The Third AFSS(1998). 326−330

LMI-Based Design of Fuzzy Controllers for Takagi-Sugeno Fuzzy Systems

Jinsung Kim[a], Jooyoung Park[b], and Daihee Park[c]

a. Department of Applied Electronics, Graduate School, Korea University
208 Seochang-Dong, Chochiwon, Chungnam, 339-800, Korea.
Tel: +82-415-60-1444 Fax: +82-415-865-1820 E-mail: grjskim@tiger.korea.ac.kr
b. Department of Control and Instrumentation Engineering, Korea University
208 Seochang-Dong, Chochiwon, Chungnam, 339-800, Korea.
Tel: +82-415-60-1444 Fax: +82-415-865-1820 E-mail: jpark@tiger.korea.ac.kr
c. Department of Computer Science, Korea University
208 Seochang-Dong, Chochiwon, Chungnam, 339-800, Korea.
Tel: +82-415-60-1344 E-mail: dhpark@tiger.korea.ac.kr

Abstract

There have been several recent studies concerning the stability of fuzzy control systems and the synthesis of stabilizing fuzzy controllers. This paper reports on a related study of the TS (Takagi-Sugeno) fuzzy systems, and it is shown that the controller synthesis problems for the nonlinear systems described by the TS fuzzy model can be reduced to convex problems involving LMIs (linear matrix inequalities). After classifying the TS fuzzy systems into two families based on how diverse their input matrices are, different controller synthesis procedure is given for each of these families. A numerical example is presented to illustrate the synthesis procedures developed in this paper.

Keywords : Control theory, Fuzzy control, Takagi-Sugeno fuzzy model, Linear matrix inequalities

1. Introduction

In the TS fuzzy model [4], the overall system is described by fuzzy IF-THEN rules, each of which represents local linear state equation $\dot{\mathbf{x}} = A_i\mathbf{x} + B_i\mathbf{u}$ in a different state space region. In this paper, we are concerned with stabilizing the nonlinear systems described by the TS fuzzy model with the TS fuzzy controllers or a modified version of the TS fuzzy controller. For clear and convenient presentation of our results, we classify the TS fuzzy systems into two families based on how diverse their input matrices B_i are, and different controller synthesis procedure is developed for each of these families. First, the family of the TS fuzzy systems with the common input matrices is considered, and it is observed that the TS fuzzy controllers which stabilize the systems in the family can be found by solving a simple set of LMIs (linear matrix inequalities). Next, the complement set of the first family is considered, and a new fuzzy controller is proposed for the stabilization of the family. The control signal of the proposed controller is produced by postfiltering the output of the TS fuzzy control. A stabilizability criterion for the proposed fuzzy controller is given in terms of LMIs. Also, it is shown that to find the parameters of the new fuzzy controller can be cast as an LMI problem.

Formulation of the controller synthesis problems with LMIs is of great practical value because they can be solved by reliable and efficient convex optimization techniques [2], e.g. the LMI Control Toolbox for use with Matlab [3].

This paper is organized as follows: Section II gives preliminaries regarding Takagi-Sugeno fuzzy model, quadratic stability and linear matrix inequalities. The fuzzy controller synthesis problems are formally stated and solved in Section III. In Section IV, a numerical example is presented to illustrate the controller synthesis procedures proposed in this paper. Finally, concluding remarks are given in Section V.

2. Preliminaries: TS fuzzy model, Quadratic stability and LMIs

The focus of this paper is on the design of stabilizing fuzzy controllers for nonlinear systems described by the TS fuzzy model. The general element of IF-THEN implications of the TS fuzzy model is given in the following form:

Plant Rule i:
IF $z_1(t)$ is M_{i1} and $\cdots$ and $z_g(t)$ is M_{ig},
THEN

$$\dot{\mathbf{x}}(t) = A_i\mathbf{x}(t) + B_i\mathbf{u}(t). \quad (1)$$

$i = 1, \cdots, r$.

