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The Properties of Uniform Probabilistic Relaxation Systems

Gi Y. Lim
Department of Control & Instrumentation Engineering
Taejon National University of Technology, Korea
fax: +82 42-625-1485
e-mail: limgy@hyunam.tnut.ac.kr
Alan M. N. Fu and Hong Yan
Department of Electrical Engineering
University of Sydney, NSW 2006, Australia
phone: +61 2 9351-4824
fax: +61 2 9351-3847
e-mail: alanf@ee.usyd.edu.au

Abstract

In this paper we first show that uniform PR systems and half independent PR systems have same dynamics, and then an important property of this two kinds of systems is derived. The most important property of uniform PR systems is that they have the ability of classifying m-dimensional probabilistic vectors into m classes. The significance of studying the dynamics of uniform PR systems lies in the fact that many learning rules for PR systems are tried from the beginning with a uniform PR system.

Key words: Probabilistic relaxation, Compatibility coefficient matrix,
Uniform components

1 Introduction

The relaxation labeling technique was first proposed by Rosenfeld *et al.* [1] to deal with ambiguity and noise in vision systems. Probabilistic relaxation (PR) methods have been successfully applied to many image processing tasks, such as scene labeling, pixel labeling, shape matching, line and curve enhancement, handwritten character recognition, breaking substitution ciphers, optical flow - template matching and image segmentation [2-7]. Efforts have also been made towards the understanding of the properties of the method from mathematical analysis [8-11]. In this paper, we study the dynamics of a uniform PR system, i.e. a special case of the PR systems in which both the influence coefficient matrix and the compatibility coefficient matrix have uniform elements.

The paper is organized as follows. In Section 2, the dynamics of uniform PR systems is derived. Summary is given in Section 3.

2 Uniform PR Systems

The PR system considered in this paper is one given in [11], in which the consistency or inconsistency measures between initial certainty measures and compatibility coefficient matrix (CCM) are defined exactly based on Bayes' formula and a probability space partition. The PR systems in [11] is given by the following updating equation:

$$p_i^{(k+1)}(\lambda) = p_i^{(k)}(\lambda) + p_i^{(k)}(\lambda)\frac{b_i^{(k)}(\lambda)}{R_i^{(k)}}, \quad i = 1, \cdots, n \tag{1}$$

where n denote the total number of objects to be labelled, $p_i^{(t)}(\lambda)$ is the λth component of the m-dimensional probabilistic vector $\mathbf{p}_i^{(t)}$ associated with the ith object at the tth step of an iterative process and

$$\begin{aligned}
R_i^{(t)} &= \sum_{\lambda'=1}^{m} \left(p_i^{(t)}(\lambda') + p_i^{(t)}(\lambda')v_i^{(t)}(\lambda')\right), \\
b_i^{(k)}(\lambda) &= q_i^{(k)}(\lambda)v_i^{(k)}(\lambda) - \sum_{z\neq\lambda}^{m} p_i^{(k)}(z)v_i^{(k)}(z), \qquad (2)\\
q_i^{(k)}(\lambda) &= \sum_{z\neq\lambda}^{m} p_i^{(k)}(z), \\
v_i^{(t)}(z) &= p_i^{(t)}(z) - s_i^{(t)}(z), \\
s_i^{(t)}(z) &= \sum_{j=1}^{n} d_{ij}\left(\sum_{\lambda'=1}^{m} c_{ij}(z\,,\lambda')p_j^{(t)}(\lambda')\right) \qquad (3)
\end{aligned}$$

where m is the total number of labels that each labeled object can be assigned. The λth component $p_i^{(0)}(\lambda)$ of $\mathbf{p}_i^{(0)}$ is the probability that the ith object is assigned the labeling value being λ initially. d_{ij} is the weight of the influence on the ith object label from the jth object label and satisfys: $0 \leq d_{ij} \leq 1$ and $\sum_{j=1}^{n} d_{ij} = 1$, $c_{ij}(q, r)$ represents the compatibility measure of object i having label q when object j having label r and satisfys: $0 \leq c_{ij}(q, r) \leq 1$ and $\sum_{q=1}^{m} c_{ij}(q, r) = 1$. Let D denote the influence weight matrix of labeled objects and C_{ij} denote the CCM between the ith and jth objects $(i, j = 1, \cdots, n)$. If D and C_{ij} are uniform matrixes, i.e., the element of D, $d_{ij} = \frac{1}{n}$, $i, j = 1, \cdots, n$, and the element of C_{ij}, $c_{ij}(x, y) = \frac{1}{m}$, where $i, j = 1, \cdots, n$ and $x, y = 1, \cdots, m$, then the PR system is known as a uniform PR system. For simplicity, the CCM of uniform PR systems is denoted by C whose element is represented by $c(x, y)$ rather than $c_{ij}(x, y)$. If D is an identity matrix and CCMs are uniform, then the PR system is referred to as a half independent PR system. What follow will show that the uniform PR systems and half independent PR systems have identical dynamics, and then derive one of their important property.

