The Third AFSS(1998), 143−147

An Elliptic Approach to Learning Discriminants

Fatiha KARBOU(*), Fatima KARBOU(**), M. KARBOU(***)

(*):ENSEM, Dept. E.G, B. P. 8118, Oasis, Route d'El Jadida, Casablanca, Morocco
(**) IAV HASSAN II, dept. Topography, B.P. 6202, Rabat-instituts, Rabat, Morocco
(***): Sophia Concept, 47, rue Planquette Belvédère, Casablanca , Morocco

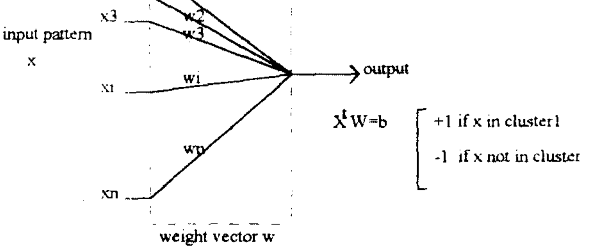
Abstract
It is wisely stated that the most valuable knowledge that a person can acquire is the knowledge of how to learn. The human's learning is characterized by the ability to extract relationships between the different characters of a given situation. The ellipse is a first approach of comparison. We assimilate each character to a half axis of the ellipse and the result is a geometrical figure that varies according to values of the two characters. Thus, we take into account the two characters as an alone entity.

Keywords: Fuzzy sets, learning, pattern recognition

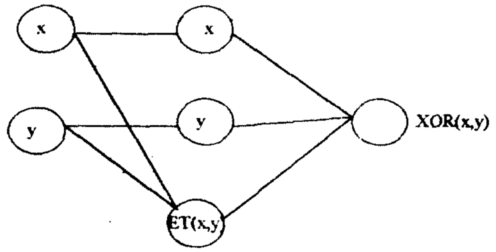
1. Introduction

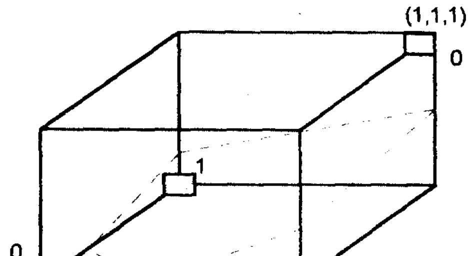
A concept central to the practice of pattern recognition is that of discriminants [1,2]. The idea a pattern-recognition system learns adaptively from experience and distills various discriminants, each appropriate for its purpose. In pattern recognition's quintessential form, both the learning and recognition phases would be achieved with concurrent distributed processing and the entire procedure would be powerful and rapid. This seems to be the case in biological neural systems and should also be the case for computer based adaptive pattern-recognition systems.

One of the most exciting developments during of the early days of pattern recognition was the Perceptron. It can be represented schematically in the form of an array of multipliers and summing junctions as shown in the next figure. Each input is connected to the output by a link containing a multiplier, such that the input to the output node is the sum of all the appropriately weighted inputs. The task is to learn a set of weights so that all patterns can be classified correctly using a single set of weights.



However, this system presents some limitations. Indeed, the Perceptron can learn only problems of classifications with a linear separation. It can not therefore solve the problem of XOR in two dimension. Nevertheless, if we considers an extension in three dimension of this problem we can have a linear separation using the back-propagation or the functional link net as shown in the next figures.





Or, the separation is not immediate [1,2]. So, we can suppose that if the Perceptron arrives badly to solve the problem of XOR, it is because we have not described it or given it relationships that exist between elements and tools to put them in obviousness and to valorize them. Indeed, we wonder why the human arrives to extract relationships who exist within an example of data without needing to make an extension of dimensions? We can suppose that he possesses a powerful and instantaneous system of extraction of information and relationships that allows him to realize a separation of subset better than the most powerful of machine.

It would be interesting to try to imitate the human separation manner taking into account the totality of characters unite and targeting the parameters of discrimination. Indeed, if we want to compare two persons, the man is not going to compare the length of arms, the length of legs, the width of the torso, ..., but first he is going to target a criterion of comparison, for example the size. Then, he is going to tell that a person A is higher than a person B and that B is slimmer than A without detailing the comparison. I.e. he is not going to say that the person A possesses an arm more long than B, or that the face of B is slimmer than A, but he is going to make the comparison by taking into account all characters unite.

The goal of this paper is first to make architectures of systems that permit to extract relationships between elements and to make decision based on these relationships immediately. Then, to put in obviousness the power of the generalization of these systems

2. The elliptic approach

It is wisely stated that the most valuable knowledge that a person can acquire is the knowledge of how to learn. The human's learning is characterized by the ability to extract relationships between the different characters of a given situation. In other words, humans learn relationships as entities rather than values of characters. What gives a certain power to recognize a situation already seen.

