The Third AFSS(1998). 69 – 74

DEA with Interval Efficiency Values

Yutaka MAEDA, Tomoe ENTANI and Hideo TANAKA

Dept. of Industrial Engineering, College of Engineering, Osaka Prefecture University
1-1, Gakuencho, Sakai, Osaka, 599-8531 Japan
Tel:+81-722-54-9354 Fax:+81-722-54-9915 E-mail:{maeda, enatni, tanaka}@ie.oskakfu-u.ac.jp

Abstract:
In this paper, we formulate the DEA model with interval efficiency. There exist two phases of efficiency evaluation with respect to the upper limit and the lower limit. From these viewpoints, we can define two extreme points of efficiency. As a result, an interval efficiency for each DMU can be obtained. We also formulate the interval cross-efficiency.

1 Introduction

DEA (Data Environment Analysis) is a non-parametric technique for measuring and evaluating the relative efficiencies of DMUs (Decision Making Units) with common input and output terms [1,2]. Most of DEA models regard the maximum of relative ratios of weighted outputs to weighted inputs as the efficiency. In general, if there exists one set of weights for output and input, we can compute one relative efficiency. In case that the relative efficiency is 1, there exist many sets of weights. It is reasonable to suppose that the efficiency is located in some interval.

Evaluating DMU by interval efficiency value is useful when there is a peculiar DMU and there are a lot of DMUs whose efficiencies from CCR model are 1. Peculiar DMUs are tend to be evaluated efficient from CCR model [5]. If each of the upper and lower limits is not good, the DMU might need to be improved. There are two kinds of efficiencies with respect to the upper and lower limits so that there are two ways to improve a DMU. The optimistic view is to improve the upper limit and the pessimistic view is to improve the lower limit. The numerical examples are shown to illustrate the proposed method for interval efficiency.

2 A fundamental DEA model

In the DEA framework, DMUs are regarded as decisional entities responsible for converting multiple inputs to outputs. All DMUs are enveloped by the efficient frontier. The efficiency of DMU is obtained as a ratio of the weighted sum of outputs to that of inputs subject to the constraint conditions that the similar ratio for every DMU is less than equal to unity. A fundamental DEA model called CCR model, which gives the efficiency of the o-th DMU, is formulated as follows:

$$\left.\begin{array}{llll} \max\limits_{\boldsymbol{u},\boldsymbol{v}} & \theta & = \frac{\boldsymbol{u}^t\boldsymbol{y}_o}{\boldsymbol{v}^t\boldsymbol{x}_o} & \\ \text{sub. to} & \frac{\boldsymbol{u}^t\boldsymbol{y}_j}{\boldsymbol{v}^t\boldsymbol{x}_j} & \leq 1 & \text{for all j} \\ & \boldsymbol{u} & \geq \boldsymbol{0} & \\ & \boldsymbol{v} & \geq \boldsymbol{0} & \end{array}\right\} \quad (1)$$

where $\boldsymbol{y} \in \Re^k$ is an output vector and $\boldsymbol{x} \in \Re^m$ is an input vector. The number of DMUs is n. The number of inputs of the DMU is m and that of outputs is k. (1) is considered as following LP-problem:

$$\left.\begin{array}{llll} \max\limits_{\boldsymbol{u},\boldsymbol{v}} & \theta & = \boldsymbol{u}^t\boldsymbol{y}_o & \\ \text{sub. to} & \boldsymbol{v}^t\boldsymbol{x}_o & = 1 & \\ & \boldsymbol{u}^t\boldsymbol{y}_j - \boldsymbol{v}^t\boldsymbol{x}_j & \leq 0 & \text{for all j} \\ & \boldsymbol{u} & \geq 0 & \\ & \boldsymbol{v} & \geq 0 & \end{array}\right\} \quad (2)$$

The dual problem of (2) is obtained as follows by θ, $\boldsymbol{\lambda} \in \Re^n$, $\boldsymbol{s}_x \in \Re^m$ and $\boldsymbol{s}_y \in \Re^k$.

