The Third AFSS(1998), 63－68

FUZZY REGRESSION ANALYSIS WITH NON-SYMMETRIC FUZZY COEFFICIENTS BASED ON QUADRATIC PROGRAMMING APPROACH

Haekwan Lee, Hideo Tanaka

Department of Industrial Engineering, Osaka Prefecture University
Gakuencho 1-1, Sakai, Osaka 599-8531, JAPAN
Tel: + 81-722-54-9353 Fax: +81-722-54-9915
E-mail:{leehk, tanaka}@ie.osakafu-u.ac.jp

Abstract

This paper proposes fuzzy regression analysis with non-symmetric fuzzy coefficients. By assuming non-symmetric triangular fuzzy coefficients and applying the quadratic programming formulation, the center of the obtained fuzzy regression model attains more central tendency compared to the one with symmetric triangular fuzzy coefficients. For a data set composed of crisp inputs-fuzzy outputs, two approximation models called an upper approximation model and a lower approximation model are considered as the regression models. Thus, we also propose an integrated quadratic programming problem by which the upper approximation model always includes the lower approximation model at any threshold level under the assumption of the same centers in the two approximation models. Sensitivities of weight coefficients in the proposed quadratic programming approaches are investigated through real data.

keywords: Fuzzy approximation models, fuzzy regression analysis, quadratic programming, central tendency.

1. Introduction

In fuzzy regression analysis originated by Tanaka et al. [5], to deal with a vague and uncertain phenomenon, a fuzzy structure of the given phenomenon is represented as a fuzzy linear function whose parameters are fuzzy numbers. Therefore, a fuzzy linear function is used as a regression model to describe fuzziness in the given phenomenon. Several regression analyses had been developed based on linear programming (LP) and quadratic programming (QP) [5-10]. Recent developments on fuzzy regression are shown in Inuiguchi et al. [1] and Ishibuchi and Nii [3].

Savic and Pedrycz [4] proposed a method where center coefficients a_i, $(i=0, \dots, n)$ are obtained by least squares and then spread coefficients c_i, $(i=0, \dots, n)$ are obtained by LP problem [6]. Due to symmetric coefficients, this method gives too wide spreads in spite of the reasonable central tendency.

In this paper, we propose fuzzy regression analysis with non-symmetric fuzzy coefficients by QP. The proposed QP approach can integrate the property of central tendency in least squares and the possibilistic property in fuzzy regression analysis. The characteristic of the proposed QP approach is that it allows us to obtain the center and spread coefficients simultaneously with one optimization problem while the method [4] is not. Also, the proposed QP approach gives some trade-off between minimum spreads and central tendency in the regression model.

For a data set composed of crisp inputs-fuzzy outputs, we can consider two approximation models, an upper approximation model and a lower approximation model which are similar to the possibility and necessity concepts. It is necessary that the upper approximation model should include the lower approximation model for any input vector. If the two approximation models are obtained by solving two separate optimization problems, there is a possibility that the upper approximation model does not include the lower approximation model for some input vector as discussed in [2]. Thus, we also propose an integrated QP problem where centers of two approximation models are assumed to be identical. The advantage of assuming a same center for two approximation models is that if the upper approximation model includes the lower approximation model at some h-level, then this inclusion relation is satisfied at any level between 0 and 1. Using the GDP data, the proposed methods are illustrated and also sensitivities of weight coefficients in the QP problems are investigated.

2. Formulation of fuzzy regression model with non-symmetric fuzzy coefficients by QP

Former regression analyses [5, 6] are based on the fuzzy linear system whose coefficients are assumed to be symmetric fuzzy numbers. Differently to our former approaches, in this paper, we propose a fuzzy linear regression model with non-symmetric fuzzy coefficients.

Let us assume a fuzzy regression model as

$$Y(x) = A_0 + A_1 x_1 + \cdots + A_n x_n = A x \tag{1}$$

where $x = (1, x_1, \dots, x_n)^t$ is an input vector,

$A = (A_0, \dots, A_n)$ is a fuzzy coefficient vector, and $Y(x)$ is the estimated fuzzy output. If the coefficients A_i, $(i = 0, \dots, n)$ are assumed to be non-symmetric triangular fuzzy numbers, A_i denoted as $A_i = (a_i, c_i, d_i)_T$ can be defined by

$$\begin{aligned} \mu_{A_i}(x) &= 1 - (a_i - x)/c_i, \quad \text{if } a_i - c_i \le x \le a_i, \\ &= 1 - (x - a_i)/d_i, \quad \text{if } a_i \le x \le a_i + d_i, \\ &= 0, \quad \text{otherwise}, \end{aligned} \tag{2}$$

where a_i is a center, c_i is a left-spread, and d_i is a right-spread.

