurocomputational approaches, however, also have a limitation that they often give only approximate relations as local optimal solutions for the given input and output fuzzy data.

To alleviate these drawbacks, Bien and Chun [3] proposed a form of fuzzy relational mapping which handles fuzzy knowledge given as fuzzy input and output data and supports an approximate reasoning. The inference network performs a forward and a backward reasoning in knowledge base system.

2 OWA Operator and Equality Measure

Yager[1] suggested the OWA operator for aggregations lying between the logical OR and AND. It is defined as follows.

Definition 1: An OWA operator of dimension n is a mapping $f : R^n \rightarrow R$ that has an associated n vector W

$$W = [w_1, w_2, \cdots, w_n]^T$$

such that (1) $w_i \in [0, 1]$, (2) $\sum_i w_i = 1$. Furthermore $f(a_1, a_2, \cdots, a_n) = \sum_j w_j b_j$ where b_j is the jth largest of the a_i.

It is noted that different OWA operators are distinguished by their weighting function. If we choose $W = W_{max} = [1\,0\,0 \cdots 0]$ and $W = W_{min} = [0\,0\,0 \cdots 1]$, then $OWA_u(a_1, a_2, \cdots, a_n) = \mathrm{Max}_i(a_i)$ and $OWA_l(a_1, a_2, \cdots, a_n) = \mathrm{Min}_i(a_i)$, respectively. When $W = W_{avg} = [\frac{1}{n}\ \frac{1}{n} \cdots \frac{1}{n}]$, the OWA operator performs the averaging operation such as $OWA_a(a_1, a_2, \cdots, a_n) = \frac{1}{n} \sum_i (a_i)$. From the definition of the OWA operator, we can show

$$OWA_l(a_1, a_2 \cdots a_n) \leq OWA(a_1, a_2 \cdots a_n) \leq OWA_u(a_1, a_2 \cdots a_n) \qquad (1)$$

Now, let us consider the measure of equality or similarity between two fuzzy vectors $x \in [0, 1]^n$ and $y \in [0, 1]^n$. Pedrycz[7][8] proposed an equality index $q = (x \equiv y)$ for two fuzzy values $x \in [0, 1]$ and $y \in [0, 1]$ having a strong logical background expressed as

$$q = (x \equiv y) = \frac{1}{2}[(x \rightarrow y) \wedge (y \rightarrow x) + (\bar{x} \rightarrow \bar{y}) \wedge (\bar{y} \rightarrow \bar{x})] \qquad (2)$$

where "$\rightarrow$" denotes an implication and $\bar{x} = 1 - x$ is the complement of x. In the sequel, we shall simply adopt the Lukasiewicz implication:

$$x \rightarrow y = \min(1, 1 + y - x). \qquad (3)$$

Now, let us consider an equality measure $S \in [0, 1]$ between two n-dimensional fuzzy vectors $X = [x_1, x_2, \cdots, x_n]$ and $Y = [y_1, y_2, \cdots, y_n]$. Bien and Chun [3] defined a S as

$$S = (X \equiv Y) = 1/n \sum_i^n \frac{1}{2}[(x_i \rightarrow y_i) \wedge (y_i \rightarrow x_i) + (\bar{x}_i \rightarrow \bar{y}_i) \wedge (\bar{y}_i \rightarrow \bar{x}_i)] \quad (4)$$

The above equlity measure, however, is the only arithmatic averge of the equality indeces. So, we propose a new type of equality measure based on the OWA operator from a logical foundation.

Definition 2: For two fuzzy vector $X = [x_1, x_2, \cdots, x_n]$ and $Y = [y_1, y_2, \cdots, y_n]$, its fuzzy measure is

$$E = (X \equiv Y) = OWA(q_1, q_2, \cdots, q_n) \tag{5}$$

where q_i is the equality index between x_i and y_i given from (2).

For the proposed equality measure, when we select $W = W_{max} = [1\ 0\ \cdots\ 0]^T$ and $W = W_{min} = [0\ 0\ \cdots\ 1]^T$, $E = E_u(q_1, q_2 \cdots q_n) = \text{Max}_i(q_i)$ and $E_l(q_1, q_2 \cdots q_n) = \text{Min}_i(q_i)$, respectively. Similarily, if we choose $W = W_{avg} = [\frac{1}{n} \frac{1}{n} \cdots \frac{1}{n}]^T$, then $E = E_a(q_1, q_2 \cdots q_n) = \frac{1}{n} \sum_i (q_i)$ which is equal to (4). Therefore, we can easily derive follolwing relation:

$$E_l(q_1, q_2 \cdots q_n) \leq E(q_1, q_2 \cdots q_n) \leq E_u(q_1, q_2 \cdots q_n) \tag{6}$$