$$\left.\begin{array}{lll} \min\limits_{\boldsymbol{s}_x,\boldsymbol{s}_y,\boldsymbol{\lambda}} & \theta & \\ \text{sub. to} & \theta\boldsymbol{x}_o - X\boldsymbol{\lambda} - \boldsymbol{s}_x & = \boldsymbol{0} \\ & Y\boldsymbol{\lambda} - \boldsymbol{s}_y & = \boldsymbol{y}_o \\ & \boldsymbol{\lambda} & \geq \boldsymbol{0} \\ & \boldsymbol{s}_x & \geq \boldsymbol{0} \\ & \boldsymbol{s}_y & \geq \boldsymbol{0} \end{array}\right\} \quad (3)$$

First we minimize θ then maximize $\boldsymbol{e}^t\boldsymbol{s}_x + \boldsymbol{e}^t\boldsymbol{s}_y$ where $\boldsymbol{e}^t$ is equal to $\{1,\ldots,1\}$. The optimal solitions are regarded as $(\theta^*, \boldsymbol{\lambda}^*, \boldsymbol{s}_x{}^*, \boldsymbol{s}_y{}^*)$. In case that θ^* is 1, $\boldsymbol{s}_x{}^*$ is 0 and $\boldsymbol{s}_y{}^*$ is 0, the DMU is evaluated efficient. In other cases, it is evaluated inefficient.

If the DMU is evaluated inefficient, the input $\boldsymbol{x}_o$ and output $\boldsymbol{y}_o$ can be improved.

$$\boldsymbol{x}_o{}^* = \theta^*\boldsymbol{x}_o - \boldsymbol{s}_x{}^*, \quad \boldsymbol{y}_o{}^* = \boldsymbol{y}_o + \boldsymbol{s}_y{}^* \quad (4)$$

The efficiency value from improved input $\boldsymbol{x}_o{}^*$ and improved output $\boldsymbol{y}_o{}^*$ is 1. This model assume the set of possible inputs and outputs.

$$P = \{(\boldsymbol{x},\boldsymbol{y}) | \boldsymbol{x} \geq X\boldsymbol{\lambda},\ \boldsymbol{y} \leq Y\boldsymbol{\lambda},\ \boldsymbol{\lambda} \geq \boldsymbol{0}\} \quad (5)$$

All poits on the efficienct frontier have the same efficiencies.

3 Interval efficiency for crisp data

The efficiency of o-th DMU is evaluated by the maximum efficiency of all DMUs. To determine the relative efficiency for o-th DMU with input-output vector pair (x_o, y_o) is formulated as follows:

$$\left.\begin{array}{lll} \max_{u,v} & \theta_o^* & = \dfrac{\frac{u^t y_o}{v^t x_o}}{\max_j \frac{u^t y_j}{v^t x_j}} \\ \text{sub. to} & u & \geq 0 \\ & v & \geq 0 \end{array}\right\} \quad (6)$$

which stems from the original meanings of CCR model (1). When the maximum ratio of weighted output to weighted input is fixed to 1, (6) can be reduced to the following problem:

$$\left.\begin{array}{lll} \max_{u,v} & \theta_o^* & = \frac{u^t y_o}{v^t x_o} \\ \text{sub. to} & \max_j \frac{u^t y_j}{v^t x_j} & = 1 \\ & u & \geq 0 \\ & v & \geq 0 \end{array}\right\} \quad (7)$$

The solution space limited under the constraint conditions in (7) is boundary of that of (1). Therefore the upper limit of efficiency interval for o-th DMU can be obtained by solving the problem (1).

Considering the inverse concept of (6) the lower limit of interval efficiency for o-th DMU can be defined as:

$$\left.\begin{array}{lll} \min_{u,v} & \theta_{o*} & = \dfrac{\frac{u^t y_o}{v^t x_o}}{\max_j \frac{u^t y_j}{v^t x_j}} \\ \text{sub. to} & u & \geq 0 \\ & v & \geq 0 \end{array}\right\} \quad (8)$$

