# A TSK Fuzzy Controller for Underwater Robots

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#### Abstract

Underwater robotic vehicles (URVs) have been an important tool for various underwater tasks because they have greater speed, endurance, depth capability, and safety than human divers. As the use of such vehicles increases, the vehicle control system becomes one of the most critical subsystems to increase autonomy of the vehicle. The vehicle dynamics are nonlinear and their hydrodynamic coefficients are often difficult to estimate accurately. In this paper a new type of fuzzy model-based controller based on Takagi-Sugeno-Kang fuzzy model is designed and applied to the control of an underwater robotic vehicle. The proposed fuzzy controller: 1) is a nonlinear controller, but a linear state feedback controller in the consequent of each local fuzzy control rule; 2) can guarantee the stability of the closed-loop fuzzy system; 3) is relatively easy to implement. Its good performance as well as its robustness to the change of parameters have been shown and compared with the results of conventional linear controller by simulation .

**Keywords:** URV, Fuzzy model, Fuzzy controller, State feedback controller, Stability, Robustness.

### 1. Introduction

Underwater robotic vehicles are used for various work assignments such as pipe-lining, inspection, data collection, drill support, hydrography mapping, construction, maintenance and repairing of undersea equipment, etc. As the use of such vehicles increases, the development of vehicles having greater autonomy becomes highly desirable. The vehicle control system is one of the most critical subsystems to increase autonomy of the vehicle. As discussed by many researchers[1,2], the vehicle dynamics are nonlinear and their hydrodynamic coefficients are often difficult to estimate accurately. Furthermore, some of work assignments often require the vehicle

to handle payloads of different shapes and weights. The varying payloads during the operation change the total vehicle mass as well as the centers of gravity and buoyancy. Therefore, the conventional linear controller such as PID may not be able to handle these changes promptly and result in poor performance.

In this paper we design a fuzzy model-based controller to control an underwater vehicle and provide the superiority to the conventional linear controller. Most fuzzy controllers in the literature have been designed by linguistic rules without an explicit model of the system. A major drawback of this approach is a lack of systematic method for the analysis and design of the fuzzy control system. In this paper we use simple mathematical fuzzy models of dynamic systems based on fuzzy sets and fuzzy inference [3-5] in order to design a new type of fuzzy controller. This fuzzy model consists of a small set of fuzzy rules whose consequents are linear equations, not fuzzy sets. This type of fuzzy model is often called Takagi-Sugeno-Kang (TSK) fuzzy model. The advantage of this approach is that analysis is simpler, and a model structure can often be obtained with minimum modeling effort. [6-8]. We deal with the TSK fuzzy model with constant terms and present the design algorithm of fuzzy controller based on the TSK fuzzy model. The proposed fuzzy controller is basically a nonlinear controller and can guarantee the stability of the overall fuzzy system.

## 2. Fuzzy Controller Based on TSK Fuzzy Model

In this section we suggest a fuzzy controller which is based on the TSK fuzzy model and can guarantee the stability of the closed-loop fuzzy system. The TSK fuzzy controller have the same number of rules and premises as those of the TSK fuzzy model. The most important characteristic of the TSK fuzzy controller is that its consequents are linear state feedback controllers in the structure.

### 2.1. State Space Description of TSK Fuzzy Model

The TSK fuzzy model describing the behavior of a single-input single-output continuous dynamic system can be constructed from the following fuzzy rules:

$$R^{i}: \text{ if } z_{1} \text{ is } F_{1}^{i}, z_{2} \text{ is } F_{2}^{i}, \cdots, z_{m} \text{ is } F_{m}^{i} \text{ then}$$

$$\frac{d^{n}y^{i}}{dt^{n}} = a_{0}^{i} + a_{1}^{i}y + a_{1}^{i}y + \cdots + a_{n}\frac{d^{n-1}y}{dt^{n-1}} + b_{1}^{i}u$$
(1)