Let the input-output data be given as $(x_j, y_j) = (1, x_{j1}, \dots, x_{jn}; y_j)$, $j = 1, \dots, m$. Since the regression coefficients $A_i = (a_i, c_i, d_i)_T$, $(i = 0, \dots, n)$ in (1) are assumed to be non-symmetric triangular fuzzy numbers, the estimated output $Y(x_j)$ also becomes a non-symmetric triangular fuzzy number which can be calculated by fuzzy arithmetic. Therefore, parametric representation of (1) becomes

$$Y(x_j) = \left(\theta_C(x_j), \theta_L(x_j), \theta_R(x_j)\right)_T \tag{3}$$

where

$$\begin{aligned} \theta_C(x_j) &= \sum_{i=0}^{n} a_i x_{ji}, \\ \theta_L(x_j) &= \sum_{x_{ji} \ge 0} c_i x_{ji} - \sum_{x_{ji} < 0} d_i x_{ji}, \\ \theta_R(x_j) &= \sum_{x_{ji} \ge 0} d_i x_{ji} - \sum_{x_{ji} < 0} c_i x_{ji}, \end{aligned} \tag{4}$$

represent a center, a left-spread, and a right-spread of the fuzzy output $Y(x_j)$, respectively. It should be noted that $\theta_L(x_j)$ and $\theta_R(x_j)$ in (4) are positive since c_i and d_i $(i = 0,\dots,n)$ are assumed to be positive. Then, the membership function of $Y(x)$ can be defined as

$$\begin{aligned} \mu_{Y(x)}(y) &= 1 - \frac{\theta_C(x) - y}{\theta_L(x)}, \quad \text{if } \theta_C(x) - \theta_L(x) \le y \le \theta_C(x), \\ &= 1 - \frac{y - \theta_C(x)}{\theta_R(x)}, \quad \text{if } \theta_C(x) \le y \le \theta_C(x) + \theta_R(x), \\ &= 0, \quad \text{otherwise}. \end{aligned} \tag{5}$$

The h-level set of $Y(x)$ can be expressed as an interval

$$[Y(x)]_h = \left\{ y \,\middle|\, \mu_{Y(x)}(y) \ge h \right\} = [y_h^-, y_h^+] \tag{6}$$

where

$$\begin{aligned} y_h^- &= \theta_C(x) - (1-h)\,\theta_L(x), \\ y_h^+ &= \theta_C(x) + (1-h)\,\theta_R(x), \end{aligned} \tag{7}$$

represent the bounds of $[Y(x)]_h$, respectively.

To determine the optimal fuzzy coefficients $A_i = (a_i, c_i, d_i)_T$, $(i = 0, \dots, n)$ of the fuzzy regression model (1), the sum of spreads of the estimated outputs can be considered as an objective function, that is, the sum of spreads of $[Y(x)]_h$ for all data is taken as an objective function:

$$(1-h)\sum_{j=1}^{m} \left(\theta_L(x_j) + \theta_R(x_j)\right) = (1-h)\sum_{j=1}^{m} \left(c'|x_j| + d'|x_j|\right) \tag{8}$$

where $c = (c_0, \dots, c_n)'$ and $d = (d_0, \dots, d_n)'$ are left and right spread coefficient vectors, and m is a data size.

Let us consider minimization of sum of squared distances between the estimated output centers and the observed outputs, denoted as

$$\sum_{j=1}^{m} (y_j - a'x_j)^2 \tag{9}$$

which corresponds to the least squares concept and $a = (a_0, \dots, a_n)'$ is a center vector. Thus we suggest a new objective function by combining (8) and (9):

$$J = k_1 \sum_{j=1}^{m} (y_j - a'x_j)^2 + k_2 (1-h) \sum_{j=1}^{m} (c'|x_j| + d'|x_j|) \tag{10}$$

where k_1 and k_2 are weight coefficients. Given a threshold h, the given output y_j should be included in the h-level set of the estimated fuzzy output $Y(x_j)$, that is, satisfies

$$\left\{ \begin{aligned} \theta_C(x_j) + (1-h)\,\theta_R(x_j) \ge y_j, \\ \theta_C(x_j) - (1-h)\,\theta_L(x_j) \le y_j \end{aligned} \right\}, \quad j = 1, \dots, m, \tag{11}$$

which is regarded as one of possibilistic properties of fuzzy regression.

