Analysis and Auto-tuning of Scale Factors of Fuzzy Logic Controller

Chul Heui LEE , Seon Hak SEO

Dept. of Electrical Engineering, Kangwon National University
Chuncheon, Kangwon-do, 200-701, Korea
Tel:+82-361-250-6296 Fax:+82-361-241-3775 E-mail:chlee@cc.kangwon.ac.kr

Abstract

In this paper, we analyze the effects of scaling factors on the performance of a fuzzy logic controller(FLC). The quantitative relation between input and output variables of FLC is obtained by using a quasi-linear fuzzy model, and an approximate transfer function of FLC is derived from the comparison of it with the conventional PID controller. Then we analyze in detail the effects of scaling factor using this approximate transfer function and root locus method. Also we suggest an on-line tuning method for scaling factors which employs an sample performance function and a variable reference for tuning index.

Keywords: Scaling Factor, Quasi-linear Fuzzy model, PID Control, On Line Tuning

1.Introduction

In recent years fuzzy logic control has emerged as one of the most active and fruitful areas in the application of fuzzy set theory. Fuzzy logic control appears very useful when the mathematical treatment of system is difficult because of its nonlinearities and complexity[1].

Since fuzzy logic controller(FLC) determines the control action nonlinearly due to the fuzzy rule base and the inference mechanism, it can cover broader range of operational conditions than conventional PID controller, and give better and robust performance[2]. However, it doesn't mean that there exist tractable systematic methods for designing FLC.

In general, The parameters of FLC such as linguistic rules and membership functions are usually tuned through trial and error, and tuning of FLC is more difficult than that of a conventional controller[3-7].

However, for easy implementation and satisfactory performance, systematic and excellent tuning mechanism should be provided. If possible, we had better know beforehand the effect of these parameters on the system characteristics to develop a systematic tuning method which is capable of improving system performance.

Thus, in this paper, we present a quantitative analysis of the effect of scaling factors on the performance of a FLC as well as qualitative one, and also propose a new auto-tuning method of them based on the evaluation of system response.

We derive by using Sugeno's quasi-linear fuzzy model(QLFM) a quantitative formula of input-output relation which is a function of scaling factors, and examine the effect of scaling factor by applying root locus method to the approximate transfer function of FLC. Through this analysis, we obtain the relation between scaling factors and system response.

The proposed on-line tuning method uses error ratio as a tuning index and uses a variable reference for tuning index to handle both coarse control mode and fine control mode. The sample performance function is introduced to measure the system performance at each data step.

2. Analysis of the Effect of Scaling Factors

2.1 Fuzzy Logic Controller and Scaling Factors

FLC receives crisp values as input and produces crisp output as control input applied to controlled system, while it internally handles fuzzy values described by membership functions. Also FLC has a mechanism which can convert a linguistic control strategy into an automatic control strategy. Therefore FLC usually consists of scaling factors, fuzzifier, rule base, inference engine and defuzzifier, as shown in Fig.1

The operation of FLC requires the fuzzification that attaches each crisp input to fuzzy subsets with grades of membership depending on a priori chosen membership functions. The membership functions of the input and output variables are usually defined within normalized. universe of discourse for simplicity and generality. Thus, to match physical signal world with fuzzy world inside FLC, an appropriate choice of scaling factors is indispensable.

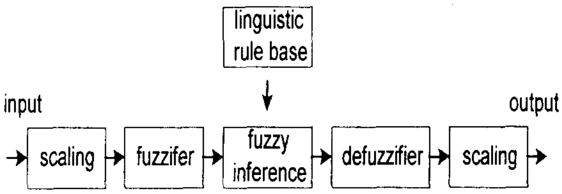


Fig. 1 Typical Structure of Fuzzy Controller

The scaling factors play a role similar to that of the gain coefficients in a conventional controller. In other words, they are the source of possible instabilities, oscillation problems, and deteriorated damping effects.

2.2 Qualitative Analysis of Scaling Factors

If scaling factor of a fuzzy variable is changed, the definition of each membership function will be changed by same ratio. For example, if the input value is scaled by 1/6 as shown in Fig.2, an input value of 2.5 is classified as PS and PM. On the other hand, with scaling factor of 1/3 a value of 2.5 is now classified as PM and PB. It implies the meaning of either the antecedent or the consequents in the rules is changed.

Hence a change of scaling factor may affect all of the linguistic rules in rule base. As a result, the behavior characteristics of FLC becomes quite different.

