The Third AFSS(1998). 526 – 530

# On (the product of) normal F-subpolygroups

Abbas HASANKHANI[a], Mohammad Mehdi ZAHEDI[b]

a. Dept. of Math., Sistan and Baluchestan University, Zahedan, Iran
b. Dept. of Math., Shahid Bahonar University of Kerman, Kerman, Iran
Tel & Fax: +98-341-263244
E-mail: Zahedi@arg3.uk.ac.ir

**Abstract**
In this note by considering the notions of F-polygroups, the product of F-polygroups, F-subpolygroups and (weak) normal F-subpolygroups two questions are given. Then by an example it is shown that the answer of one of the questions (posed in the paper [12]) is in general negative. In other words, the product of two normal F-subpolygroups need not be normal.
**Keywords:** Fuzzy Sets, Polygroups, F-polygroups, (weak) normal F-subpolygroups.

## 1. Introduction and Preliminaries

Zadeh in 1965 [9] introduced the notion of fuzzy subsets of a nonempty set $A$ as a function from $A$ to $[0,1]$. Rosenfeld in 1971 [8] defined fuzzy subgroups and obtained some basic results. A polygroup is a completely regular, reversible-in-itself multigroup in the sense of Dresher and Ore [5]. These systems occur naturally in the study of algebraic logic [2,3]. Ioulidis in 1981 [7] studied the concept of polygroup, which is a generalization of the concept of ordinary group. Zahedi, Bolurian, and Hasankhani in 1995 [10] introduced the concept of a fuzzy subpolygroup, which is a generalization of the concept of a fuzzy subgroup. Zahedi and Hasankhani [11] defined the notion of $F$-polygroup, which is a generalization of polygroups.

**Definition 1.1.** Let $A \neq \emptyset$ and "$o$" be a function from $A \times A$ to $P^*(A) = P(A)\backslash\{\emptyset\}$. Then "$o$" is called a hyperoperation on $A$.

**Definition 1.2.** If $X, Y \in P^*(A)$, then we define $XoY$ as:

$$XoY = \bigcup_{(x,y)\in X\times Y} xoy.$$

**Notation.** Let "$o$" be a hyperoperation on $A$ and $a \in A, X \in P^*(A)$. Then by $aoX$ and $Xoa$ we mean $\{a\}oX$ and $Xo\{a\}$ respectively.

**Definition 1.3** (see [2,3,6,7]). Let "$o$" be a hyperoperation on $H$. Then $(H, o)$ is called a polygroup or quasi-canonical hypergroup iff

(*i*) $xo(yoz) = (xoy)oz \ \ \forall\, x, y, z \in H$,

(*ii*) There exists an element $e \in H$ such that

$$xoe = eox = \{x\}, \ \ \forall\, x \in H.$$

($e$ is called the identity element of $H$.),

(*iii*) for each $x \in H$, there exists a unique element $x' \in H$ such that

$$e \in xox' \cap x'ox.$$

($x'$ is called the inverse of $x$ and is denoted by $x^{-1}$.),

(*iv*) $z \in xoy \Rightarrow x \in zoy^{-1} \Rightarrow y \in x^{-1}oz$, $\forall x, y, z \in H$.

**Definition 1.4.** [9]. Let $X$ be a set. A fuzzy subset of $X$ is a function $\mu : X \to [0,1]$.

**Remark 1.5.** Throughout this note $I$ is the unit interval $[0,1] \subseteq \mathbb{R}$, and $I^A$ is the set of all fuzzy subsets of $A$. If $\mu \in I^A$, then by $supp(\mu)$ we mean the set $\{x \in A : \mu(x) \neq 0\}$. Let $\mu, \eta \in I^A$. Then $\mu \leq \eta$ iff $\mu(x) \leq \eta(x)$, for all $x \in A$.