Based on the above assumptions, fuzzy regression by QP is to determine the optimal fuzzy coefficients $A_i = (a_i, c_i, d_i)_T$, $(i = 0, \dots, n)$ that minimize the objective function J (10) subject to the constraint conditions (11). This can be expressed as the following QP problem:

$$\begin{aligned} \min_{a, c, d} \; J &= k_1 \sum_{j=1}^{m} (y_j - a'x_j)^2 \\ &+ k_2 (1-h) \sum_{j=1}^{m} \left(c'|x_j| + d'|x_j|\right) + \xi (c'c + d'd) \\ \text{s.t.} \quad & (11), \\ & c_i \ge 0,\ d_i \ge 0, \quad i = 0, \dots, n, \end{aligned} \tag{12}$$

where ξ is a small positive number such that $k_1, k_2 >> \xi$. The term $\xi(c'c + d'd)$ is added to (10) so that the objective function in (12) becomes a quadratic function with respect to decision variables a, c, and d. This is a well-known technique in obtaining the optimal solution by QP. By this approach, we can obtain a regression model with more central tendency comparing to the ones with symmetric triangular fuzzy coefficients by the LP problem [6].

3. Integrated QP approach for upper and lower approximation models

Let us consider a data set composed of crisp inputs-fuzzy outputs denoted as

$$(x_j; Y_j) = (1, x_{j1}, \dots, x_{jn}; Y_j), \quad j = 1, \dots, m \tag{13}$$

where the fuzzy output is defined by $Y_j = (y_j, e_j)_T$ with a center (y_j) and a spread (e_j), and m is a data size. For the data set (13), two approximation models called as a lower approximation model (LAM) $Y_*(x_j)$ and an upper approximation model (UAM) $Y^*(x_j)$ can be considered as

$$Y_*(x_j) = A_{*0} + A_{*1}x_{j1} + \cdots + A_{*n}x_{jn} = A_* x_j \tag{14}$$

$$Y^*(x_j) = A_0^* + A_1^* x_{j1} + \cdots + A_n^* x_{jn} = A^* x_j \tag{15}$$

where coefficients A_{*i} and A_i^* are non-symmetric triangular fuzzy numbers.

From the concept of lower and upper approximation models, the following inclusion relation between coefficients A_{*i} and A_i^* should be satisfied

$$A_i^* \supseteq A_{*i}, \quad i = 0, \dots, n, \tag{16}$$

which are defined as

$$[A_i^*]_h \supseteq [A_{*i}]_h, \quad i = 0, \dots, n, \text{ for any } h. \tag{17}$$

Therefore A_{*i} of LAM and A_i^* of UAM ($i = 0, \dots, n$) can be defined as

$$\begin{aligned} A_{*i} &= (b_i, f_i, g_i)_T, \\ A_i^* &= (b_i, f_i + p_i, g_i + q_i)_T, \quad i = 0, \dots, n, \end{aligned} \tag{18}$$

which are depicted in Fig. 1.

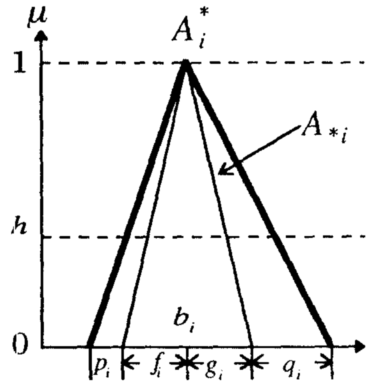

Fig. 1. Fuzzy coefficients A_i^* and A_{*i}

By simple fuzzy arithmetic, the inclusion relation between A_{*i} and A_i^* can be extended to the inclusion relation between LAM $Y_*(x_j)$ and UAM $Y^*(x_j)$, that is

$$Y^*(x) \supseteq Y_*(x) \text{ for any } x = (1, x_1, \dots, x_n)' \tag{19}$$

if $A_i^* \supseteq A_{*i}$, ($i = 0, \dots, n$).

Using the coefficients $A_{*i} = (b_i, f_i, g_i)_T$, ($i = 0, \dots, n$) in (18), LAM $Y_*(x_j)$ (14) can be expressed as

$$Y_*(x_j) = \left(b'x_j,\ \theta_{*L}(x_j),\ \theta_{*R}(x_j)\right)_T$$

$$\begin{aligned} \text{where } \theta_{*L}(x_j) &= \sum_{x_{ji} \ge 0} f_i x_{ji} - \sum_{x_{ji} < 0} g_i x_{ji} \\ \theta_{*R}(x_j) &= \sum_{x_{ji} \ge 0} g_i x_{ji} - \sum_{x_{ji} < 0} f_i x_{ji} \end{aligned} \tag{20}$$