Here, $z_i(t)$ and M_{ij}, $i = 1, \cdots, r$, $j = 1, \cdots, g$ are premise variables and fuzzy sets, respectively, and r is the number of IF-THEN rules. Following the usual inference method of the TS fuzzy model, we obtain the state equation of the overall system represented in the form of weighted average along the trajectories $\mathbf{z}(t)$:

$$\dot{\mathbf{x}}(t) = \frac{\sum_{i=1}^{r} w_i(\mathbf{z}(t))\{A_i\mathbf{x}(t) + B_i\mathbf{u}(t)\}}{\sum_{i=1}^{r} w_i(\mathbf{z}(t))}, \quad (2)$$

where $\mathbf{u}(t) \in \Re^p, \mathbf{x}(t) \in \Re^n$ and $\mathbf{z}(t) \in \Re^g$. In the equation (2), the weight functions are defined as

$$w_i(\mathbf{z}(t)) = \prod_{j=1}^{g} M_{ij}(z_j(t)),$$

where $M_{ij}(z_j(t))$ is the grade of membership of $z_j(t)$ in the fuzzy set M_{ij}. The weight functions w_i, which are nonnegative measurable, usually satisfy

$$\sum_{i=1}^{r} w_i(\mathbf{z}(t)) > 0 \text{ for all } t > 0. \tag{3}$$

Throughout this paper, it is assumed that (3) holds always, and that the vectors $\mathbf{x}(t)$ and $\mathbf{z}(t)$ can be measured in real time. With the normalization of weight functions $h_i(\mathbf{z}(t)) \stackrel{\triangle}{=} w_i(\mathbf{z}(t))/\sum_{i=1}^{r} w_i(\mathbf{z}(t))$, the state equation (2) can be written in the "polytopic" form:

$$\dot{\mathbf{x}}(t) = \sum_{i=1}^{r} h_i(\mathbf{z}(t))\{A_i\mathbf{x}(t) + B_i\mathbf{u}(t)\}, \tag{4}$$

where the normalized weights h_i satisfy $h_i(\mathbf{z}(t)) \geq 0, i = 1, \cdots, r$ and $\sum_{i=1}^{r} h_i(\mathbf{z}(t)) = 1$ for $\forall t \geq 0$. In general, premise variables do not depend on the input $\mathbf{u}(t)$, but are heavily dependent on the state $\mathbf{x}(t)$, thus the dynamics of the TS fuzzy system is basically of nonlinear nature.

When $\mathbf{u}(t) = 0, \forall t \geq 0$, the TS fuzzy system (4) becomes an input-free polytopic system:

$$\dot{\mathbf{x}}(t) = \sum_{i=1}^{r} h_i(\mathbf{z}(t))A_i\mathbf{x}(t). \tag{5}$$

As is well-known from the stability theory, an autonomous dynamical system is stable if there exists a positive definite quadratic function $V(\mathbf{x}) = \mathbf{x}^T Q^{-1}\mathbf{x}$ which decreases along every nonzero trajectory of the system, and a system having such Lyapunov function is called quadratically stable. In the polytopic system (5), the derivative of V along a nonzero trajectory $\mathbf{x}(\cdot)$ is given by

$$\begin{aligned}\frac{dV}{dt}(t) &= \frac{d}{dt}\{\mathbf{x}^T(t)Q^{-1}\mathbf{x}(t)\} \\ &= \mathbf{x}^T(t)\{\textstyle\sum_{i=1}^{r} h_i(\mathbf{z}(t))A_i^TQ^{-1} \\ &\qquad +Q^{-1}\textstyle\sum_{i=1}^{r} h_i(\mathbf{z}(t))A_i\}\mathbf{x}(t) \\ &= \textstyle\sum_{i=1}^{r} h_i(\mathbf{z}(t))\mathbf{x}^T(t)\{A_i^TQ^{-1} + Q^{-1}A_i\}\mathbf{x}(t).\end{aligned}$$

Since $A_i^TQ^{-1} + Q^{-1}A_i < 0$ is equivalent to $A_iQ + QA_i^T < 0$ when Q is positive definite, we can see that the polytopic system (5) is quadratically stable if there exists a symmetric matrix Q satisfying the following inequalities [2], [6]:

$$Q > 0,\ A_iQ + QA_i^T < 0,\ i = 1, \cdots, r. \tag{6}$$

Note that the left-sides of these inequalities are all linear in the matrix variable Q. Also, note that, if each of the fuzzy IF-THEN rules (1) truly represents the local dynamics, i.e. for each $i \in \{1, \cdots, r\}$, there exists $\mathbf{z}(t) \in \Re^g$ such that $\sum_{i=1}^{r} h_i(\mathbf{z}(t))A_i = A_i$, then the set of inequalities (6) becomes necessary as well as sufficient for the quadratic stability of the TS fuzzy system (5).