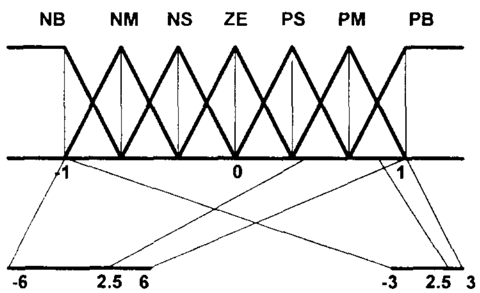


Fig. 2. Change of Meaning of fuzzy variables by scaling factor

Now we provide another interpretation of the effect of scaling factors. Assume that input variable e and Δe is scaled by k_e and k_d individually. If scaling factor are changed to k_e' and k_d', the slope of switching line in phase plane is changed as shown in Fig.3. Thus ill scaling results in either shifting the operating area to the boundaries or inclining the operational range toward a certain specific area.

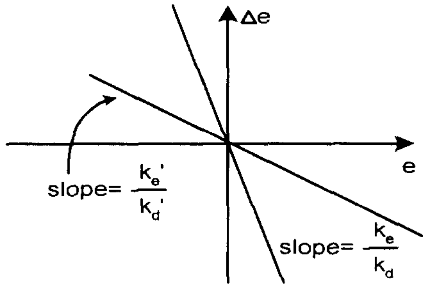


Fig.3 Shift of Switching Line by Scaling Factor

Remark : Nonlinear scaling factors yield the basically identical effect even though their mapping relations are more complicated.

In the same manner, the adjustment of the output sealing factor affects the closed loop gain of overall system.

As seen from above qualitative analysis, scaling factors have significant influences on the characteristics and the performance of FLC.

2.3 Quantitative Analysis of Scaling Factors

In general, FLC has the linguistic rules of the following type;

$$\text{IF } e \text{ is } A_i \text{ and } \Delta e \text{ is } B_i \text{ and } \Delta^2 e \text{ is } C_i \quad \text{THEN } \Delta u \text{ is } D_i \tag{1}$$

But the linguistic control rule of the type in eq.(1) can't furnish the clue for the quantitative relation between input and output variable of FLC. Therefore, in order to obtain the quantitative scope of the effect of scaling factors, we introduce Sugeno's quasi-linear fuzzy model (QLFM) where the consequents employ the functions of input variable instead of fuzzy subsets;

$$R_i : \text{IF } e^* \text{ is } A_i \text{ and } \Delta e^* \text{ is } B_i \text{ and } \Delta^2 e^* \text{ is } C_i \quad \text{THEN } \Delta u_i^* = \alpha_i e^* + \beta_i \Delta e^* + \gamma_i \Delta^2 e^* \tag{2}$$

We can show with ease that the control surface of QLFM-FLC is almost equal to the conventional FLC. In Fig.4 the control surface of the typical PI-like FLC with 7x7 rules[2] is compared with that of QLFM where back propagation is used in learning the parameters in consequents. It offers the validity of substitution of (1) by (2). Thus we can proceed the analysis using QLFM without loss of generality.

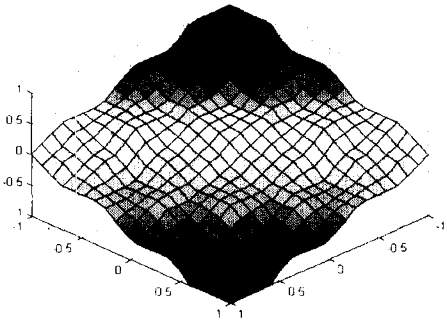


Fig. 4a Control Surface by FLC

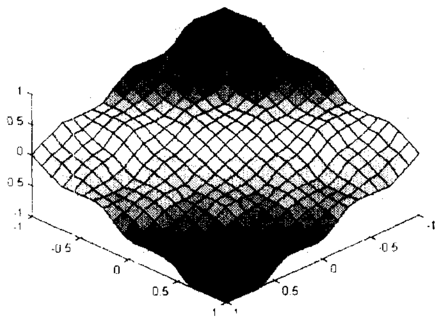


Fig. 4b Control surface by QLFM

Let's denote scaling factors of input variable e, Δe, and Δ^2e by $k_e(=1/s_e)$, $k_d(=1/s_d)$, and $k_s(=1/s_s)$, respectively. Also let's denote scaling factor of output variable u by $k_u(=s_u)$. Then the relation between fuzzy variable and corresponding variable is given by

$$e^* = k_e e \qquad -S_e \le e \le S_e \qquad (2.a)$$
$$\Delta e^* = k_d \Delta e \qquad -S_d \le \Delta e \le S_d \qquad (2.b)$$
$$\Delta^2 e^* = k_s \Delta^2 e \qquad -S_s \le \Delta^2 e \le S_s \qquad (2.c)$$
$$\Delta u^* = \Delta u / k_u \qquad -S_u \le \Delta u \le S_u \qquad (2.d)$$