and $b = (b_0, \dots, b_n)'$. The spread of $[Y_*(x_j)]_h$ can be denoted as

$$\begin{aligned} &(1-h)\left(\theta_{*L}(x_j) + \theta_{*R}(x_j)\right) \\ &= (1-h)(f'|x_j| + g'|x_j|) \end{aligned} \tag{21}$$

where $f = (f_0, \dots, f_n)'$ and $g = (g_0, \dots, g_n)'$. Using the coefficients $A_i^* = (b_i, f_i + p_i, g_i + q_i)_T$, ($i = 0, \dots, n$) in (18), UAM $Y^*(x_j)$ (15) can be expressed as

$$Y^*(x_j) = \left(b'x_j,\ \theta_{*L}(x_j),\ \theta_{*R}(x_j)\right)_T$$

where (22)

$$\theta_L^*(x_j) = \sum_{x_{ji} \ge 0} f_i x_{ji} + \sum_{x_{ji} \ge 0} p_i x_{ji} - \sum_{x_{ji} < 0} g_i x_{ji} - \sum_{x_{ji} < 0} q_i x_{ji},$$

$$\theta_R^*(x_j) = \sum_{x_{ji} \ge 0} g_i x_{ji} + \sum_{x_{ji} \ge 0} q_i x_{ji} - \sum_{x_{ji} < 0} f_i x_{ji} - \sum_{x_{ji} < 0} p_i x_{ji}.$$

The spread of $[Y^*(x_j)]_h$ can be denoted as

$$\begin{aligned} &(1-h)\left(\theta_L^*(x_j) + \theta_R^*(x_j)\right) \\ &= (1-h)\left(f'|x_j| + g'|x_j| + p'|x_j| + q'|x_j|\right) \end{aligned} \tag{23}$$

where $p = (p_0, \dots, p_n)'$ and $q = (q_0, \dots, q_n)'$.

The h-level set of the given output Y_j should be included in the h-level set of the estimated UAM $Y^*(x_j)$, i.e., satisfies $[Y^*(x_j)]_h \supseteq [Y_j]_h$. We should consider to minimize the sum of spreads of the estimated $[Y^*(x_j)]_h$ for all data. Thus, the following objective function should be minimized:

$$\begin{aligned} J_U &= t_1 \sum_{j=1}^{m} (y_j - b'x_j)^2 \\ &+ t_2 (1-h) \sum_{j=1}^{m} \left(f'|x_j| + g'|x_j| + p'|x_j| + q'|x_j|\right) \end{aligned} \tag{24}$$

where the term $\sum_{j=1}^{m} (y_j - b'x_j)^2$ is inserted to obtain the property of central tendency in least squares. Thus, the optimization problem for obtaining UAM $Y^*(x_j)$ can be described as follows:

$$[\text{UAM}]: \min_{b, f, g, p, q} \ (24)$$

$$\text{s. t. } [Y^*(x_j)]_h \supseteq [Y_j]_h \tag{25}$$

$$f_i \ge 0,\ g_i \ge 0,\ p_i \ge 0,\ q_i \ge 0, \quad i = 0, \dots, n.$$

On the contrary to UAM, the h-level set of the estimated LAM $Y_*(x_j)$ should be included in the h-level set of the given output Y_j, i.e., satisfies $[Y_*(x_j)]_h \subseteq [Y_j]_h$. We should consider to maximize the sum of spreads of the estimated $[Y_*(x_j)]_h$ for all data. Thus, the following objective function should be maximized:

$$J_L = t_2 (1-h) \sum_{j=1}^{m} (f'|x_j| + g'|x_j|) - t_1 \sum_{j=1}^{m} (y_j - b'x_j)^2. \tag{26}$$

Thus, the optimization problem for obtaining LAM $Y_*(x_j)$ can be described as follows:

$$[\text{LAM}]: \max_{b, f, g} \ (26)$$

$$\text{s. t. } [Y_*(x_j)]_h \subseteq [Y_j]_h \tag{27}$$

$$f_i \ge 0,\ g_i \ge 0, \quad i = 0, \dots, n.$$

If the coefficients A_i^* of UAM and A_{*i} of LAM ($i = 0, \dots, n$) are obtained separately by solving two optimization problems (25) and (27), there is a possibility that the inclusion relation

$$[Y_*(x_j)]_h \subseteq [Y^*(x_j)]_h \tag{28}$$

does not be satisfied for some new input vector as discussed in [2]. Thus, in order to integrate these two (min, max) optimization problems into a single problem, letting weight coefficients $k_1 = 2t_1$ and $k_2 = t_2$, a new objective function combining (24) and (26) can be introduced as

$$\min \left(J_U - J_L\right) \tag{29}$$
$$= \min\ k_1 \sum_{j=1}^{m} (y_j - \boldsymbol{b}'\boldsymbol{x}_j)^2 + k_2 (1-h) \sum_{j=1}^{m} \left(\boldsymbol{p}'|\boldsymbol{x}_j| + \boldsymbol{q}'|\boldsymbol{x}_j|\right).$$