When the maximum ratio of weighted output to weighted input is fixed to 1, (8) can be reduced to the following problem:

$$\left.\begin{array}{lll} \min_{u,v} & \theta_{o*} & = \frac{u^t y_o}{v^t x_o} \\ \text{sub. to} & \max_j \frac{u^t y_j}{v^t x_j} & = 1 \\ & u & \geq 0 \\ & v & \geq 0 \end{array}\right\} \quad (9)$$

(9) can not be replaced with the equivalent LP-problem. By assuming that $u^t y_j / v^t x_j = 1$ for each j, (9) can be devided into n problems as follows:

$$\left.\begin{array}{lll} \min_{u,v} & \theta_j & = \frac{u^t y_o}{v^t x_o} \\ \text{sub. to} & \frac{u^t y_j}{v^t x_j} & = 1 \\ & u & \geq 0 \\ & v & \geq 0, \qquad j = 1, ..., n \end{array}\right\} \quad (10)$$

Therefore we can solve n problems. Then the minimum of them is the optimal solution of (9) which is the lower limit of interval efficiency. Mathematically, we can write the lower limit of interval efficiency as:

$$\theta_{o*} = \min_j \ \theta_j \quad (11)$$

All efficiencies for each DMU are between θ_{o*} and θ_o^*. Through (1) and (11), we obtain the upper and lower limits of efficiency for o-th DMU's input and output data and an interval efficiency as follows:

$$\theta_{o*} \leq \theta_o \leq \theta_o^* \quad (12)$$

4 Interval efficiency models for interval data

Let us formulate DEA model with interval data. The data are given as intervals:

$$x_{ij} \in [x_{ij*}, x_{ij}^*], \quad y_{rj} \in [y_{rj*}, y_{rj}^*]$$

The upper limit of interval efficiency for o-th DMU, θ_o^*, is defined as:

$$\left.\begin{array}{lll} \max_{u,v} & (\max_{x_o, y_o} \theta_o^* & = \dfrac{\frac{u^t y_o}{v^t x_o}}{\max_j \frac{u^t y_j}{v^t x_j}}) \\ \text{sub. to} & u & \geq 0 \\ & v & \geq 0 \end{array}\right\} \quad (13)$$

where x_{ij} which is i-th element of a vector x_j is whithin an interval $[x_{ij*}, x_{ij}^*]$. When the maximum ratio of weighted output to weighted input is fixed to 1, the maximum of (13) with respect to interval data can be reduced to:

$$\left.\begin{array}{ll} \max_{u,v} & \theta_o^* = \frac{u^t y_o^*}{v^t x_{o*}} \\ \text{sub. to} & \max\left(\max_{j \neq o} \frac{u^t y_{j*}}{v^t x_j^*}, \frac{u^t y_o^*}{v^t x_{o*}}\right) = 1 \\ & u \geq 0 \\ & v \geq 0 \end{array}\right\} \quad (14)$$

where the lower bounds of input intervals and the upper bounds of output intervals are used for o-th DMU and the upper bounds of input intervals and the lower bounds of output intervals are used for other DMUs. This is from optimistic viewpoint for o-th DMU, because the efficiency of o-th DMU is calculated by a pair of data which gives o-th DMU the maximum efficiency. The solution space limited under the constraint conditions in (14) is a boundary of that of the following problem:

$$\left.\begin{array}{llll} \max_{u,v} & \theta_o^* & = \frac{u^t y_o^*}{v^t x_{o*}} & \\ \text{sub. to} & \frac{u^t y_{j*}}{v^t x_j^*} & \leq 1 & \text{for } j \neq o \\ & \frac{u^t y_o^*}{v^t x_{o*}} & \leq 1 & \\ & u & \geq 0 & \\ & v & \geq 0 & \end{array}\right\} \quad (15)$$

The optimal solutions of (14) and (15) are the same. Therefore the upper limit of interval efficiency for o-th DMU, $\theta_o{}^*$, can be obtained by solving the problem (15).