In fact, the pattern recognition in computer takes generally in consideration only one character at each comparison. Nevertheless, the human vision, takes in consideration the totality of characters at the same time. Consequently, the decision is taken globally without dividing the comparison. For example, if we want to compare the form of two objects, the human vision takes into account the parameter's width/length/height at the same time as a relation. However, the machine needs to compare widths then lengths and finally height. In each comparison the machine considers only one character.

The ellipse is a first approach of comparison because it provides us a general and a simple relation that can link two parameters that are the half axis of the ellipse. Indeed, we assimilate each character to a half axis of the ellipse and the result is a geometrical figure that varies according to values of the two characters. Thus, we take into account the two characters as an alone entity.

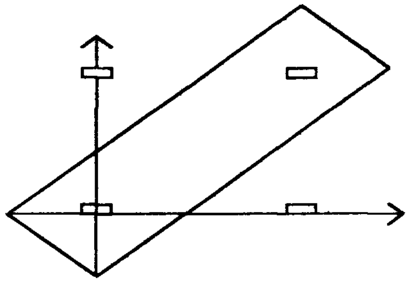
Instead of separating the two characters, we use an ellipse and we assimilate each character to a half axis of the ellipse. In the case of intervals, the center of the ellipse is the point formed by the two inferior interval extremities, and in the case of values, the center is arbitrary and is common to all ellipses .

The eccentricity of the ellipse is a general relation of comparison. Its value belongs to [0,1], and measures the similarity of the form of the considered attributes. So, the notion of memberships degree and fuzzy sets is implicit. Instead of using the Euclidean distance, we use the eccentricity because it measures a fuzzy relationship between the elements. We have no need to use other functions to calculate the fuzzy membership.

3. Architecture for the XOR resolution

3.1. In two-dimensional space

The even-parity patterns have an associated output value of 1 and the odd-parity have an output of 0.



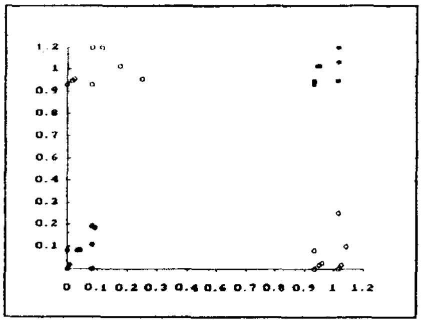


Figure 3.1. Patterns of even and odd parity

We can see that a regular metric would not allow us to solve the problem of XOR. However, if we represent each point by an ellipse we will see that close elements to (0, 0) and (1,1) would be ellipses whose eccentricity is neighbor 0. And close elements to (0,1) and (1,0) will have ellipses whose eccentricity is neighbor 1. Thus, we have succeeded to separate two clusters only by using properties of ellipses. The generalization is immediate as shown in the figures (3.1)and (3.2).

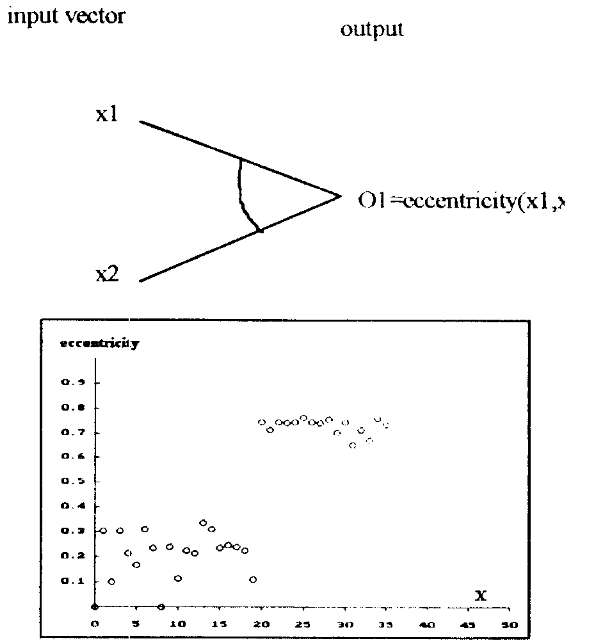
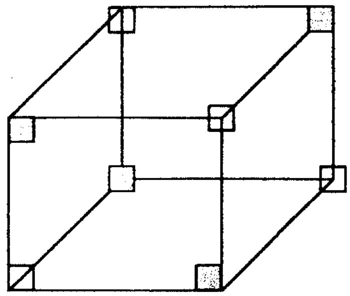