Using (29), to determine the optimal fuzzy coefficients A_{*i} of LAM and A_i^* of UAM ($i = 0, \ldots, n$) simultaneously, we propose the following Integrated QP (IQP) problem by combining two optimization problems (25) and (27):

$$[\text{IQP}]: \min_{\boldsymbol{b}, \boldsymbol{f}, \boldsymbol{g}, \boldsymbol{p}, \boldsymbol{q}} \ J = k_1 \sum_{j=1}^{m} (y_j - \boldsymbol{b}'\boldsymbol{x}_j)^2$$
$$+ k_2 (1-h) \sum_{j=1}^{m} \left(\boldsymbol{p}'|\boldsymbol{x}_j| + \boldsymbol{q}'|\boldsymbol{x}_j|\right) + \xi (\boldsymbol{f}'\boldsymbol{f} + \boldsymbol{g}'\boldsymbol{g} + \boldsymbol{p}'\boldsymbol{p} + \boldsymbol{q}'\boldsymbol{q})$$
$$\text{s.t. } \boldsymbol{b}'\boldsymbol{x}_j + (1-h)\,\theta_R^*(\boldsymbol{x}_j) \geq y_j + (1-h)\,e_j,$$
$$\boldsymbol{b}'\boldsymbol{x}_j - (1-h)\,\theta_L^*(\boldsymbol{x}_j) \leq y_j - (1-h)\,e_j,$$
$$\boldsymbol{b}'\boldsymbol{x}_j + (1-h)\,\theta_{*R}(\boldsymbol{x}_j) \leq y_j + (1-h)\,e_j,$$
$$\boldsymbol{b}'\boldsymbol{x}_j - (1-h)\,\theta_{*L}(\boldsymbol{x}_j) \geq y_j - (1-h)\,e_j, \quad j = 1, \ldots, m,$$
$$f_i \geq 0,\ g_i \geq 0,\ p_i \geq 0,\ q_i \geq 0, \qquad i = 0, \ldots, n, \tag{30}$$

where ξ is a small positive number such that $k_1, k_2 >> \xi$. The term $\xi(\boldsymbol{f}'\boldsymbol{f} + \boldsymbol{g}'\boldsymbol{g} + \boldsymbol{p}'\boldsymbol{p} + \boldsymbol{q}'\boldsymbol{q})$ is inserted to (29) so that the objective function in (30) becomes a quadratic function with respect to decision variables $\boldsymbol{b}, \boldsymbol{f}, \boldsymbol{g}, \boldsymbol{p}$, and $\boldsymbol{q}$. The obtained UAM and LAM by the above IQP always satisfy the inclusion relation $Y_*(\boldsymbol{x}) \subseteq Y^*(\boldsymbol{x})$ at h^*-level ($0 \leq h^* \leq 1$) for any new input vector $\boldsymbol{x} = (1, x_1, \ldots, x_n)'$.

4. Discussion on sensitivities of weight coefficients in QP (12) & IQP (30)

In this section, let us investigate sensitivities of weight coefficients in the proposed QP approaches in Sections 2 and 3. The gross domestic product (GDP) data are given in Table 1 where inputs are income (x_1), working population (x_2), and output (y) is GDP of Japan during 1975 - 1992. All inputs-outputs are ratios formed by assigning the year 1970 a value of 100. Furthermore, to apply IQP to the fuzzy data, we formed fuzzy outputs $Y_j = (y_j, e_j)_T$ in the last column of Table 4 by assigning 5 % of each output y_j as the corresponding spread e_j. It should be noted that $Y_j = (y_j, e_j)_T$ is a symmetric triangular fuzzy number.