To show the qualitative aspect of the proposed equality measure, we denote the equality measure $E(q_1, q_2)$ in Figure 1 between $X = [x_1, x_2]$ and $Y = [y_1, y_2]$ for each equality index $q_1 = (x_1 \equiv y_1) \in [0\ 1]$ and $q_2 = (x_2 \equiv y_2) \in [0\ 1]$. Figure 1 (b), (c), and (d) show E_u, E_l, and E_a, respectively. At the Figure, more bright part have more higher equality value. From the point of qualitavive, we can consider E_u as an optimistic aggregator because it takes the maximum value among equality index. Similarily, we can take E_l a pessimistic aggregator. On the other hand, E_a belongs between E_l and E_u. The above interpretation can be adopted to the case of n-dimentional fuzzy vectors.

3 Interval Valued Decision Making System

Let us consider an interval valued decision making system defined as follows.

Definition 3: For a set T of l pairs of fuzzy input and output data $\{(x_j, y_j) |\ x_j \in [0,1]^n,\ y_j \in [0,1]^m,\ j = 1, 2, \cdots, l\}$, an interval valued decision making system performs a mapping $I_f : [0,1]^n \rightarrow [0,1]^m$ and $I_b : [0,1]^m \rightarrow [0,1]^n$. For a given x which is similar to x_j, I_f maps the output y is to be similar to y_j, which corresponds to a forward approximate reasoning. As the same token, For a given y which is similar to y_j, I_b maps the output x is to be similar to x_j, which corresponds to a backward approximate reasoning. Here, the interval valued decision making system give also the upper value x_u and y_u

Now, to construct the inference network, denote the fuzzy input and output data as

$$X_j = [x_1^j, x_2^j, \cdots, x_n^j] \text{ and } Y_j = [y_1^j, y_2^j, \cdots, y_m^j] \quad j = 1, 2, \cdots, l,$$

and encode the weighting values in the inference network as follows:

$$W_j^f = X_j \text{ and } R_j^f = Y_j, \quad j = 1, 2, \cdots, l. \tag{7}$$

$$W_j^b = Y_j \text{ and } R_j^b = X_j, \quad j = 1, 2, \cdots, l. \tag{8}$$

From the above relations, it is easy to show that $R^f = W^b$ and $W^f = R^b$.

Let us consider the behavior of the inference network which performs a bidirectional approximate reasoning.

Step 1: When a fuzzy input vector $X \in [0, 1]^n$ is presented to the layer 1, x_i becomes an input value for the ith unit in layer 1. If we denote $S^f = [s_1^f, s_2^f, \cdots s_l^f]$ as the equality measure vector, then S^f can be found as follows.

$$s_j^f = (X \equiv W_j^f), \quad j = 1, 2, \cdots, l. \tag{9}$$

Step 2: After computing $S^f = (s_1^f, s_2^f, \cdots, s_l^f)$, the outputs of layer 3, $Z^f = (z_1^f, z_2^f, \cdots, z_l^f)$, are given as

$$z_j^f = \begin{cases} s_j^f & \text{if } s_j^f \geq \theta_j^f, \; j = 1, 2, \cdots, l \\ 0 & \text{otherwise} \end{cases} \tag{10}$$

Here, the threshold value θ_j^f is given to determine the recalling level. That is, if the equality measure between a fuzzy input data X and the fuzzy data X_j in X^t is less than the threshold value θ_j^f, then the fuzzy data X_j is discarded in constructing the fuzzy output Y.

Step 3: Finally, the output of layer 4 is obtained by performing the generalized max-min composition as

$$Y = Z^f \circ_p R^f \tag{11}$$

which can be rewritten as

$$y_j = \max_{1 \leq i \leq l}(\min{}_p(z_i^f, r_{ij}^f)), \quad 1 \leq j \leq m$$

where the generalized intersection $\min_p$ introduced by Yager is defined in [1] as follows:

$$\min{}_p(x, y) = 1 - \min[1, \{(1-x)^p + (1-y)^p\}^{\frac{1}{p}}] \quad \text{for } p \geq 1$$

4 Concluding Remarks

We have proposed an inference network as a tool for bidirectional approximate reasoning. If a fuzzy input is given for the inference network, then the network renders a reasonable fuzzy output after performing the approximate reasoning based on an equality measure. Also, from its bidirectional structure, if a fuzzy output is given, then the network can find its corresponding reasonable fuzzy input. In defining the equality measure, we adopted the Lukasiewicz implication operator. However any given problem can be handled in a different manner by choosing other implication operators.