**Definition 1.6.** Let $\mu, \eta$ and $\mu_\alpha \in I^A$ where $\alpha$ is in the index set $\Lambda$. We define the fuzzy subsets $\mu \cap \eta, \mu \cup \eta, \bigcap_{\alpha\in\Lambda} \mu_\alpha$ and $\bigcup_{\alpha\in\Lambda} \mu_\alpha$

as follows:

(i) $(\mu \cap \eta)(x) = \min\{\mu(x), \eta(x)\}$,

(ii) $(\mu \cup \eta)(x) = \max\{\mu(x), \eta(x)\}$,

(iii) $(\bigcap_{\alpha \in \Lambda} \mu_\alpha)(x) = \inf_{\alpha \in \Lambda} \mu_\alpha(x)$,

(iv) $(\bigcup_{\alpha \in \Lambda} \mu_\alpha)(x) = \sup_{\alpha \in \Lambda} \mu_\alpha(x)$, for all $x \in A$.

**Definition 1.7.** Let $f : A \to B$ be a function and $\mu \in I^A, \eta \in I^B$. Then the functions $f(\mu)$ and $f^{-1}(\eta)$ which belong, respectively, to $I^B$ and $I^A$ are defined as follows:

(i)

$$f(\mu)(b) = \begin{cases} \sup\limits_{a \in f^{-1}(b)} \mu(a) & \text{if } f^{-1}(b) \neq \emptyset \\ 0 & \text{otherwise} \end{cases},$$

for all $b \in B$,

(ii) $f^{-1}(\eta)(a) = \eta(f(a))$, for all $a \in A$.

**Definition 1.8.** Let $a \in A, t \in I$. Then by a fuzzy point $a_t$ of $A$ we mean the fuzzy subset of $A$ given below:

$$a_t(x) = \begin{cases} t & \text{if } x = a \\ 0 & \text{otherwise} \end{cases}.$$

## 2. F-polygroups

For any subset $A$ of $X$, we let $\chi_A$ denotes the characteristic function of $A$.

**Definition 2.1.** Let $A \neq \emptyset$ and $I_*^A = I^A \backslash \{0\}$. Then

(i) by an F-hyperoperstion "$*$" on $A$ we mean a function from $A \times A$ to $I_*^A$, in other words for any $a, b \in A, a * b$ is a non-empty fuzzy subset of $A$.

(ii) if $\mu, \eta \in I_*^A$, then $\mu * \eta \in I_*^A$ is defined by

$$\mu * \eta = \bigcup_{x \in supp(\mu), y \in supp(\eta)} x * y.$$

**Notation 2.2.** Let $\mu \in I_*^A, B, C \in P^*(A)$ and $a \in A$. Then

(i) $a * \mu$ and $\mu * a$ denote $\chi_{\{a\}} * \mu$ and $\mu * \chi_{\{a\}}$ respectively,

(ii) $a * B, B * a, \mu * B, B * \mu$ and $B * C$ denote $\chi_{\{a\}} * \chi_B, \chi_B * \chi_{\{a\}}, \mu * \chi_B, \chi_B * \mu$ and $\chi_B * \chi_C$ respectively.

**Definition 2.3.** Let " $*$ " be an F-hyperoperation on the non-empty set $A$. Then $(A, *)$ is called an F-polygroupoid.

**Definition 2.4.** Let $(H, *)$ be an F-polygroupoid. Then $(H, *)$ is called a semi-F-polygroup iff

$$x * (y * z) = (x * y) * z, \quad \forall x, y, z \in H.$$

**Definition 2.5.** Let $\mathcal{F}$ be a non-empty set. Then $(\mathcal{F}, *)$ is called a non-reversible F-polygroup iff

(i) $(\mathcal{F}, *)$ is a semi-F-polygroup,

(ii) there exists an element $e_\mathcal{F} \in \mathcal{F}$ such that

$$x \in supp(x * e_\mathcal{F} \cap e_\mathcal{F} * x), \quad \forall x \in \mathcal{F}$$

(In this case we say that $e_\mathcal{F}$ is an F-identity element of $\mathcal{F}$.),

(iii) for each $x \in \mathcal{F}$, there exists a unique element $x' \in \mathcal{F}$ such that

$$e_\mathcal{F} \in supp(x * x' \cap x' * x).$$

($x'$ is called the F-inverse of $x$ and it is denoted by $x_\mathcal{F}^{-1}$.)