To find Q satisfying (6) or determine that there does not exist such Q is a convex problem called LMI feasibility problem. An LMI is any constraint of the form

$$A(\mathbf{x}) \stackrel{\triangle}{=} A_0 + x_1A_1 + \cdots + x_NA_N < 0, \tag{7}$$

where $\mathbf{x} \stackrel{\triangle}{=} (x_1, \cdots, x_N)$ is the variable, $A_0, \cdots, A_N$ are given symmetric matrices and "$<$" stands for "negative definite". It is well-known that LMI-based optimization problems as well as LMI feasibility problems can be solved in polynomial time [2], and a toolbox of Matlab which is dedicated to convex problems involving LMIs is now available [3].

3. Fuzzy Controller Synthesis Using LMIs

In this section, we present LMI-based solutions to the fuzzy controller synthesis problems for nonlinear systems described by the TS fuzzy model. For the sake of clarity and convenience, we classify the TS fuzzy systems into two families based on how diverse their input matrices B_i are, and present an LMI-based solution for each of these families.

First, we consider the family of the TS fuzzy systems with the common input matrix property:

$$B_1 = \cdots = B_r = B. \tag{8}$$

We call this family $TS(B)$. The state equation of the TS fuzzy systems in $TS(B)$ can be described by

$$\dot{\mathbf{x}}(t) = \sum_{i=1}^{r} h_i(\mathbf{z}(t))A_i\mathbf{x}(t) + B\mathbf{u}(t). \tag{9}$$

Thus, if we apply a TS fuzzy controller

$$\mathbf{u}(t) = \sum_{i=1}^{r} h_i(\mathbf{z}(t))K_i\mathbf{x}(t), \tag{10}$$

then we have the closed-loop described by

$$\dot{\mathbf{x}}(t) = \{\sum_{i=1}^{r} h_i(\mathbf{z}(t))(A_i + BK_i)\}\mathbf{x}(t). \tag{11}$$

Note that the controller (10) is derived from the following TS fuzzy rules which share the same fuzzy sets with the TS fuzzy model of the plant (1):

Controller Rule i:
IF $z_1(t)$ is M_{i1} and $\cdots$ and $z_g(t)$ is M_{ig},
THEN

$$\mathbf{u}(t) = K_i\mathbf{x}(t).$$

$i = 1, \cdots, r$.

Since the closed-loop (11) is again in the form of a polytopic system, the stability criterion (6) can be used to obtain the following: If there exists a symmetric matrix Q such that

$$Q > 0,\ (A_i + BK_i)Q + Q(A_i + BK_i)^T < 0, \quad i = 1, \cdots, r, \tag{12}$$

then the closed loop (11) is stable. Note that, with the definition of new variables $Y_i = K_iQ, i = 1, \cdots, r$, the stability problem (12) can be transformed to LMIs. Thus, under the condition (8), stabilizing TS fuzzy controllers can be found as follows:

Synthesis procedure for the family $TS(B)$:

- Find $Q = Q^T \in \Re^{n\times n}$ and $Y_i \in \Re^{p\times n}$, $i = 1, \cdots, r$ satisfying

$$Q > 0,\ A_iQ + QA_i^T + BY_i + Y_i^TB^T < 0, \quad i = 1, \cdots, r. \tag{13}$$