Choosing the state variables  $x_1 = y$ ,  $x_2 = \frac{dy}{dt}$ ,

 $\cdots$ ,  $x_n = \frac{dy^{n-1}}{dt^{n-1}}$  yields the state space description of the form

$$R^{i}$$
: IF  $z_{1}$  is  $F_{1}^{i}$ ,  $z_{2}$  is  $F_{2}^{i}$ , ...,  $z_{m}$  is  $F_{m}^{i}$ 

THEN  $\dot{\mathbf{x}}^{i} = A^{i} \mathbf{x} + \mathbf{b}^{i} u + \mathbf{d}^{i}$  (2)

where

$$A^{i} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ a_{1}^{i} & a_{2}^{i} & a_{3}^{i} & \cdots & a_{n}^{i} \end{bmatrix}, \quad \mathbf{b}^{i} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ b_{1}^{i} \end{bmatrix}, \quad \mathbf{d}^{i} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ a_{0}^{i} \end{bmatrix},$$

$$\mathbf{c} = \begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix}$$

The state vector x is inferred as

$$\dot{\mathbf{x}} = \sum_{i=1}^{k} w^{i}(z) \quad \dot{\mathbf{x}}^{i} / \sum_{i=1}^{k} w^{i}(z)$$
 (3)

$$w^{i}(z) = \prod_{j=1}^{m} F_{j}^{i}(z_{j}) \tag{4}$$

where k is the number of rules and  $F_j^i(z_j)$  is the membership value of  $z_j$  in the fuzzy set  $F_j^i$ . The consistency of the two descriptions (1) and (3) can be seen easily.

# 2.2. Design of TSK Fuzzy Controller by Pole Placement

The TSK fuzzy controller is a nonlinear controller and is composed of the TSK fuzzy rules whose consequents are state feedback controllers. We can arbitrarily assign all poles of the closed-loop fuzzy system to any desirable locations so that all states asymptotically converge to zero as  $t \to \infty$ . Furthermore, the fuzzy controller can guarantee the stability of the controlled system.

The i-th rule of the fuzzy controller corresponding to that of the fuzzy model is given by

$$C^{i}$$
: IF  $z_{1}$  is  $F_{1}^{i}$ ,  $z_{2}$  is  $F_{2}^{i}$ , ...,  $z_{m}$  is  $F_{m}^{i}$  (5)  
THEN  $u^{i} = -\mathbf{g}^{i} \mathbf{x} + g_{0}^{i}$ 

where  $\mathbf{g}^i = [g_1^i \ g_2^i \ \cdots \ g_n^i]$  is a  $1 \times n$  feedback vector of constant gains and  $g_0^i$  is a scalar quantity.

The method of inferring the control input u from the fuzzy control rule (5) is slightly different from that of inferring the fuzzy model (3).

**Theorem 1:** The control input u is inferred from the consistency condition

$$\sum_{i=1}^{k} w^{i}(z) \quad \mathbf{b}^{i} u = \sum_{i=1}^{k} w^{i}(z) \quad \mathbf{b}^{i} u^{i}$$
 (6)

if the feedback gain vector  $\mathbf{g}^{i}$  and scalar quantity  $\mathbf{g}_{0}^{i}$  in (15) are determined such that

$$\mathbf{\Phi} = A^i - \mathbf{b}^i \mathbf{g}^i \tag{7}$$

$$\mathbf{b}^{i}g_{0}^{i} = -\mathbf{d}^{i} \tag{8}$$

where  $\Phi$  is a state transition matrix whose eigenvalues are the desired poles of the closed-loop fuzzy system. Then every equilibrium state of the closed-loop fuzzy system is asymptotically stable...