**Defintion 2.6.** Let $(\mathcal{F}, *)$ be a non-reversible F-polygroup. Then $(\mathcal{F}, *)$ is an F-polygroup iff

$$\begin{aligned} z \in supp(x * y) \;&\Rightarrow\; x \in supp(z * y_\mathcal{F}^{-1}) \\ &\Rightarrow\; y \in supp(x_\mathcal{F}^{-1} * z); \\ &\quad\;\; \forall x, y, z \in \mathcal{F}. \end{aligned}$$

(This property is called the F-reversibility of $\mathcal{F}$ with respect to " $*$ ".)

When there is no ambiguity, for simplicity of notation we use $e$ and $x^{-1}$ instead of $e_F$ and $x_F^{-1}$ respectively.

**Example 2.7.** Let $\mathcal{F} = \{a, b\}$. Then the following table denotes an F-polygroup structure on $\mathcal{F}$.

| $*$ | $a$ | $b$ |
|---|---|---|
| $a$ | $\frac{a}{0.7}, \frac{b}{0}$ | $\frac{a}{0}, \frac{b}{0.3}$ |
| $b$ | $\frac{a}{0}, \frac{b}{0.3}$ | $\frac{a}{0.7}, \frac{b}{0}$ |

**Example 2.8.** Let $\alpha$ be an arbitrary element of $I \backslash \{0\}$ and $G$ be a group such that $x^2 = e, \ \forall x \in G$. Then it is easily seen that the F-hyperoperation "$*$" which is defined as

$$(x * y)(z) = e_\alpha(xyz) \ \ \forall x, y, z \in G$$

induces an F-polygroup structure on $G$, where $e_\alpha$ is a fuzzy point of $G$.

Throughout this note $\mathcal{F}$ will denote an F-polygroup with F-hyperoperation "$*$" and $e$ will denote the F-identity of $\mathcal{F}$.

**Remark 2.9.** Let $\mu_1, \mu_2 \in I_*^\mathcal{F}$. Then we have

(i) If $\mu_1(t) > 0 \quad (\mu_2(t) > 0)$, then

$$t * \mu_2 \leq \mu_1 * \mu_2 \quad (\mu_1 * t \leq \mu_1 * \mu_2).$$

In particular,

$$e * \mu_2 \leq (x * x^{-1}) * \mu_2, \quad \forall\, x \in \mathcal{F}.$$

(ii) If $\mu_1 \leq \mu_2$, then

$$\mu_1 * x \leq \mu_2 * x, \quad x * \mu_1 \leq x * \mu_2, \quad \forall\, x \in \mathcal{F}.$$

**Lemma 2.10.** Let $\mu \in I_*^{\mathcal{F}}, z \in \mathcal{F}$. Then

$$supp(\mu * z) = \bigcup_{t \in supp(\mu)} supp(t * z)$$

and

$$supp(z * \mu) = \bigcup_{t \in supp(\mu)} supp(z * t).$$

**Theorem 2.11.** (i) Let $(A, o)$ be a polygroup. Then $(A, *)$ is an F-polygroup where

$$x * y = \chi_{xoy}, \quad \forall\, x, y \in A.$$

($*$ is called the F-hyperoperation induced by "$o$".)

(ii) Let $(A, *)$ be an F-polygroup and $supp(x * e) = supp(e * x) = \{x\}$ $\forall x \in A$. Then $(A, \odot)$ is a polygroup where

$$x \odot y = supp(x * y), \quad \forall\, x, y \in A.$$

($\odot$ is called the hyperoperation extracted from $*$.)