- Compute $K_i = Y_iQ^{-1}$, $i = 1, \cdots, r$.
- Set

$$\mathbf{u}(t) = \sum_{i=1}^{r} h_i(\mathbf{z}(t))K_i\mathbf{x}(t).$$

Next, we consider the stabilization of the family of the TS fuzzy systems whose input matrices are not all the same. We call this family $TS(B_i)$. Note that $TS(B_i)$ is the complement set of $TS(B)$. We propose a new fuzzy controller which has the following state equation:

$$\dot{\mathbf{u}}(t) = A_c\mathbf{u}(t) + \sum_{i=1}^{r} h_i(\mathbf{z}(t))K_i\mathbf{x}(t). \tag{14}$$

Note that, in this strategy, the output of the typical TS fuzzy controller

$$\mathbf{u}_{TS}(\cdot) \triangleq \sum_{i=1}^{r} h_i(\mathbf{z}(\cdot))K_i\mathbf{x}(\cdot)$$

is postfiltered by a linear system $G_c(s) \triangleq (sI - A_c)^{-1}$. This is a simple modification of the original law of the TS fuzzy control or the strategy of [1], but will lead to a significant convenience in the controller synthesis. With the control input $\mathbf{u}(t)$ defined by (14) applied to the TS fuzzy system (4), we have the following:

$$\begin{aligned} \dot{\mathbf{x}}(t) &= \textstyle\sum_{i=1}^{r} h_i(\mathbf{z}(t))\{A_i\mathbf{x}(t) + B_i\mathbf{u}(t)\} \\ \dot{\mathbf{u}}(t) &= A_c\mathbf{u}(t) + \textstyle\sum_{i=1}^{r} h_i(\mathbf{z}(t))K_i\mathbf{x}(t) \\ &= \textstyle\sum_{i=1}^{r} h_i(\mathbf{z}(t))\{K_i\mathbf{x}(t) + A_c\mathbf{u}(t)\}. \end{aligned}$$

Thus, the resulting closed-loop is described by

$$\dot{\mathbf{X}}(t) = \sum_{i=1}^{r} h_i(\mathbf{z}(t)) \begin{bmatrix} A_i & B_i \\ K_i & A_c \end{bmatrix} \mathbf{X}(t), \tag{15}$$

where X is the augmented state vector defined by

$$\mathbf{X}(t) = \begin{bmatrix} \mathbf{x}^T(t) & \mathbf{u}^T(t) \end{bmatrix}^T.$$

By applying the stability condition (6) to this polytopic system, we can see that the closed-loop (15) is stable if there exists a symmetric matrix Q satisfying:

$$\begin{cases} Q > 0 \\ \begin{bmatrix} A_i & B_i \\ K_i & A_c \end{bmatrix} Q + Q \begin{bmatrix} A_i & B_i \\ K_i & A_c \end{bmatrix}^T < 0, \\ \qquad\qquad i = 1, \cdots, r. \end{cases} \tag{16}$$

Note that the inequalities (16) are not linear in the variables $Q, A_c, K_i, i = 1, \cdots, r$. But, by applying the technique of elimination of matrix variables, we can express (16) with the following LMIs in which the controller matrices A_c and K_i are eliminated:

$$\begin{cases} Q > 0 \\ \mathcal{N}^T\left(\begin{bmatrix} A_i & B_i \\ 0 & 0 \end{bmatrix} Q + Q \begin{bmatrix} A_i & B_i \\ 0 & 0 \end{bmatrix}^T\right)\mathcal{N} < 0, \\ \qquad\qquad i = 1, \cdots, r, \end{cases} \tag{17}$$

where $\mathcal{N}$ is any matrix of maximum rank satisfying $[\ 0_{p\times n} \quad I_p\ ]\mathcal{N} = 0$. Of course, the most convenient choice for $\mathcal{N}$ will be $\mathcal{N} = [\ I_n \quad 0_{n\times p}\ ]^T$, by which

$$\mathcal{N}^T \begin{bmatrix} A_i & B_i \\ 0 & 0 \end{bmatrix} \text{ and } \begin{bmatrix} A_i & B_i \\ 0 & 0 \end{bmatrix}^T \mathcal{N}$$

can be simplified as $[\ A_i \quad B_i\ ]$ and $[\ A_i \quad B_i\ ]^T$, respectively. Once the matrix Q which satisfies the LMIs (17) is obtained, the controller matrices A_c and K_i can be computed from (16). Hence, we have the following design procedure, which provides stabilizing controllers in the form of a postfiltered TS fuzzy controller for the family $TS(B_i)$:

Synthesis procedure for the family $TS(B_i)$:

- Find a symmetric matrix $Q \in \Re^{(n+p)\times(n+p)}$ satisfying

$$\begin{cases} Q > 0, \\ [A_i\ B_i]Q\begin{bmatrix} I_n \\ 0_{p\times n} \end{bmatrix} + [I_n\ 0_{n\times p}]Q[A_i\ B_i]^T < 0, \\ \qquad\qquad i = 1, \cdots, r. \end{cases} \tag{18}$$

- Compute the controller matrices A_c and K_i, $i = 1, \cdots, r$ by solving the LMIs (16).
- Set the TS fuzzy controller by

$$\mathbf{u}_{TS}(t) = \sum_{i=1}^{r} h_i(\mathbf{z}(t))K_i\mathbf{x}(t),$$

and the postfilter by

$$\frac{U(s)}{U_{TS}(s)} = (sI - A_c)^{-1}.$$

4. A Numerical Example

In this section, we present an example that illustrates a synthesis procedure developed in this paper. The example considers a simple nonlinear mass-spring-damper system which is adapted from the design example 2 of [5]. The dynamic equation of the system is given by

$$M\ddot{x} + g(x, \dot{x}) + f(x) = \phi(\dot{x})u, \tag{19}$$

where M is the mass, $g(x,\dot{x}) = D(c_1x + c_2\dot{x})$ is the nonlinear term with respect to the damper, $f(x) = c_3x + c_4x^3$ is the nonlinear term with respect to the spring, and $\phi(\dot{x}) = 1 + c_5\dot{x}^3$ is the nonlinear term with respect to the input force. In the example, the parameters are set as follows:

$$M = 1, D = 1,$$
$$c_1 = 0, c_2 = 1, c_3 = 0.01, c_4 = 0.1, c_5 = 0.13.$$

Also, it is assumed that $x \in [-1.5, 1.5]$ and $\dot{x} \in [-1.5, 1.5]$. With $x_1 \triangleq \dot{x}$, $x_2 \triangleq x$, and $\mathbf{x} \triangleq [\ x_1 \quad x_2\ ]^T$, the system (19) can be described by

$$\dot{\mathbf{x}} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -x_1 - 0.01x_2 - 0.1x_2^3 \\ x_1 \end{bmatrix} + \begin{bmatrix} 1 + 0.13x_1^3 \\ 0 \end{bmatrix} u. \qquad (20)$$

This system can be represented by the following TS fuzzy model :

Rule 1 : IF x_1 is about 0 and x_2 is about 0
THEN $\dot{\mathbf{x}} = A_1\mathbf{x} + B_1u$,
Rule 2 : IF x_1 is about 0 and x_2 is about ± 1.5
THEN $\dot{\mathbf{x}} = A_2\mathbf{x} + B_2u$,
Rule 3 : IF x_1 is about 1.5 and x_2 is about 0
THEN $\dot{\mathbf{x}} = A_3\mathbf{x} + B_3u$,
Rule 4 : IF x_1 is about 1.5 and x_2 is about ± 1.5
THEN $\dot{\mathbf{x}} = A_4\mathbf{x} + B_4u$,
Rule 5 : IF x_1 is about -1.5 and x_2 is about 0
THEN $\dot{\mathbf{x}} = A_5\mathbf{x} + B_5u$,
Rule 6 : IF x_1 is about -1.5 and x_2 is about ± 1.5
THEN $\dot{\mathbf{x}} = A_6\mathbf{x} + B_6u$,

where the weights w_{ij} are nonnegative functions defined by

$$w_{11}(x_1) = \begin{cases} 1 - \frac{8}{27}x_1^3 & \text{for } x_1 \geq 0 \\ 1 + \frac{8}{27}x_1^3 & \text{for } x_1 \leq 0, \end{cases}$$
$$w_{12}(x_1) = \begin{cases} \frac{8}{27}x_1^3 & \text{for } x_1 \geq 0 \\ 0 & \text{for } x_1 \leq 0, \end{cases}$$
$$w_{13}(x_1) = \begin{cases} 0 & \text{for } x_1 \geq 0 \\ -\frac{8}{27}x_1^3 & \text{for } x_1 \leq 0, \end{cases}$$
$$w_{21}(x_2) = 1 - \tfrac{4}{9}x_2^2, \quad w_{22}(x_2) = \tfrac{4}{9}x_2^2.$$