### Proof

The closed-loop fuzzy system with the controller and consistency condition (5) - (8) can be expressed as follows:

$$\dot{\mathbf{x}} = \sum_{i=1}^{k} w^{i}(z) \quad \dot{\mathbf{x}}^{i} / \sum_{i=1}^{k} w^{i}(z)$$

$$= \frac{\sum_{i=1}^{k} w^{i}(z) (A^{i} \mathbf{x} + \mathbf{d}^{i}) + \sum_{i=1}^{k} w^{i}(z) \mathbf{b}^{i} u^{i}}{\sum_{i=1}^{k} w^{i}(z)}$$

$$= \sum_{i=1}^{k} w^{i}(z) (A^{i} - \mathbf{b}^{i} \mathbf{g}^{i}) / \sum_{i=1}^{k} w^{i}(z)$$

$$= \mathbf{0} \mathbf{x}$$
(9)

It is obvious that the structure of the closed-loop fuzzy system (9) is a linear system. If we choose the state transition matrix  $\boldsymbol{\Phi}$  as a stable matrix whose all the eigenvalues have negative real parts, every equilibrium state of (9) is asymptotically stable.

### 2.3. Integral Action on TSK Fuzzy Controller

If there exists the modeling error between the fuzzy model and the process or the process has constant disturbances, the steady-state error occurs. In this case an integral action is required to eliminate the steady-state error. The state space design approach will not produce the integral action unless a special step is introduced. We define the augmented state vector  $\mathbf{x}_a$  with an integral state  $x_I$  through

$$\mathbf{x}_a = \begin{bmatrix} x_I \\ \mathbf{x} \end{bmatrix} \quad \text{where} \quad \dot{x_I} = y = \mathbf{c}\mathbf{x} \tag{10}$$

to write the fuzzy model (2) as an augmented state space description

R<sup>i</sup>: IF 
$$z_1$$
 is  $F_1^i$ ,  $z_2$  is  $F_2^i$ , ...,  $z_m$  is  $F_m^i$ 

THEN  $\dot{\mathbf{x}}_a^i = A_a^i \mathbf{x}_a + \mathbf{b}_a^i u + \mathbf{d}_a^i$ 
 $y = \mathbf{c}_a \mathbf{x}_a$ 

(11)

where

$$A_a^i = \begin{bmatrix} 0 & c \\ 0 & A^i \end{bmatrix}, \quad \mathbf{b}_a^i = \begin{bmatrix} 0 \\ \mathbf{b}^i \end{bmatrix}, \quad \mathbf{d}_a^i = \begin{bmatrix} 0 \\ \mathbf{d}^i \end{bmatrix},$$

$$\mathbf{c}_a = \begin{bmatrix} 0 & \mathbf{c} \end{bmatrix}$$

The fuzzy controller for the augmented fuzzy model can be designed by using the same method as described in (5) - (8).

# 3. Fuzzy Model and Controller for Underwater Vehicle

Underwater vehicle model can be described by the following vector equation:

$$M(\mathbf{x}) \ddot{\mathbf{x}} + A(\dot{\mathbf{x}}) \dot{\mathbf{x}} + \mathbf{h}(\mathbf{x}) = \mathbf{F}$$
 (12)

where  $\mathbf{x} \in R^6$  is position and orientation in the vehicle coordinates;  $M \in R^{6 \times 6}$  is an inertia matrix (rigid body inertia + added mass);  $A \in R^{6 \times 6}$  includes all the nonlinear dynamic terms with inertia velocity terms associated with the forces and torques exerted on the vehicle by fluid motion, drag forces and torque;  $\mathbf{h} \in R^6$  is a vector including gravity, buoyancy and other disturbance terms; and is a vector representing the forces and torques  $\mathbf{g} \cdot \mathbf{F} \in R^6$  enerated by the thruster forces.

### 3.1 Dynamic Model of ODIN

In this paper we use the dynamic model for the depth and pitch motion of the Omni-Directional Intelligent Navigator (ODIN) developed at the Autonomous Systems Laboratory of the University of Hawaii[8]. ODIN's dynamic equations for the depth and pitch motion can be described as:

$$\begin{bmatrix} \alpha(m-z_{w^{-}}) & -mx_{c} \\ -\alpha mx_{c} & (I_{y}-M_{q^{-}}) \end{bmatrix} \begin{bmatrix} \dot{w} \\ \dot{q} \end{bmatrix}$$

$$+ \begin{bmatrix} \alpha^{2}z_{ww} & |w| & -mz_{c}q \\ mz_{c}q & m_{qq}|q \end{bmatrix} \begin{bmatrix} w \\ q \end{bmatrix} +$$

$$\begin{bmatrix} (\rho V - mg)\cos\theta \\ (x_c mg - x_b \rho V)\cos\theta + (z_c mg - z_b \rho V)\sin\theta \end{bmatrix} = \begin{bmatrix} \beta F_z \\ \gamma T \end{bmatrix}$$
(13)

where  $F_z$  and T are the force and torque, respectively. The depth z and angular displacement  $\theta$  in global coordinates can be obtained by integrating the following equation:

The numerical values of the system parameters in (13) are: m = 145 kg,  $z_w = 0.675$ ,  $I_v = 0.418$ ,  $M_q = 3.594$ ,  $z_{ww} = 116.96$ ,  $m_{qq} = 4.2034$   $\rho V = 1421.65$ , g = 9.8,  $\alpha = 0.305$ ,  $\eta = 4.45$ ,  $\gamma = 1.35725$ , the center of gravity  $(x_c, z_c) = (0,0)$ , the center of buoyancy  $(x_b, z_b) = (0,0)$ .

### 3.2. TSK Fuzzy Model of ODIN

We obtained the TSK fuzzy models for the depth and pitch motion described by (15) and (17). Fig. 1 and Fig. 2 show membership functions of the depth motion with premise variables z and  $\theta$ , and the pitch motion with premise variable  $\theta$ , respectively. The fuzzy model of the depth motion consists of four rules as follows:

$$R_{z}^{1}$$
: IF  $z$  is  $V_{1}$  and  $\theta$  is  $T_{1}$   
THEN  $z = a_{0}^{1} + a_{1}^{1}z + a_{2}^{1}z + b_{1}^{1}F_{z} + c^{1}\theta$   
 $R_{z}^{2}$ : IF  $z$  is  $V_{1}$  and  $\theta$  is  $T_{2}$   
THEN  $z = a_{0}^{2} + a_{1}^{2}z + a_{2}^{2}z + b^{2}F_{z} + c^{2}\theta$   
 $R_{z}^{3}$ : IF  $z$  is  $V_{2}$  and  $\theta$  is  $T_{1}$   
THEN  $z = a_{0}^{3} + a_{1}^{3}z + a_{2}^{3}z + b^{3}F_{z} + c^{3}\theta$   
 $R_{z}^{4}$ : IF  $z$  is  $V_{2}$  and  $\theta$  is  $T_{2}$   
THEN  $z = a_{0}^{4} + a_{1}^{4}z + a_{2}^{4}z + b^{4}F_{z} + c^{4}\theta$ 

where

$$\begin{bmatrix} a_0^1 & a_1^1 & a_2^1 & b_1^1 & c^1 \\ a_0^2 & a_1^2 & a_2^2 & b_1^2 & c^2 \\ a_0^3 & a_1^3 & a_2^3 & b_1^3 & c^3 \\ a_0^4 & a_1^4 & a_2^4 & b_1^4 & c^4 \end{bmatrix} =$$

$$\begin{bmatrix} -0.019 & 0.022 & -0.009 & 0.094 & -0.022 \\ 0.000 & -0.010 & -0.015 & 0.101 & -0.028 \\ 0.003 & 0.032 & -0.062 & 0.097 & 0.047 \\ 0.000 & 0.003 & -0.023 & 0.101 & 0.012 \end{bmatrix} \tag{16}$$

The fuzzy model of the pitch motion with two rules is described by

$$R_{\theta}^{1}: \text{ IF } \theta \text{ is } O_{1} \text{ THEN}$$

$$\theta = \overline{a_{0}}^{1} + \overline{a_{1}}^{1}\theta + \overline{a_{2}}^{1}\theta + \overline{b_{1}}^{1}T$$