The lower limit of interval efficiency for o-th DMU, θ_{o*}, can be formulated as:

$$\left.\begin{array}{lll} \min\limits_{u,v} & \left(\min\limits_{x_o,y_o} \theta_{o*} = \dfrac{\frac{u^t y_o}{v^t x_o}}{\max\limits_j \frac{u^t y_j}{v^t x_j}}\right) & \\ \text{sub. to} & u \geq 0 & \\ & v \geq 0 & \end{array}\right\} \quad (16)$$

When the maximum ratio of weighted output to weighted input is fixed to 1, (16) can be recuced to:

$$\left.\begin{array}{ll} \min\limits_{u,v} & \theta_{o*} = \frac{u^t y_{o*}}{v^t x_o{}^*} \\ \text{sub. to} & \max\left(\max\limits_{j\neq o} \frac{u^t y_j{}^*}{v^t x_{j*}}, \frac{u^t y_{o*}}{v^t x_o{}^*}\right) = 1 \\ & u \geq 0 \\ & v \geq 0 \end{array}\right\} \quad (17)$$

where the upper bounds of input intervals and the lower bounds of output intervals are used for o-th DMU and the lower bounds of input intervals and the upper bounds of output intervals are used for other DMUs. This is from pessimistic viewpoint for o-th DMU, because the efficiency of o-th DMU is calculated by a pair of data which gives o-th DMU the minimum efficiency. To reach the optimal solution , we reconsider (17) as n problems:

$$\left.\begin{array}{lll} \min\limits_{u,v} & \theta_j = \frac{u^t y_{o*}}{v^t x_o{}^*} & \\ \text{sub. to} & \frac{u^t y_j{}^*}{v^t x_{j*}} = 1 & \\ & u \geq 0 & \\ & v \geq 0, & j = 1, \ldots, n \end{array}\right\} \quad (18)$$

When j is o, the first constraint condition is $u^t y_{o*}/v^t x_o{}^* = 1$. We can solve n problems. Then the minimum of them is the optimal solution of (17) which is the lower limit of interval efficiency. Mathematically, we can write the lower limit of interval efficiency as:

$$\theta_{o*} = \min_j \ \theta_j \quad (19)$$

5 Cross-efficiency

The cross-efficiency for a-th DMU is evaluated by the weights which give the efficiency for o-th DMU [3,4]. There are two ways to compute cross-efficiency, because of two kinds of efficiency with respect to the upper and lower limits. So, one is computed by the weights which give o-th DMU its upper limit of interval efficiency. The other is computed by the weights which give o-th DMU its lower limit of interval efficiency.

First we think about the cross-efficiency from the weigths of the upper limit of interval efficiency. In general, there exists one set of weights for input and output, we can compute one cross-efficiency for each DMU. If there exist many sets of weights, cross-efficiencies for each DMU can not be calculated uniquely. In this case, it is reasonable to suppose that the cross-efficiency is located in some interval. So there are two cases that the cross-efficiency is given as a value or an interval.

If the efficiency of o-th DMU is not 1, the cross-efficiency is given as a value. Because we get one set of weights by solving (7). Then we regard the optimal solutions of (7) as u^* and v^*. The cross-efficiency of a-th DMU from the upper limit of interval efficiency value is obtained as follows:

$$\bar{\theta}_{ao} = \frac{u^{*t} y_a}{v^{*t} x_a} \quad (20)$$

If the efficiency of o-th DMU is 1, the cross-efficiency might be given as an interval, because many sets of weights might be given from (7). When the efficiency of o-th DMU is 1, we regard it as one of the constraint conditions. Tnen the upper limit of cross-efficiency is obtained by the following problem:

$$\left.\begin{array}{llll} \max\limits_{u,v} & \bar{\theta}_{ao}{}^* & = \frac{u^t y_a}{v^t x_a} & \\ \text{sub. to} & \frac{u^t y_j}{v^t x_j} & \leq 1 & \text{for all j} \\ & \frac{u^t y_o}{v^t x_o} & = 1 & \\ & u & \geq 0 & \\ & v & \geq 0 & \end{array}\right\} \quad (21)$$

When the weighted input is fixed to 1, (21) can be reduced to the following LP-problem:

$$\left.\begin{array}{llll} \max\limits_{u,v} & \bar{\theta}_{ao}{}^* & = u^t y_a & \\ \text{sub. to} & v^t x_a & = 1 & \\ & -v^t x_j + u^t y_j & \leq 0 & \text{for all j} \\ & -v^t x_o + u^t y_o & = 0 & \\ & u & \geq 0 & \\ & v & \geq 0 & \end{array}\right\} \quad (22)$$