This fuzzy model can be rewritten in the polytopic form

$$\dot{\mathbf{x}}(t) = \sum_{i=1}^{6} h_i(\mathbf{x}(t))\{A_i\mathbf{x}(t) + B_iu\}, \qquad (21)$$

where the state space matrices and the weight functions h_i are given by

$$A_1 = A_3 = A_5 = \begin{bmatrix} -1 & -0.01 \\ 1 & 0 \end{bmatrix}, \ A_2 = A_4 = A_6 = \begin{bmatrix} -1 & -0.235 \\ 1 & 0 \end{bmatrix},$$

$$B_1 = B_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \ B_3 = B_4 = \begin{bmatrix} 1.4388 \\ 0 \end{bmatrix}, \ B_5 = B_6 = \begin{bmatrix} 0.5613 \\ 0 \end{bmatrix},$$

$$h_1(\mathbf{x}(t)) = w_{11}(x_1(t))w_{21}(x_2(t)),$$
$$h_2(\mathbf{x}(t)) = w_{11}(x_1(t))w_{22}(x_2(t)),$$
$$h_3(\mathbf{x}(t)) = w_{12}(x_1(t))w_{21}(x_2(t)),$$
$$h_4(\mathbf{x}(t)) = w_{12}(x_1(t))w_{22}(x_2(t)),$$
$$h_5(\mathbf{x}(t)) = w_{13}(x_1(t))w_{21}(x_2(t)),$$
$$h_6(\mathbf{x}(t)) = w_{13}(x_1(t))w_{22}(x_2(t)),$$

and the fuzzy sets are defined as shown in Fig.1,2. In this fuzzy model, the state variables x_1 and x_2 are taken to be the premise variables, and the dimensions of the state vector $\mathbf{x}$ and the input u are $n = 2$ and $p = 1$, respectively. Also, $h_i(\mathbf{x}(t)) \geq 0$, $\forall i$ and $\sum_{i=1}^{6} h_i(\mathbf{x}(t)) = 1$ hold always. Since the input matrices B_i of the TS fuzzy model are not all the same, the procedure for $TS(B_i)$ is readily applicable to the design of stabilizing fuzzy controllers. In the following, we illustrate this synthesis procedure, in which the software LMI Control Toolbox[3] is used to compute the solutions of LMIs.

By solving the LMIs (16) and (18) , we obtain

$$A_c = -1.1629,$$
$$K_1 = [\,-3.4860\ -1.4063\,],\ K_2 = [\,-3.7336\ -1.4520\,],$$
$$K_3 = [\,-4.4792\ -1.5896\,],\ K_4 = [\,-4.7268\ -1.6354\,],$$
$$K_5 = [\,-2.4930\ -1.2229\,],\ K_6 = [\,-2.7406\ -1.2686\,].$$

Then the control signal $u(t)$ is obtained from the following:

$$\dot{u}(t) = A_cu(t) + \sum_{i=1}^{6} h_i(\mathbf{x}(t))K_i\mathbf{x}(t). \qquad (22)$$

Applying this controller to the system (20), we obtain the result of Fig. 3 for the initial condition $\mathbf{x}(0) = [\ -1 \quad -1\ ]^T$, in which $u(0) = 0$ is assumed for simplicity. Note that the trajectory of Fig. 3 reflects the design strategy enforcing the closed-loop stability only. The result can be made better by incorporating other performance requirements (on decay rate, bound on the output, etc.) in the process of controller synthesis.