Figure 3.2. The net for the two dimensional XOR problem and The results obtained



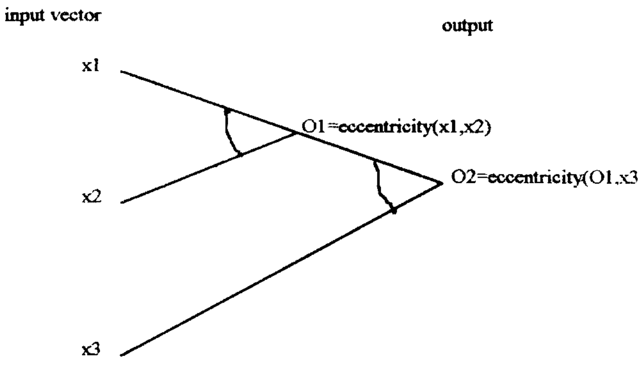


Figure 3.3. The net for the three dimensional XOR problem

Remark

The eccentricity make the output of the even parity 0. So, if we want to have 1 as an associated output we use (1-eccenticity).

To make the eccentricity defined while elements are close to (0, 0), we make a translation of all the set.

So, the relation between the two abscissas has been detected only by the eccentricity. In fact, we construct a net of relations. So, the conjunction of each two knots gives an information. Indeed. the link between the two inputs to produce the target output is the eccentricity.

3.2. In three-dimensional space

The XOR problem in the three dimensional space is as illustrated in the figure (3.5). So, the elements (0 1 0), (1 0 0), (0 0 1), (1 1 1) would have the same output 1 and the elements (0 0 0), (1 0 1), (0 1 1), (1 1 0) would have the same output 0, (or inversely) . So, let x1,x2,x3 a vector represented the elements of this set.

eccentricity(x1,x2)=0 if x1=x2.

eccentricity(x1,x2)=1 if x1≠x2

then eccentricity(eccentricity(x1,x2),x3) is an XOR problem between eccentricity(x1,x2) and x3. So, we find the case of two-dimensional XOR problem's as it illustrated in the next figure (3.6). In fact, we construct a net of relations. So, the conjunction of each two knots gives an information.

3.3. In (n+1)-dimensional space

The XOR problem in (n+1)-dimensional space is the generalization of the case of three-dimensional problem. So, let x1,x2,x3,...,xn a vector represented the elements of this set. then

eccentricity(x1,x2)=0 if x1=x2,

eccentricity(x1,x2)=1 if x1≠x2

then

eccentricity(eccentricity(x1,x2),x3) is an XOR problem between eccentricity(x1,x2) and x3. So,

eccentricity(eccentricity(eccentricity(x1,x2),x3),x4) is an XOR problem between eccentricity(eccentricity(x1,x2),x3) and x4. ...etc. as it illustrated in the next figure (3.7).

So, we find at the end the case of two-dimensional XOR problem's.

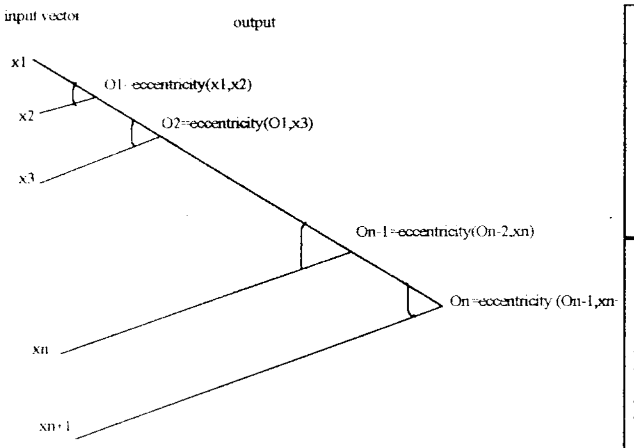


Figure 3.4. The net for the (n+1) dimensional XOR problem

4. Learning a separation of the Iris data of Anderson

We have already, put in obviousness during the using of fuzzy curves [3] that there is a relation that links elements of this set (figure 4.1).

The Iris data of Anderson[3] has often been used as a standard set for testing the performances of algorithms and discrimination's criteria. In this case, 50 plants in each of two varieties of Iris represented in the data: Virginia iris and Versicolor iris. Therefore, we are concerned by the comparison of the form of theses plants. So, it logic to compute the perimeter or the surface of the ellipses to determine the most similar elements.

By examining the graph, that represents the evolution of characters of the two varieties, we notice that there is a certain relationship that links parameters. No regular metric can separate correctly these two varieties. Therefore, we have represented varieties by ellipses whose the half center are characters (i, j) where i≠j.

So, we construct a net that takes into account the difference of the size. In this net, we compute the perimeter or the surface of the six combination of the characters ((1 2), (1 3), (1 4), (2 3), (2 4), (3 4)) for two compared vectors, as shown in the figure (4.4). Then we choose a relation and we compute the eccentricity between the two vector relatively to this relation. The results of all relations for the Iris data of Anderson are shown in the figure(4.3) (We have compared all the elements with an alone vector: the first one).