Since the inference network can be designed directly from the given fuzzy input and output data, it is easy to add or delete knowledge to the inference network. Moreover, the developed scheme requires only simple arithmetic operations, it is possible to perform real-time decision making with applications to control and diagnostic systems in real situations.

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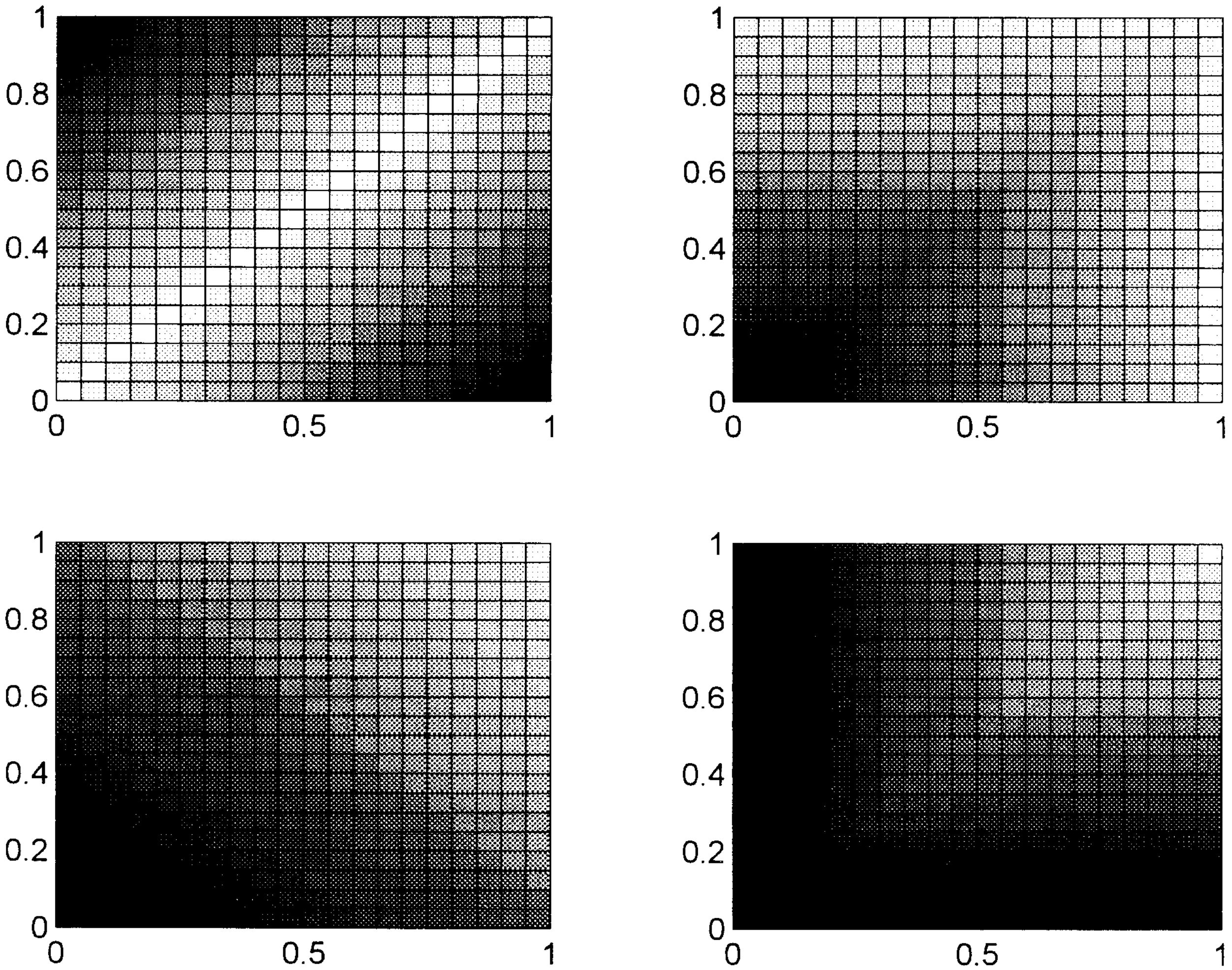

Figure 1. Similarity Measure

(a) Pedrycz's similarity measure
(b) Maximum similarity measure
(c) Average similarity measure
(d) Minimum similarity measure