Theorem 1 The uniform PR systems and half independent PR systems possess the same dynamics.

Proof Based on the updating equation (1) we need to show that the quantity $b_i^{(k)}(\lambda)$ in uniform PR systems is the same as that in half independent PR systems. Since equations (2) and (3) we only need to show that they have an identical certainty measure $s_i^{(t)}(z)$. In fact, it is easy to show that in both cases we have $s_i^{(t)}(z) = \frac{1}{m}$. So Theorem 1 is achieved.

Theorem 2 Let $\mathrm{p}_i^{(0)}$ be an m-dimensional probabilistic vector. If the initial feature-factor[1] $\mathrm{N}_i = 1$, and we

[1] N_i denotes the total number of labels λ which satisfy $v_i^{(0)}(\lambda) > 0$, i.e., $\mathrm{N}_i = \sum_\lambda \mathrm{Y}_i(\lambda)$ where $Y_i = 1$ as $v_i^{(0)}(\lambda) > 0$ and $Y_i = 0$ otherwise.

assume $p_i^{(0)}(\lambda^*) > \frac{1}{m}$, i.e., $\nu_i^{(0)}(\lambda^*) > 0$ (since $s_i^{(0)}(\lambda^*) = \frac{1}{m}$), then

(a) $p_i^{(k)}(\lambda^*)$ converges to 1 as k increases infinitely,

(b) $p_i^{(k)}(\lambda)$ converges to 0, for $\lambda \neq \lambda^*$ as k increases infinitely. In other words, p_i converges to the basic unit vector U_{λ^*} with λ^*th component being 1.

Proof Since C is uniform, i.e., $c(x, y) = \frac{1}{m}$, $x, y = 1, \cdots, m$, we have $\mathrm{Min}\{c(\lambda, \lambda^*) : \lambda \neq \lambda^*\} \geq \mathrm{Max}\{c(\lambda, \lambda') : \lambda' \neq \lambda\}$, and therefore the condition of Theorem 3 in [11] is satisfied. Hence, based on the Theorem 3 the conclusions (a) and (b) are true.

3 The Learning Rule

Based on the previous analyses of dynamics of uniform PR systems, we have the following learning rule to adjust the compatibility coefficients for a m-dimensional initial probabilistic vector converging to a specific a m-dimensional basic unit vector under the PR process.

Learning Rule

If an initial certainty support, a m-dimensional probabilistic vector $p_i^{(0)}$ should converge to a basic unit vector Uk under the uniform PR system, but it does not, then

(a) set $c(k, k) = 0$;

(b) set $c(j, k) = \frac{2}{m}$, where the index j satisfies $p_i^{(0)}(j) = \mathrm{Min}\{p_i^{(0)}(z) : z \neq k, z = 1, \cdots, m\}$;

(c) other components of CCM remain being $\frac{1}{m}$.

If there is another initial certainty support $p_l^{(0)}$, $l \neq i$, should converge to a basic unit vector Uq, $q \neq k$ under the uniform PR system, but it does not, then following the same procedures, i.e.,

(a') set $c(q, q) = 0$;

(b') set $c(r, q) = \frac{2}{m}$, where the index r satisfies $p_i^{(0)}(r) = \mathrm{Min}\{p_i^{(0)}(z) : z \neq q, z = 1, \cdots, m\}$;

(c') other components of CCM keep being $\frac{1}{m}$ except $c(k, k) = 0$ and $c(j, k) = \frac{2}{m}$.

Experiments have shown that PR systems after training can classify similar initial probabilistic vectors (i.e., similar patterns) into a same class (i.e., an identical m-dimensional basic unit vector) and different initial probabilistic vectors into different classes.

4 Summary

In Section 2 we have shown that the uniform PR systems and half independent PR systems have same dynamics. We have also derived their important dynamics. The important property of the two kinds of PR systems is that they can classify m-dimensional probabilistic vectors into m classes [11]. This property plays a central role in an application of PR systems to solve practical problems. Many learning rule for PR systems can be yielded from the beginning with a uniform PR system. Thus, with better understanding of the dynamics of uniform PR systems, more effective learning rules for PR systems will be found.

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