**Definition 2.12.** If

$$(x * e)(x) = (e * x)(x) = 1, \quad \forall\, x \in \mathcal{F},$$

then we say $e$ is of degree 1.

**Corollary 2.13.** Let $(A, o)$ be a polygroup. Then the F-identity of F-polygroup $(A, *)$, which is defined in Theorem 2.11 is of degree 1.

**Theorem 2.14.** Let $(\mathcal{F}, *)$ be an F-polygroup. Then

(i) $e^{-1} = e$ and $e$ is unique. Also $supp(e * e) = \{e\}$,

(ii) $(x^{-1})^{-1} = x, \quad \forall\, x \in \mathcal{F}$,

(iii) $\bigcup_{x \in supp(\mu_1)} x * \mu_2 = \mu_1 * \mu_2 = \bigcup_{y \in supp(\mu_2)} \mu_1 * y, \forall \mu_1, \mu_2 \in I_*^{\mathcal{F}}$

(iv) $(\mu_1 * \mu_2) * \mu_3 = \mu_1 * (\mu_2 * \mu_3)$, $\forall \mu_1, \mu_2, \mu_3 \in I_*^{\mathcal{F}}$

**Definition 2.15.** Let $\mathcal{F}_1, \mathcal{F}_2$ be two F-polygroups and $f : \mathcal{F}_1 \to \mathcal{F}_2$ be a function such that $f(e_1) = e_2$. Then

(i) $f$ is called a homomorphism iff

$$f(x * y) \leq f(x) * f(y), \quad \forall\, x, y \in \mathcal{F}_1$$

(ii) $f$ is called a strong homomorphism iff

$$f(x * y) = f(x) * f(y), \quad \forall\, x, y \in \mathcal{F}_1$$

## 3. The normal F-subpolygroups

**Definition 3.1.** Let $\emptyset \neq H \subseteq \mathcal{F}$. Then $H$ is called an F-subpolygroup iff

(i) if $x \in H$, then $x^{-1} \in H$

(ii) $supp(x * y) \subseteq H, \quad \forall\, x, y \in H$.

In this case we write: $H <_{F-P} \mathcal{F}$.

Note that condition (ii) of above definition is equivalent to $x * y \leq \chi_H, \forall\, x, y \in H$.

**Example 3.2.** Let $\mathcal{F} = \{e, a, b\}$. Then the following table shows an F-polygroup structure on $\mathcal{F}$.

| $*$ | $e$ | $a$ | $b$ |
|---|---|---|---|
| $e$ | $\frac{e}{1}, \frac{a}{0}, \frac{b}{0}$ | $\frac{e}{0}, \frac{a}{1}, \frac{b}{0}$ | $\frac{e}{0}, \frac{a}{0}, \frac{b}{1}$ |
| $a$ | $\frac{e}{0}, \frac{a}{1}, \frac{b}{0}$ | $\frac{e}{0}, \frac{a}{1}, \frac{b}{0}$ | $\frac{e}{1}, \frac{a}{1}, \frac{b}{1}$ |
| $b$ | $\frac{e}{0}, \frac{a}{0}, \frac{b}{1}$ | $\frac{e}{1}, \frac{a}{1}, \frac{b}{1}$ | $\frac{e}{0}, \frac{a}{0}, \frac{b}{1}$ |

Now let $H = \{a\}$. Then $(H, *')$ is an F-polygroup, where "$*'$" is the restriction of "$*$" to $H \times H$. However $H$ is not an F-subpolygroup of $\mathcal{F}$, because $a^{-1} = b$ in $\mathcal{F}$ and $b \notin H$.

**Lemma 3.3.** Let $\emptyset \neq H \subseteq \mathcal{F}$. Then $H <_{F-P} \mathcal{F}$ if and only if $supp(x * y^{-1}) \subseteq H, \ \forall\, x, y \in H$.