Crisp inputs - outputs:

First, let us analyze crisp inputs-outputs data with the following fuzzy linear system:

$$Y(x) = A_0 + A_1 x_1 + A_2 x_2 \tag{31}$$

where the coefficients $A_i = (a_i, c_i, d_i)_T$ ($i = 0, 1, 2$) are non-symmetric triangular fuzzy numbers. By solving QP (12) with three combinations of weight coefficients k_1 and k_2 ($h = 0$), optimal coefficients are obtained as shown in Table 2. It can be noticed in Table 2 that the obtained center vector $\boldsymbol{a}$ is not so sensitive to weight coefficients k_1 and k_2 in QP (12), while the spread

Table 1. GDP of Japan related to income and working population (1970: 100)

No (j)	Year	Income (x_1)	Working population (x_2)	GDP (y_j)	GDP $Y_j = (y_j, e_j)_T$
1	1975	137.0	102.5	124.5	(124.5, 6.23)
2	1976	138.1	103.5	129.4	(129.4, 6.47)
3	1977	141.5	104.8	135.1	(135.1, 6.76)
4	1978	144.8	106.1	142.3	(142.3, 7.12)
5	1979	148.1	107.5	150.1	(150.1, 7.51)
6	1980	146.0	108.7	154.3	(154.3, 7.72)
7	1981	148.8	109.5	159.2	(159.2, 7.96)
8	1982	151.0	110.7	164.0	(164.0, 8.20)
9	1983	151.4	112.5	167.9	(167.9, 8.40)
10	1984	153.7	113.2	174.4	(174.4, 8.72)
11	1985	155.6	114.0	182.1	(182.1, 9.11)
12	1986	158.8	114.9	187.4	(187.4, 9.37)
13	1987	161.9	116.0	195.2	(195.2, 9.76)
14	1988	166.1	118.0	207.3	(207.3, 10.37)
15	1989	169.1	120.3	217.3	(217.3, 10.87)
16	1990	173.0	122.6	228.3	(228.3, 11.42)
17	1991	176.0	125.0	237.0	(237.0, 11.85)
18	1992	174.8	126.3	239.4	(239.4, 11.97)

coefficients $\boldsymbol{c}$ and $\boldsymbol{d}$ are slightly related to weight coefficients.

As the proposed QP method (12) combines the properties of least squares and fuzzy regression where $M_1 = \sum_j (y_j - \boldsymbol{a}'\boldsymbol{x}_j)^2$ represents a measure of central tendency in least squares and $M_2 = \sum_j \left(\boldsymbol{c}'|\boldsymbol{x}_j| + \boldsymbol{d}'|\boldsymbol{x}_j|\right)$ represents a measure of the possibilistic property in fuzzy regression analysis, it is meaningful to check values of M_1 and M_2 as weight coefficients k_1 and k_2 in QP (12) change. Thus, using the optimal coefficients in Table 2, M_1 and M_2 are shown in Table 3. Comparing the case (a) against the case (b) in Table 3, M_1 is reduced by 0.1 % while M_2 is increased by 0.2 %. On the other hand, comparison of the case (c) against the case (b) in Table 3 shows that M_1 is increased by 0.4 % while M_2 is reduced by 0.3 %. Thus, it can be said that fluctuations of M_1 and M_2 corresponding to change of weight coefficients are very small. Furthermore, it can be noticed that values of $M_1 + M_2$ for the cases (a), (b), and (c) in Table 3 are almost same.

To summarize results in Tables 2 and 3, weight coefficients k_1 and k_2 in QP (12) are not so sensitive in determining an optimal model. Insensitivity of weight coefficients are caused by constraint conditions in QP and the assumption of non-symmetric fuzzy coefficients. Considering that value of M_1 in conventional least squares is 34.5, the optimal models of the cases (a), (b), and (c) in Table 2 represent good central tendency.

For the case (b) in Table 2, the optimal model can be denoted as

Table 2. Optimal coefficients by QP (12) for GDP data with crisp inputs-outputs

Case	Weights k_1	Weights k_2	Optimal coefficient vectors a^t	c^t	d^t
(a)	10	1	(-341.571, 1.418, 2.641)	(0.383, 0, 0.018)	(0.393, 0, 0.013)
(b)	1	1	(-341.507, 1.417, 2.643)	(0.455, 0, 0.018)	(0.541, 0, 0.012)
(c)	1	10	(-340.588, 1.430, 2.617)	(2.305, 0, 0)	(1.967, 0, 0)

Table 3. Comparison results using the optimal coefficients in Table 2

Case	k_1	k_2	$M_1=\sum_j (y_j-a^t x_j)^2$		$M_2=\sum_j \left(c^t\lvert x_j\rvert+d^t\lvert x_j\rvert\right)$		M_1+M_2
(a)	10	1	34.704	(99.9 %)	77.293	(100.2 %)	111.997
(b)	1	1	34.724	(100.0 %)	77.175	(100.0 %)	111.899
(c)	1	10	34.872	(100.4 %)	76.909	(99.7 %)	111.781

$$Y(x)=(-341.507, 0.455, 0.541)_T + (1.417, 0, 0)_T x_1 + (2.643, 0.018, 0.012)_T x_2 \quad (32)$$

which is depicted in Fig. 2.