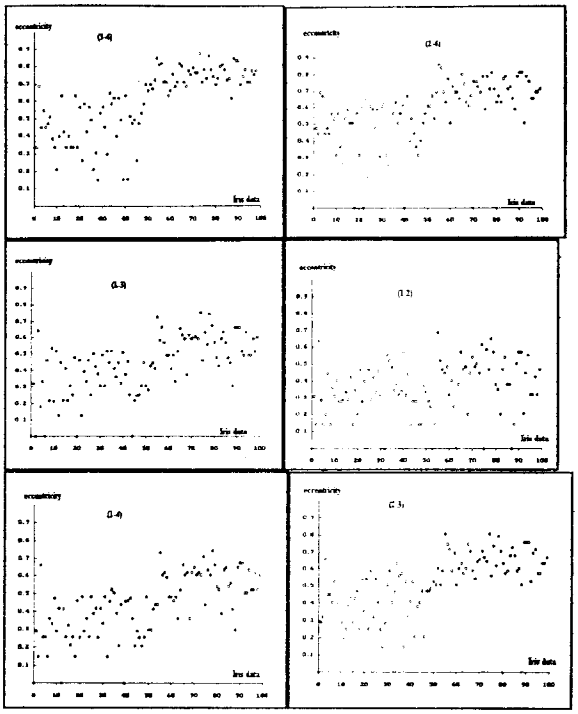


Figure 4.3. The results of net for the iris data of Anderson

We can use another layer in order to determinate the most discriminate relation.

Conclusion

The separation obtained by the eccentricity of ellipses show that even if there are little close elements or more similar elements, the system arrives to classify them by dispersed eccentricity. Therefore, we tend to the human vision because the human itself is incapable to decide in front of two appeared similar elements.

So, we give to the machine necessary tools in order to decide immediately, without making iterations.

we see that the ellipse gives us magical resolutions to different processed problems [5]. Is it a chance or a coincidence or simply an ellipse's secret that escapes us, because we see the ellipse everywhere around us; even the planets have choose it as path?

5. References and bibliography

1. YOH-HAN PAO, Adaptive pattern recognition and neural networks
   Addison-Wesley, 1989.
2. Eric DAVALO, Patrick NAIM
   Des réseaux de neurones
   Eyrolles Paris 1990, 2ème edition.
3. F. KARBOU, New Fuzzy Concepts, FUZZ-IEEE'97, Six IEEE international Conference on Fuzzy Systems, Barcelona, Spain, 1-5 JULLY, 1997.

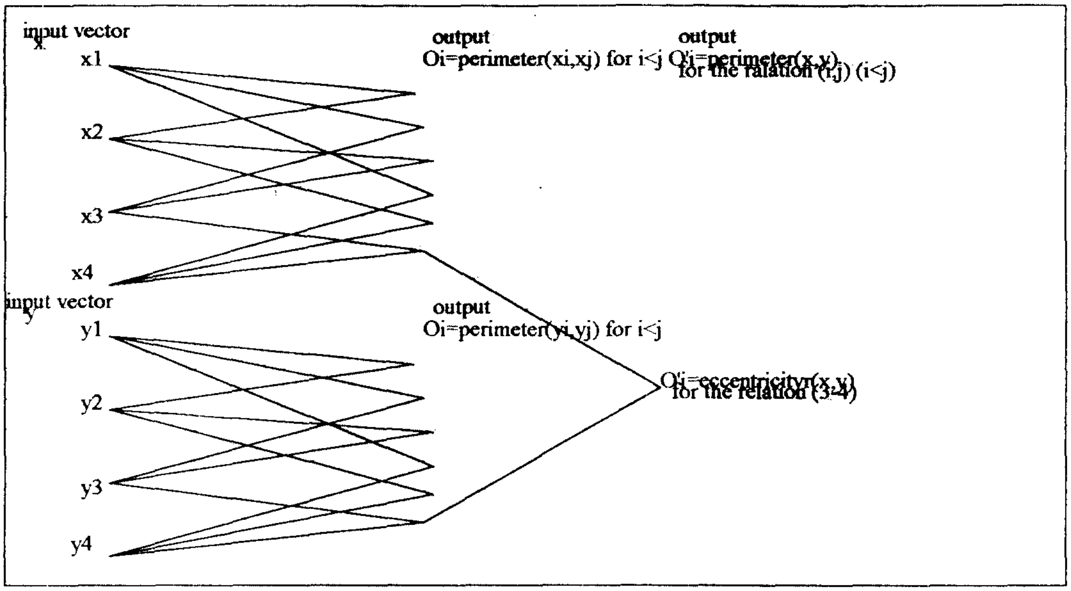


Figure 4.1. The net for the resolution for the iris data