**Corollary 3.4.** $\{e\} <_{F-P} \mathcal{F}$.

**Definition 3.5.** Let $H <_{F-P} \mathcal{F}$. Then

(i) $H$ is said to be weak normal in $\mathcal{F}$ ($H \lhd^{w}_{F-P} \mathcal{F}$) iff

$$x * H * x^{-1} \leq \chi_H, \quad \forall\, x \in \mathcal{F}, \qquad (1)$$

(ii) $H$ is said to be normal in $\mathcal{F}$ ($H \lhd_{F-P} \mathcal{F}$) iff

$$x * H * x^{-1} = \chi_H, \quad \forall\, x \in \mathcal{F}.$$

Note that (1) is equivalent to:

$$supp(x * H * x^{-1}) \subseteq H, \quad \forall\, x \in \mathcal{F}.$$

The following example shows a weak normal F-subpolygroup which is not normal.

**Example 3.6.** Consider the Example 2.7. Let $H = \mathcal{F}$. Then it is obvious that $H \lhd^{w}_{F-P} \mathcal{F}$. Since

$$(x * H * x^{-1})(a) \leq 0.7 < 1, \quad \forall\, x, a \in \mathcal{F}.$$

we conclude that $H \ntriangleleft_{F-P} \mathcal{F}$.

Now we give an example of a normal F-subpolygroup.

**Example 3.7.** Consider a group $G$ with order greater than 1 and $x^2 = e, \forall x \in G$. Let $\alpha \in (0,1)$. Define the hyperoperation "$*$" on $G$ by

$$(x * y)(z) = \begin{cases} e_1(xyz) & \text{if } \ z = e \\ e_\alpha(xyz) & \text{if } \ z \neq e, \end{cases}$$

where $e_1, e_\alpha$ are fuzzy points of $G$. After some manipulation it can be seen that $(G, *)$ is an F-polygroup, $e_F = e$ and $x_F^{-1} = x$, $\forall\, x \in G$. Now let $H = \{e\}$. Then by Corollary 3.4 we have $H <_{F-P} G$. And it is not difficult to check that

$$x * H * x = \chi_H, \quad \forall\, x \in G.$$

Thus $H \lhd_{F-P} G$.

Note that in the above example the identity is not of degree 1, because if $x \neq e$, then $(x * e)(x) = \alpha \neq 1$.

**Theorem 3.8.** Let $e \in \mathcal{F}$ be of degree 1. Then

$$N \lhd^{w}_{F-P} \mathcal{F} \ \text{ iff } \ N \lhd_{F-P} \mathcal{F}.$$

**Lemma 3.9.** Let $\mu \in I^{\mathcal{F}}_{*}$. Then

$$x * \mu * y = \bigcup_{t \in supp(\mu)} x * t * y, \quad \forall\, x \in \mathcal{F}.$$

**Lemma 3.10.** Let $n \in \mathbb{N}$ and $w, a_1, \cdots, a_n \in \mathcal{F}$. If $w \in supp(a_1 * \cdots * a_n)$, then $w^{-1} \in supp(a_n^{-1} * \cdots * a_1^{-1})$.

**Theorem 3.11.** Let $H, K <_{F-P} \mathcal{F}$ and $H \odot K = \bigcup_{x \in H,\, y \in K} supp(x * y)$. Then

(*i*) $H \odot K <_{F-P} \mathcal{F}$ if and only if $H \odot K = K \odot H$,

(*ii*) if $K \lhd^{w}_{F-P} \mathcal{F}$, then $H \odot K <_{F-P} \mathcal{F}$,

(*iii*) if $H, K \lhd^{w}_{F-P} \mathcal{F}$, then $H \odot K \lhd^{w}_{F-P} \mathcal{F}$.

**Corollary 3.12.** Let $e \in \mathcal{F}$ be of degree 1 and $H, K \lhd^{w}_{F-P} \mathcal{F}$. Then $H \odot K \lhd_{F-P} \mathcal{F}$.