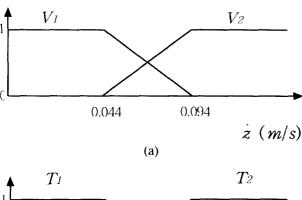
$$R_{\theta}^{2}: \text{ IF } \theta \text{ is } O_{2} \text{ THEN}$$

$$\theta = \overline{a_{0}}^{2} + \overline{a_{1}}^{2}\theta + \overline{a_{2}}^{2}\theta + \overline{b_{1}}^{2}T$$

$$(17)$$

where

$$\begin{bmatrix}
\overline{a_0}^1 & \overline{a_1}^1 & \overline{a_2}^1 & \overline{b_1}^1 \\
\overline{a_0}^2 & \overline{a_1}^2 & \overline{a_2}^2 & \overline{b_2}^2
\end{bmatrix} = 
\begin{bmatrix}
-0.006 & 0.000 & 0.068 & -0.315 \\
0.000 & 0.000 & -0.067 & -0.315
\end{bmatrix} (18)$$



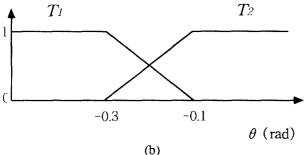
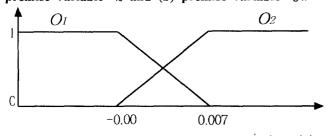


Fig. 1. Membership functions for the premise of the fuzzy model of the depth motion with (a) premise variable z and (b) premise variable  $\theta$ ..



 $\theta$  (rad/s)

Fig. 2. Membership functions for the premise of the fuzzy model of the pitch motion with premise variable  $\theta$ .

## 3.3. TSK Fuzzy Controller for ODIN

In this subsection we design the TSK fuzzy controllers for the depth and pitch motion of ODIN to track the desired trajectories, respectively. First, we define the augmented state space description of the consequent of the fuzzy model (15) of the depth motion as follows:

$$R_z^i : \quad \dot{\mathbf{z}}_a^i = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & a_1^i & a_2^i \end{bmatrix} \, \mathbf{z}_a + \begin{bmatrix} 0 \\ 0 \\ b_1^i \end{bmatrix} F_z^i + \begin{bmatrix} 0 \\ 0 \\ d_z^i \end{bmatrix}$$
 (19)

where  $i=1,\cdots,4$ , the augmented state vector  $\mathbf{z}_a = \begin{bmatrix} \tilde{z}_I & \tilde{z} & \dot{z} \end{bmatrix}^T$ ,  $\tilde{z} = z - z_r$ ,  $\tilde{z}_I = \int \tilde{z} dt$ ,  $z_r$  is the desired trajectory of the depth, and the scalar quantity  $d^i = a_0^i + a_1^i z_r + a_2^i \ \dot{z}_r - \ddot{z}_r + c^i \theta$ .

For this fuzzy model, the i-th rule of the TSK fuzzy controller is designed as follows:

$$C_z^i \colon F_z^i = - \mathbf{g}^i \mathbf{z}_a + g_0^i \tag{10}$$

where the feedback gain vector  $\mathbf{g}^i = \begin{bmatrix} \mathbf{g}_1^i & \mathbf{g}_2^i & \mathbf{g}_3^i \end{bmatrix}$  and the scalar quantity  $\mathbf{g}_0^i$  are chosen as

$$\begin{bmatrix} \mathbf{g}^{1} \\ \mathbf{g}^{2} \\ \mathbf{g}^{3} \\ \mathbf{g}^{4} \end{bmatrix} = \begin{bmatrix} 1702.13 & 766.19 & 138.20 \\ 1584.16 & 712.77 & 128.56 \\ 1649.48 & 742.60 & 133.38 \\ 1584.16 & 712.90 & 128.49 \end{bmatrix}, \quad \mathbf{g}^{i}_{0} = -d^{i}/b^{i}_{1}$$
(21)

to assign the desired poles of the closed-loop fuzzy system to  $-4 \pm i4$ , -5.