Crisp inputs-fuzzy outputs:

Let us analyze crisp inputs-fuzzy outputs data where the given outputs are fuzzy numbers $Y_j=(y_j, e_j)_T$ as shown in the last column of Table 1. The two approximation models are considered as

$$\text{LAM}: Y_*(x)=A_{*0}+A_{*1}x_1+A_{*2}x_2, \qquad \text{UAM}: Y^*(x)=A_0^*+A_1^*x_1+A_2^*x_2, \quad (33)$$

where coefficients A_{*i} and A_i^* (i = 0, 1, 2) are defined as (18). Applying IQP (30) with three combinations of weight coefficients k_1 and k_2 (h = 0), optimal coefficient vectors are obtained as shown in Table 4. In Table 4, comparison of the case (a) against the case (b) shows that weight coefficients k_1 and k_2 are not so sensitive to determine the optimal coefficient vectors, while the optimal coefficients in the case (c) are slightly changed comparing to the case (b).

Using the optimal coefficients in Table 4, values of $M_3=\sum_j (y_j-b^t x_j)^2$ and $M_4=\sum_j \left(p^t\lvert x_j\rvert+q^t\lvert x_j\rvert\right)$ are shown in Table 5. In Table 5, we can notice that M_3 decreases as the ratio k_1/k_2 increases while M_4 increases as the ratio k_1/k_2 increases. It can be noticed that values of M_3+M_4 for the cases (a), (b), and (c) in Table 5 do not indicate so much differences.

Thus, from results in Tables 4 and 5, it can be said that weight coefficients k_1 and k_2 in IQP (30) are not so critical in determining an optimal model since constraint conditions in IQP (30) and assumption of non-symmetric fuzzy coefficients allow good central tendency and minimum spreads in the obtained regression model.

For the case (a) in Table 4, estimated outputs of LAM $Y_*(x_j)$ and UAM $Y^*(x_j)$ are depicted in Fig. 3 where it can be noticed that UAM $Y^*(x_j)$ includes LAM $Y_*(x_j)$ for any j.

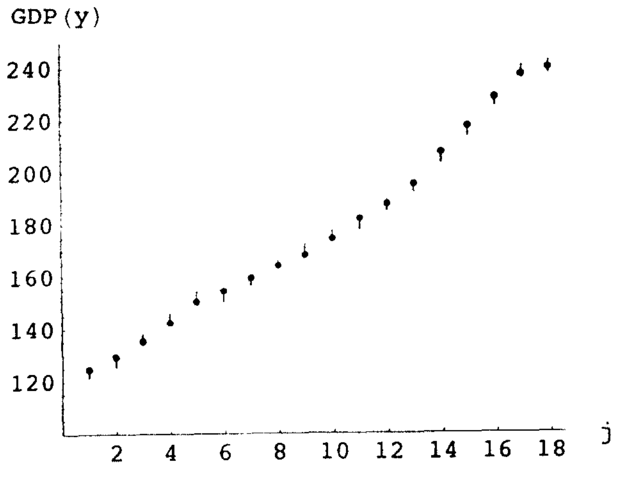

Fig. 2. Estimated outputs by QP (12) ($k_1=k_2=1$) for GDP data when the given outputs (y_j) are crisp

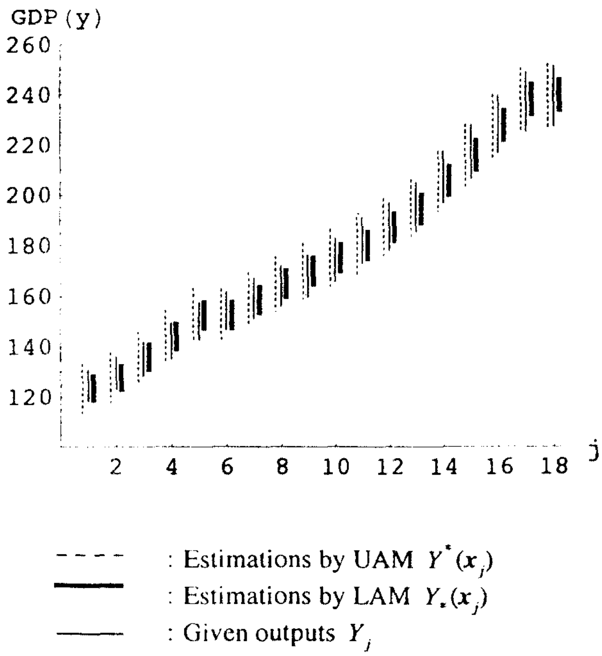

Fig. 3. Estimated UAM and LAM by IQP (k_1 = 10, k_2 = 1) and the given outputs $Y_j=(y_j, e_j)_T$