5. Concluding Remarks

In this paper, we addressed the problem of designing fuzzy controllers with guaranteed stability for nonlinear systems described by the TS fuzzy model. We classified the TS fuzzy systems into two families $TS(B)$ and $TS(B_i)$ based on how diverse their input matrices are, and presented different controller synthesis procedure for each family. The procedures provide the TS fuzzy controllers or their modified version according to which family the given system belongs to. An example of controlling a nonlinear mass-spring-damper system was considered. Since each procedure is essentially based on the LMI feasibility problem, LMI Control Toolbox in Matlab environment was effectively utilized in solving the problem, and a satisfactory simulation result was obtained. Further investigations yet to be done include the refinement of the developed procedures toward a multi-objective design tool and a detailed performance comparison with other methods.

References

[1] P. Apkarian, P. Gahinet and G. Becker, Self-scheduled H_∞ control of linear parameter-varying systems: A design example, *Automatica* **31** (1995) 1251-1261.

[2] S. Boyd, L. ElGhaoui, E. Feron and V. Balakrishnan, *Linear Matrix Inequalities in Systems and Control Theory*, SIAM Studies in Applied Mathematics, Vol. 15 (SIAM, Philadelphia, 1994).

[3] P. Gahinet, A. Nemirovski, A.J. Laub and M. Chilali, *LMI Control Toolbox* (MathWorks Inc., Natick, MA, 1995).

[4] T. Takagi and M. Sugeno, Fuzzy identification of systems and its applications to modeling and control, *IEEE Transactions on Systems, Man and Cybernetics* **15** (1985) 116-132.

[5] K. Tanaka, T. Ikeda and H. Wang, Robust stabilization of a class of uncertain nonlinear systems via fuzzy control: Quadratic stabilizability, H^∞ control theory and linear matrix inequalities, *IEEE Transactions on Fuzzy Systems* **4** (1996) 1-13.

[6] K. Tanaka and M. Sugeno, Stability analysis and design of fuzzy control system, *Fuzzy Sets and Systems* **45** (1992) 135-156.

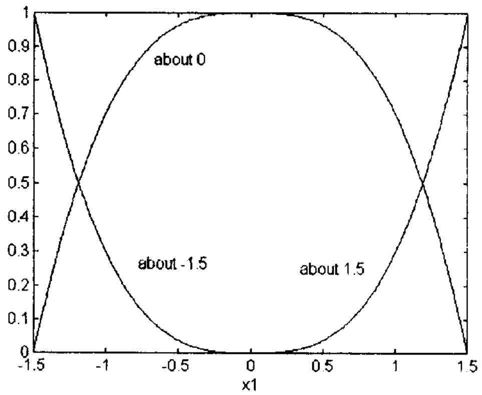

Figure 1: Fuzzy sets on the domain of x_1.

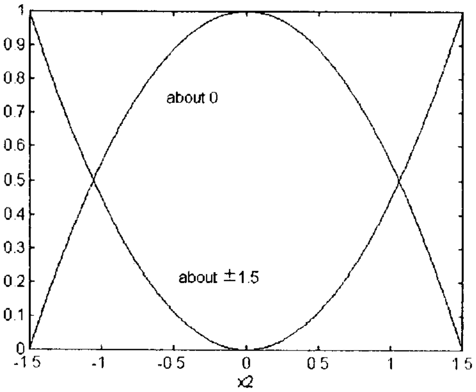

Figure 2: Fuzzy sets on the domain of x_2.

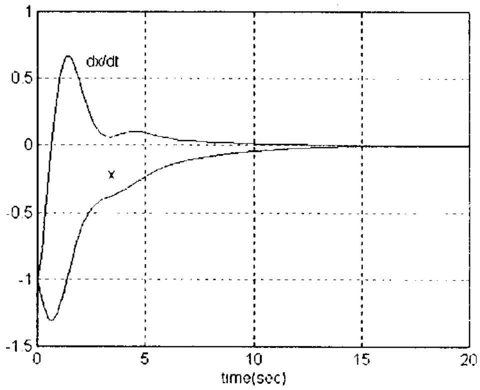

Figure 3: Simulation result for the controller obtained from (22).