The lower limit of cross-efficiency is obtained by the following problem:

$$\left.\begin{array}{llll} \min\limits_{u,v} & \bar{\theta}_{ao*} & = \frac{u^t y_a}{v^t x_a} & \\ \text{sub. to} & \frac{u^t y_j}{v^t x_j} & \leq 1 & \text{for all j} \\ & \frac{u^t y_o}{v^t x_o} & = 1 & \\ & u & \geq 0 & \\ & v & \geq 0 & \end{array}\right\} \quad (23)$$

When the weighted input is fixed to 1, (23) can be reduced to the following LP-problem:

Table 1: Crisp data

	DMU	1	2	3	4	5	6	7	8	9	10	11	12
Input	Doctors	20	19	25	27	22	55	33	31	30	50	53	38
	Nureses	151	131	160	168	158	255	235	206	244	268	306	284
Output	outpatients	100	150	160	180	94	230	220	150	190	250	260	250
	inpatients	90	50	55	72	66	90	88	80	100	100	147	120

Table 2: Efficiency for crisp data

DMU	1	2	3	4	5	6	7	8	9	10	11	12
upper	1.000	1.000	0.883	1.000	0.763	0.835	0.902	0.792	0.960	0.871	0.955	0.958
lower	0.578	0.585	0.489	0.593	0.520	0.364	0.593	0.573	0.680	0.444	0.616	0.702

Table 3: Cross-efficiency for crisp data

DMU	1	2	3	4	5	6	7	8	9	10	11	12
1	—	1.000	0.876	1.000	0.763	0.833	0.902	0.792	0.960	0.871	0.955	0.958
	—	0.585	0.489	0.593	0.667	0.364	0.593	0.573	0.688	0.444	0.616	0.702
2	1.000	—	0.883	1.000	0.763	0.835	0.902	0.719	0.960	0.871	0.934	0.958
	0.578	—	0.812	0.844	0.520	0.530	0.818	0.613	0.680	0.633	0.621	0.769
3	0.916	1.000	—	1.000	0.717	0.835	0.874	0.767	0.815	0.871	0.920	0.919
4	1.000	1.000	0.883	—	0.763	0.835	0.890	0.792	0.861	0.871	0.955	0.919
	0.946	0.981	0.868	—	0.717	0.795	0.874	0.767	0.815	0.851	0.920	0.885
5	1.000	1.000	0.876	1.000	—	0.765	0.889	0.791	0.862	0.851	0.934	0.919
6	0.916	1.000	0.883	1.000	0.717	—	0.874	0.767	0.815	0.871	0.919	0.919
7	1.000	1.000	0.819	0.902	0.745	0.561	—	0.738	0.960	0.676	0.769	0.958
8	1.000	0.981	0.868	1.000	0.763	0.833	0.874	—	0.840	0.871	0.955	0.903
9	1.000	1.000	0.819	0.902	0.745	0.561	0.902	0.738	—	0.676	0.769	0.958
10	0.916	1.000	0.883	1.000	0.717	0.835	0.874	0.767	0.815	—	0.919	0.885
11	1.000	0.981	0.869	1.000	0.763	0.833	0.874	0.792	0.840	0.871	—	0.903
12	1.000	1.000	0.819	0.902	0.745	0.561	0.902	0.738	0.960	0.676	0.769	—
max	1.000	1.000	0.883	1.000	0.763	0.833	0.902	0.792	0.960	0.871	0.955	0.958
min	0.578	0.585	0.489	0.593	0.520	0.364	0.593	0.573	0.680	0.444	0.616	0.702