Table 4. Optimal coefficients by IQP for GDP data with crisp inputs-fuzzy outputs

Case	Weights		Optimal coefficient vectors				
	k_1	k_2	b'	f'	g'	p'	q'
(a)	10	1	(-339.392, 1.465, 2.558)	(0, 0.0342, 0)	(0, 0.0340, 0)	(0, 0.0401, 0)	(0, 0.0375, 0)
(b)	1	1	(-340.009, 1.376, 2.686)	(0, 0.0362, 0)	(0, 0.0344, 0)	(0, 0.0363, 0)	(0, 0.0388, 0)
(c)	1	10	(-331.216, 1.463, 2.489)	(0, 0.0338, 0)	(0, 0, 0.0502)	(0, 0.0433, 0)	(0, 0.0310, 0)

Table 5. Comparison results using the optimal coefficients in Table 4

Case	k_1	k_2	$M_3=\sum_j (y_j-b'x_j)^2$		$M_4=\sum_j \left(p'\lvert x_j\rvert+q'\lvert x_j\rvert\right)$		M_3+M_4
(a)	10	1	34.578	(96.0 %)	216.779	(103.2 %)	251.357
(b)	1	1	36.016	(100.0 %)	209.957	(100.0 %)	245.973
(c)	1	10	40.512	(112.5 %)	207.804	(99.0 %)	248.316

5. Conclusions

In this paper, fuzzy approximation models with non-symmetric fuzzy coefficients can be obtained by QP. By assuming non-symmetric triangular fuzzy coefficients and using the QP formulation, the obtained fuzzy regression models attain more central tendency compared to the ones with symmetric triangular fuzzy coefficients. For a data set with crisp inputs-fuzzy outputs, the upper and lower approximation models can be obtained to reflect fuzziness of outputs in the analyzed phenomenon. If the two approximation models are obtained by solving two separate optimization problems, it is possible that the upper approximation model does not include the lower approximation model for some input vector. Thus, an integrated QP approach obtaining two approximation models is proposed.

Application results by GDP data showed that weight coefficients in the proposed QP approaches are not so critical in determining fuzzy approximation models while obtained models attain good central tendency and minimum spreads. Insensitivity of weight coefficients in the proposed QP approaches are due to constraint conditions in QP problems and the assumption of non-symmetric fuzzy coefficients. Also, it is shown that the proposed integrated QP approach ensures that the upper approximation model always includes the lower approximation model at any h-level for any new inputs.

References

1. M. Inuiguchi, M. Sakawa, and S. Ushiro, Mean-absolute-deviation-based fuzzy linear regression analysis by level sets automatic deduction from data, *Proc. of Sixth IEEE International Conference on Fuzzy Systems*, Barcelona, Spain, (1997), 829-834.
2. H. Ishibuchi and H. Tanaka, A unified approach to possibility and necessity regression analysis with interval regression models, *Proc. of Fifth IFSA World Congress*, Seoul, Korea, (1993), 501-504.
3. H. Ishibuchi and M. Nii, Fuzzy regression analysis with non-symmetric fuzzy number coefficients and its neural network implementation, *Proc. of Fifth IEEE International Conference on Fuzzy Systems*, New Orleans, USA, (1996), 318-324.
4. D. A. Savic and W. Pedrycz, Evaluation of fuzzy linear regression models, *Fuzzy Sets and Systems*, 39 (1991), 51-63.
5. H. Tanaka, S. Uejima, and K. Asai, Linear regression analysis with fuzzy model, *IEEE Trans. SMC*, 12 (1982), 903-907.
6. H. Tanaka, Fuzzy data analysis by possibilistic linear models, *Fuzzy Sets and Systems*, 24 (1987), 363-375.
7. H. Tanaka, H. Lee, and T. Mizukami, Identification of possibilistic coefficients in fuzzy linear systems, *Proc. of Fifth IEEE International Conference on Fuzzy Systems*, New Orleans, USA, (1996), 842-847.
8. H. Tanaka and H. Lee, Fuzzy linear regression combining central tendency and possibilistic properties, *Proc. of Sixth IEEE International Conference on Fuzzy Systems*, Barcelona, Spain, (1997), 63-68.
9. H. Tanaka and H. Lee, Exponential possibility regression analysis by identification method of possibilistic coefficients, *Fuzzy Sets and Systems*, (to appear).
10. H. Lee and H. Tanaka, Fuzzy regression analysis by quadratic programming reflecting central tendency, *Behaviormetrika*, 25-1 (1998), (to appear).