**Question 3.13.** [12]. If $H, K \lhd_{F-P} \mathcal{F}$, is $H \odot K \lhd_{F-P} \mathcal{F}$?

**Answer:** the following example shows that the answer is not affirmative in general.

**Example 3.14.** Let $V = \{e, a, b, c\}$ be the Klein 4-group and $\alpha \in (0,1)$. Define the F-hyperoperation "$*$" on $V$ by

$$(x * y)(z) = \begin{cases} e_\alpha(xyz) & \text{if } \ z = c \\ e_1(xyz) & \text{if } \ z \neq c \end{cases}$$

where $e_1$ and $e_\alpha$ are fuzzy points of $V$. Now it is easy to check that

$$supp(x * y) = \{xy\}, \quad \forall x, y \in V. \tag{2}$$

Then by using (2) and some manipulations it can be seen that $(V, *)$ is an Abelian F-polygroup and also $e_V = e$ and $x_V^{-1} = x$, $\forall x \in V$. Now let $H = \{e, a\}$ and $K = \{e, b\}$. Then from (2) and some calculations we get that $H, K <_{F-P} \mathcal{F}$. Now we show that $H, K \lhd_{F-P} V$. Let $x \in V$ be arbitrary. Then we have:

$$supp(x * e * x) = supp(x * x) = \{e\},$$

and

$$\begin{aligned} supp(x * a * x) &= supp(x * x * a) \\ &= supp(e * a) \\ &= \{a\}. \end{aligned}$$

Therefore $supp(x * H * x) = \{e, a\}$. Now

$$\begin{aligned} &(x * H * x)(e) \\ =\ & max\{(x * e * x)(e), (x * a * x)(e)\} \\ =\ & max\{(e * e)(e), (e * a)(e)\} \\ =\ & max\{e_1(e), e_1(a)\} \\ =\ & 1, \end{aligned}$$

and

$$\begin{aligned} &(x * H * x)(a) \\ =\ & max\{(x * e * x)(a), (x * a * x)(a)\} \\ =\ & max\{(e * e)(a), (e * a)(a)\} \\ =\ & max\{e_1(a), e_1(e)\} \\ =\ & 1. \end{aligned}$$

Thus

$$x * H * x = \chi_H, \quad \forall x \in V,$$

which means that $H \lhd_{F-P} V$. Similarly $K \lhd_{F-P} V$. Now we show that $H \odot K \ntriangleleft_{F-P} V$. To do this first we prove that $H \odot K = V$ and then we show that $(x * V * x)(c) < \chi_V(c)$, for all $x \in V$. We have:

$$\begin{aligned} H \odot K &= supp(e * e) \cup supp(e * b) \\ &\cup \; supp(a * e) \cup supp(a * b) \\ &= \{e\} \cup \{b\} \cup \{a\} \cup \{c\} \\ &= V, \end{aligned}$$

and

$$(x * c * x)(c) = (e * c)(c) = \alpha < 1,$$

and

$$(x * t * x)(c) = (e * e)(c) = 0 < 1, \quad \forall t = e, a, b.$$

Hence $x * V * x < \chi_V$, for all $x \in V$. In other words $H \odot K \ntriangleleft_{F-P} V$.

**Theorem 3.15.** Let $\mathcal{F}_1$ and $\mathcal{F}_2$ be two F-polygroups and $f : \mathcal{F}_1 \to \mathcal{F}_2$ be a strong homomorphism. Then $Im(f) <_{F-P} \mathcal{F}_2$.