Table 4: Interval data

		DMU	1	2	3	4	5	6	7	8	9	10	11	12
Input	Doctors	upper	21	20	26	28	23	56	34	32	31	51	54	39
		lower	19	18	24	26	21	54	32	30	29	49	52	37
	Nureses	upper	153	133	162	170	160	257	237	208	246	270	308	286
		lower	149	129	158	166	156	253	233	204	242	266	304	282
Output	outpatients	upper	102	152	162	182	96	232	222	154	192	252	262	252
		lower	98	148	158	178	92	228	218	150	188	248	258	248
	inpatients	upper	91	51	56	73	67	91	89	81	101	101	148	121
		lower	89	49	54	71	65	89	87	79	99	99	146	119

Table 5: Efficiency for interval data

DMU	1	2	3	4	5	6	7	8	9	10	11	12
upper	1.000	1.000	0.968	1.000	0.852	0.871	1.000	0.852	1.000	0.907	0.993	1.000
	1.000	1.000	0.834	0.962	0.721	0.800	0.842	0.759	0.859	0.836	0.918	0.873
lower	0.615	0.669	0.551	0.662	0.553	0.398	0.656	0.637	0.713	0.486	0.672	0.738
	0.544	0.487	0.434	0.529	0.474	0.332	0.534	0.516	0.649	0.405	0.565	0.637

Table 6: Cross-efficiency for interval data from the upper limit of efficiency

DMU	1	2	3	4	5	6	7	8	9	10	11	12
11	1.000	0.973	0.865	1.000	0.760	0.838	0.879	0.798	0.846	0.878	—	0.912
	0.947	0.928	0.825	0.952	0.721	0.799	0.837	0.759	0.804	0.836	—	0.867
12	1.000	1.000	0.882	0.998	0.762	0.748	0.924	0.799	0.935	0.830	0.917	—
	0.751	0.869	0.760	0.852	0.617	0.540	0.859	0.695	0.900	0.652	0.715	—
	0.942	0.931	0.826	0.949	0.718	0.776	0.842	0.756	0.812	0.823	0.904	—

Table 7: Cross-efficiency for interval data from the lower limit of efficiency

DMU	1	2	3	4	5	6	7	8	9	10	11	12
11	1.000	0.551	0.490	0.598	0.676	0.375	0.604	0.583	0.754	0.458	—	0.720
	0.885	0.487	0.434	0.529	0.590	0.332	0.534	0.515	0.667	0.405	—	0.637
12	1.000	0.633	0.573	0.718	0.698	0.595	0.631	0.653	0.692	0.630	0.815	—
	0.885	0.487	0.434	0.529	0.590	0.332	0.534	0.515	0.667	0.405	0.565	—

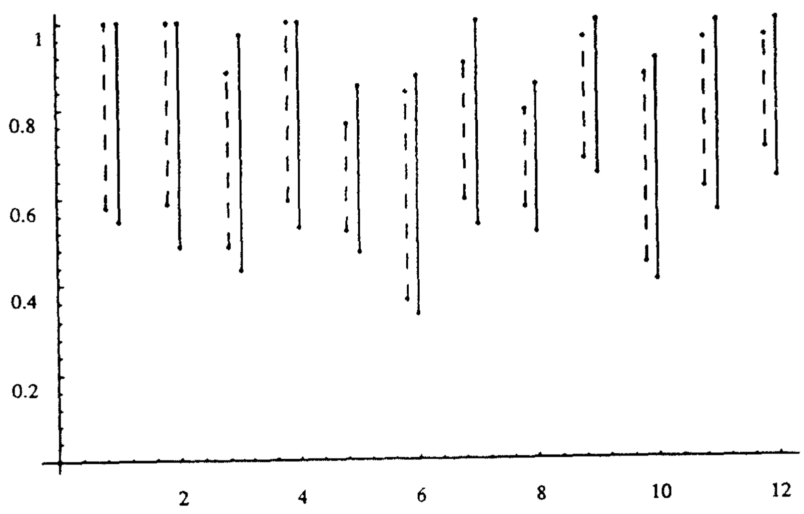

Figure 1: Interval efficiencies for crisp and interval data

$$\left.\begin{array}{llll}\min\limits_{u,v} & \overline{\theta}_{ao*} & = u^t y_a & \\ \text{sub. to} & v^t x_a & = 1 & \\ & -v^t x_j + u^t y_j & \le 0 & \text{for all j} \\ & -v^t x_o + u^t y_o & = 0 & \\ & u & \ge 0 & \\ & v & \ge 0 & \end{array}\right\} \quad (24)$$