The augmented state space description of the consequent of the fuzzy model (17) of the pitch motion is defined by

$$R_{\theta}^{i} : \quad \dot{\boldsymbol{\theta}}_{a}^{i} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & \overline{a}_{1}^{i} & \overline{a}_{2}^{i} \end{bmatrix} \quad \boldsymbol{\theta}_{a} + \begin{bmatrix} 0 \\ 0 \\ \overline{b}_{1}^{i} \end{bmatrix} T^{i} + \begin{bmatrix} 0 \\ 0 \\ \overline{d}^{i} \end{bmatrix}$$

$$(22)$$

where i=1,2, the augmented state vector  $\boldsymbol{\theta}_a = [ \stackrel{\sim}{\theta}_I \stackrel{\sim}{\theta} \stackrel{\sim}{\theta}]^T$ ,  $\stackrel{\sim}{\theta}_I = \int \stackrel{\sim}{\theta} dt$ ,  $\stackrel{\sim}{\theta}_r$  is the desired trajectory of the pitch, and the scalar quantity  $\overrightarrow{d}^i = \stackrel{\sim}{a_0}^i + \stackrel{\sim}{a_1}^i \theta_r + \stackrel{\sim}{a_2}^i \stackrel{\sim}{\theta}_r - \stackrel{\sim}{\theta}_r$ .

For this fuzzy model, the i-th rule of the fuzzy controller is designed as follows:

$$C_{\theta}^{i} : T^{i} = - \frac{\mathbf{g}^{i}}{\mathbf{g}^{i}} \boldsymbol{\theta}_{a} + \frac{\mathbf{g}_{0}^{i}}{\mathbf{g}_{0}^{i}}$$
 (23) where the feedback gain vector 
$$\frac{\mathbf{g}^{i}}{\mathbf{g}^{i}} = \begin{bmatrix} \overline{g}_{1}^{i} & \overline{g}_{2}^{i} & \overline{g}_{3}^{i} \end{bmatrix}$$
 and the scalar quantity 
$$\frac{\mathbf{g}_{0}^{i}}{\mathbf{g}_{0}^{i}}$$
 are chosen as

$$\begin{bmatrix} \overline{\mathbf{g}}^{1} \\ \overline{\mathbf{g}}^{2} \end{bmatrix} = \begin{bmatrix} -228.57 & -41.49 & -509.94 \\ -228.57 & -41.05 & -507.94 \end{bmatrix},$$

$$\overline{g}_{0}^{i} = -\overline{d}^{i} / \overline{b}_{1}^{i}$$
(24)

to assign the desired poles of the closed-loop fuzzy system to  $-4 \pm i4$ , -5.

### 4. Simulation Results

Simulation studies were conducted on the depth and pitch motion of the underwater vehicle ODIN. For comparison study, the fuzzy control system and PID control system were tested for two separate cases: 1) Case 1 - ODIN with no addition of payload; and 2) Case 2 - ODIN with addition in payload of 5 kg which makes the total mass m=150 kg and the center of gravity changes from  $(x_c, z_c) = (0,0)$  to (0.3,0) m.

We considered the desired vehicle motion with the following four segments: 1) the vehicle goes down to a desired deep depth as well as rotates; 2) the

vehicle maintains the current position; 3) the vehicle comes back to the original shallow depth and pitch angle; 4) then the vehicle maintains the position. This overall motion is a good representation of a typical set of motions during the performance ot an underwater vehicle surveying, sampling, and maintenance tasks. Desired profiles for the depth and pitch are shown in Fig. 3.

The PID controller used in the simulation can be described as

$$\begin{bmatrix} F_z \\ T \end{bmatrix} = K_P \begin{bmatrix} \tilde{z} \\ \tilde{\theta} \end{bmatrix} + K_D \begin{bmatrix} \tilde{z} \\ \tilde{\theta} \end{bmatrix} + K_I \begin{bmatrix} \tilde{z}_I \\ \tilde{\theta}_I \end{bmatrix}$$
(39)

The PID gains are chosen as

$$K_P = \begin{bmatrix} 60 & 0 \\ 0 & 60 \end{bmatrix}$$
,  $K_D = \begin{bmatrix} 20 & 0 \\ 0 & 20 \end{bmatrix}$ ,  $K_I = \begin{bmatrix} 40 & 0 \\ 0 & 40 \end{bmatrix}$ 

which provide reasonably better performance than any other gains. It is known that the optimized selection of PID gains is critical for a PID controller to produce good performances and requires much time.