**Definition 3.16.** Let $f : \mathcal{F}_1 \to \mathcal{F}_2$ be a strong homomorphism of $F$-polygroups. Then $f$ is called a zero-invariant iff

$$(x*y)(z) = 0 \Rightarrow f(x*y)(f(z)) = 0, \forall x, y, z \in \mathcal{F}_1.$$

**Theorem 3.17.** Let $f : \mathcal{F}_1 \to \mathcal{F}_2$ be a homomorphism of $F$-polygroups. Then

$(i)$ if $K <_{F-P} \mathcal{F}_2 (K \lhd^w_{F-P} \mathcal{F}_2)$, then $f^{-1}(K) <_{F-P} \mathcal{F}_1, (f^{-1}(K) \lhd^w_{F-P} \mathcal{F}_1)$,

$(ii)$ if $f$ is strong (onto and zero-invariant) and $H <_{F-P} \mathcal{F}_1 (H \lhd^w_{F-P} \mathcal{F}_1)$, then $f(H) <_{F-P} \mathcal{F}_2 (f(H) \lhd^w_{F-P} \mathcal{F}_2)$.

**Corollary 3.18.** Let $f : \mathcal{F}_1 \to \mathcal{F}_2$ be a homomorphism of $F$-polygroups, where the identities of $\mathcal{F}_1$ and $\mathcal{F}_2$ are of degree 1. Then

$(i)$ if $K \lhd^w_{F-P} \mathcal{F}_2$, then $f^{-1}(K) \lhd_{F-P} \mathcal{F}_1$,

$(ii)$ if $f$ is zero-invariant and onto and $H \lhd^w_{F-P} \mathcal{F}_1$, then $f(H) \lhd_{F-P} \mathcal{F}_2$.

**Remark 3.19.** Consider a group $G$ with order greater than 1 and such that $x^2 = e$, $\forall\, x \in G$. Then as is shown in Examples 2.8 and 3.7, there are two F-polygroup structures on $G$ as follow:

$(i)$ $(x * y)(z) = e_\alpha(xyz), \quad \forall\, x, y, z \in G,$

$(ii)$ $(x *' y)(z) = \begin{cases} e_1(xyz) & \text{if } z = e \\ e_\alpha(xyz) & \text{if } z \neq e, \end{cases}$

where $\alpha$ is a fixed element in $(0, 1)$, and $e_\alpha, e_1$ are fuzzy points in $G$.

Now let $f : (G, *) \to (G, *')$ be the identity map. Then $f^{-1}(\{e\}) \ntriangleleft_{F-P} (G, *)$, while $\{e\} \lhd_{F-P} (G, *')$.

**Question 3.20.** Is the following statement true?

Let $\mathcal{F}_1, \mathcal{F}_2$ be two arbitrary F-polygroups. If $f : \mathcal{F}_1 \to \mathcal{F}_2$ is a zero-invariant and onto, then the homomorphic image of a normal F-subpolygroup is also normal F-subpolygroup.

## REFERENCES


1. G. Birkhoff, Lattice Theory, American Mathematical Society, Providence, RI, 1984.
2. S.D. Comer, Integral relation algebras via pseudogroups, Not, Am. Math. Soc. 23:A-659, 1976.
3. S.D. Comer, Multivalued loops geometries, and algebraic logic, Houston J. Math. 2(3):373-380, 1976.
4. S.D. Comer, Combinatorial aspects of relations, Alg. Univ. 18:77-94, 1984.
5. M. Dresher and O. Ore, Theory of multigroup, Am. J. Math. 60:705-733, 1938.
6. J. Goguen, $L$-fuzzy sets, J. Math. Anal. Appl. 18:145-174, 1967.
7. S. Ioulidis, Polygroupes et certaines de leurs proprietes, Bull. Greek. Math. Soc. 22:95-104, 1981.
8. A. Rosenfeld, Fuzzy groups, J. Math. Anal. Appl. 35:512-517, 1971.
9. L.A. Zadeh, Fuzzy sets, Inf. Contr. 8:353-388, 1965.
10. M.M. Zahedi, M. Bolurian and A. Hasankhani, On polygroups and fuzzy subpolygroups, J. Fuzzy Math. 3(1):1-15, 1995.
11. M.M. Zahedi and A. Hasankhani, F-polygroups (I), J. Fuzzy Math., 4(3): 533-548 (1996).
12. M.M. Zahedi and A. Hasankhani, F-polygroups (II), J. Information Sci., 89: 225-243 (1996).