Next we think about the cross-efficiency from the weigths of the lower limit of interval efficiency. The lower limit of interval efficiency is rarely equal to 1, then we get one set of weights which give θ_* by (11). They are regarded as u_* and v_*. The cross-efficiency of a-th DMU from the lower limit of interval efficiency value is obtained as follows:

$$\underline{\theta}_{ao} = \frac{u_*{}^t y_a}{v_*{}^t x_a} \quad (25)$$

When the lower limit of interval efficiency is 1, the cross-efficiency is located in some interval. The problems are the same as (22) and (24).

If the data are intervals, there are four kinds of cross-efficiencies. Because there are two kinds of efficiencies and each of them has two viewpoints which are optimistic and pessimistic. If the efficiency is equal to 1, the cross-efficiency is located in some interval.

6 Numerical examples

Let us give an example shown in Table 1. The numbers of doctors and nureses are inputs. And the numbers of outpatients and inpatients are outputs. Table 2 illustrates the interval efficiencies for crisp data. The upper limits of interval efficiencies for 1st, 2nd and 4th DMUs are all 1. The lower limits of interval efficiencies for these are about 0.6, which are not enough good. We evaluate them efficient from their upper limits, but from their lower limits we will find that they need to be improved. The upper limits for 9th and 12th DMUs are less than those for 1st, 2nd and 4th DMUs but they are more than 0.95, and the lower limits for them are much better than 1st, 2nd and 4th DMUs. Then we find 1st, 2nd and 4th DMUs are peculiar and 9th and 12th DMUs are not peculiar. From the lower limit of interval value, we can find peculiar DMUs easily. Table 3 shows the cross-efficiencies for crisp data.

The original data are crisp. We consider all data as intervals. They are shown in Table 4. Table 5 illustrates the interval efficiencies for interval data. Optimistic and pessmistic viewpoints for the upper and lower limits of efficiencies are shown. Table 6 and 7 show the cross-efficiencies for interval data by the weights of 11-th and 12-th DMUs. In both Tables the first row is from optimistic and the second row is from pessimistic.

In Figure 1, the interval efficiencies for crisp and interval data are illustrated. The dashing lines and the lines show the efficiency for crisp and interval data. The range of efficiency for crisp data is smaller than that of interval data.

7 Conclusion and remarks

This paper proposed the interval efficiency and the interval cross-efficiency. Interval efficiency helps us to find the peculiar DMUs and gives more information on how to improve DMUs. And if there are some DMUs whose upper limits of efficiencies are 1, we can evaluate them by comparing their lower limits.

We evaluate the interval efficiency by fixed crisp data and interval data. In future, this will be extended to fuzzy inputs and outputs [6]. The efficiency will be fuzzy.

References

[1] K.Tone, Mesurement and Improvement of Efficiency by DEA, Nikkagiren, 1993 (in Japanese).

[2] A.Chanes, W.W.Cooper and E.Rhodes, "Measuring the Efficiency of Decision Making Unit", EuropeanJournal of Operational Research, **3**, 429-444, 1978.

[3] T.R.Sexton, R.H.Silkman and A.J.Hogan, "Data Envelopment Analysis: Critique and Extension, in R.H.Silkman (ed.) Measuring Efficiency", An Assessment of Data Envelopment Analysis, 73-105, 1986.

[4] E.Takahagi, "An Evaluation Method Using DEA's Cross-Efficiency and Fuzzy Mesure Choquet Integral Model", 13th Fuzzy Systems Symposium, 251-252, 1997 (in Japanese).

[5] Y.Yamada, T.Matui, M.Sugiyama, "New Analysis of Efficiency Based on DEA", Journal of Operation Reserch Society of Japan, **37**(2), 158-167, 1994 (in Japanese).

[6] F.Nagano,T.Yamaguti,T.Fukukawa, "DEA with Fuzzy Output Data", Journal of Operation Reserch Society of Japan, **40**(8), 425-429, 1995 (in Japanese).