4 show simulation results of the PID controller and the fuzzy controller for Case 1. Even shows though the PID controller reasonable performance, the fuzzy controller presents smaller errors than the PID controller. Fig. simulation results for Case 2. Even though the PID controller shown in Fig. 5(a) still maintains the stability of the system, it shows larger errors due to parameter change than Fig. 4(a). However, the results shown in Fig. 5(b) with the fuzzy controller present performances than the PID controller. better Furthermore, when the fuzzy controller is used, there is not much difference between Fig. 4(b) and Fig 5(b) except for the very first segment of the vehicle motion.

### 5. Conclusions

A new type of fuzzy controller based on the TSK fuzzy model was described. To our knowledge all of previous works are focused on a special type of the TSK fuzzy model with no constant term in the consequent of each local fuzzy rule. This fact is a strong assumption about modeling the system with fuzzy sets and fuzzy inference. In our approach the general form of the TSK fuzzy model was considered including constant terms in the consequent of the fuzzy rule. The TSK fuzzy controller: 1) is a nonlinear controller, but a linear state feedback controller in the consequent of each local fuzzy control rule; 2) can guarantee the stability of the closed-loop fuzzy system; 3) is relatively easy to implement. In this paper we also applied the TSK fuzzy controller to the control of underwater robotic vehicles since their dynamics are nonlinear and their hydrodynamic coefficients are often difficult estimate accurately. The fuzzy model and fuzzy controller of the underwater vehicle ODIN were obtained with a few rules. Its good performance as well as its robustness to the change of parameters (inertia and the center of gravity) have been shown and compared with the results of conventional linear controller by simulation .

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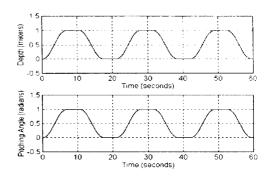


Fig. 3. Desired trajectories for the depth and pitch motion.

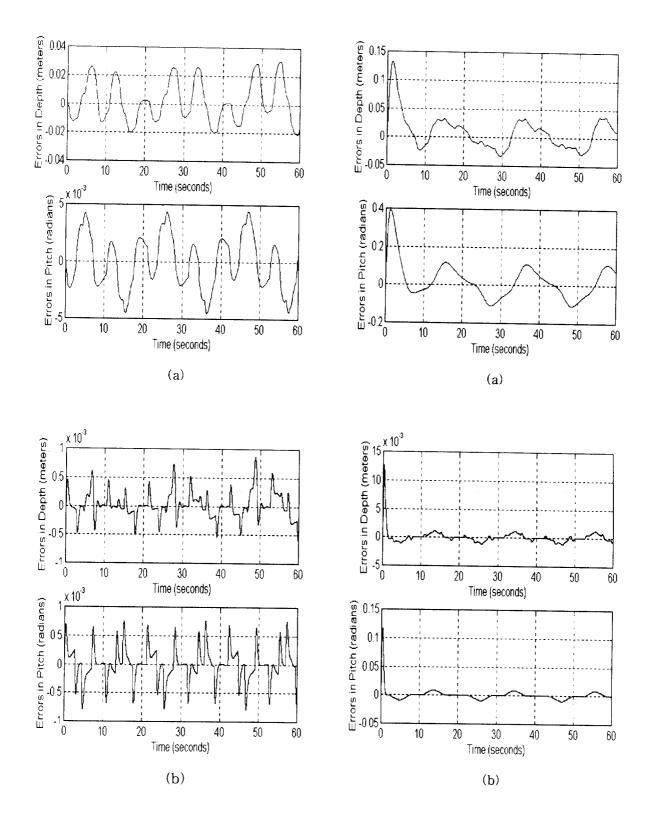


Fig. 4. Simulation results for Case 1. (a) PID controller (b) fuzzy controller

Fig. 5. Simulation results for Case 2. (a) PID controller (b) fuzzy controller