Then the output of FLC before denormalization is given by

$$\begin{aligned}\Delta u^*(n) &= \frac{\sum_{i=1}^{m} \tau_i \Delta u_i^*(n)}{\sum_{i=1}^{m} \tau_i} \\ &= \sum_{i=1}^{m} v_i(\alpha_i e^*(n) + \beta_i \Delta e^*(n) + \gamma_i \Delta^2 e^*(n)) \qquad (4) \\ &= \overline{K_I} e^*(n) + \overline{K_P} \Delta e^*(n) + \overline{K_D} \Delta^2 e^*(n)\end{aligned}$$

where τ_i is firing level of i-th rule, and v_i is weighting factor due to fuzzy reasoning.

$$v_i = \frac{\tau_i}{\sum_{i=1}^{m} \tau_i} \qquad (5)$$

And $\overline{K_P}$, $\overline{K_I}$ and $\overline{K_D}$ are defined by

$$\overline{K_P} = \sum_{i=1}^{m} v_i \alpha_i \qquad (6.a)$$

$$\overline{K_I} = \sum_{i=1}^{m} v_i \beta_i \qquad (6.b)$$

$$\overline{K_D} = \sum_{i=1}^{m} v_i \gamma_i \qquad (6.c)$$

Eq.(4) is very similar to the control law of a velocity type conventional PID controller, and $\overline{K_P}$, $\overline{K_I}$ and $\overline{K_D}$ act like PID gains.

From eq.(2) and eq.(4), we can make explicit formula describing the input-output relation in terms of scaling factors.

$$\begin{aligned}\Delta u(n) &= k_u \Delta u^*(n) \\ &= \overline{K_I} k_u k_e e(n) + \overline{K_P} k_u k_d \Delta e(n) \qquad (7) \\ &\quad + \overline{K_D} k_u k_s \Delta^2 e(n)\end{aligned}$$

The control law and transfer function of a position type PID controller are as follows.

$$u(t) = K_P e(t) + K_D \frac{de(t)}{dt} + K_I \int e(t)dt \qquad (8)$$

$$\begin{aligned}G_{PID}(s) &= K_P + K_D s + \frac{K_I}{s} \\ &= \frac{K_D s^2 + K_P s + K_I}{s}\end{aligned} \qquad (9)$$

We can convert continuous PID controller (8) to discrete one with the assumption that sampling time T is sufficiently small;

$$\Delta u(n) = K_I \cdot T \cdot e(n) + K_p \Delta e(n) + (K_D/T) \Delta^2 e(n) \qquad (10)$$

From the comparison of eq.(7) with eg.(10), we know that the conventional PID controller is equivalent to PID-like FLC, if the following equations hold.

$$K_P = \overline{K_P} k_u k_d \qquad (11.a)$$
$$K_I = \overline{K_I} k_u k_e / T \qquad (11.b)$$
$$K_D = \overline{K_D} k_u k_s T \qquad (11.c)$$

Remark : We must note that, in practice, the values of gain parameters $\overline{K_P}$, $\overline{K_I}$, and $\overline{K_D}$ of PID-like FLC vary at each time step due to fuzzy reasoning, while the conventional PID controller has fixed gains over all time steps. This point just explains the nonlinear control action of FLC.

However, by substituting eq.(11) into eq.(9), we can get the approximate transfer function of FLC.

$$G_{FC}(s) = \frac{(\overline{K_D} k_u k_s T)s^2 + (\overline{K_P} k_u k_d)s}{s} + \frac{(\overline{K_I} k_u k_e / T)}{s} = \frac{N_{FC}(s)}{s} \qquad (12)$$

Let the transfer function of the controlled system be $G_p(s) = N_p(s)/D_p(s)$, Then the transfer function of overall system becomes

$$M(s) = \frac{G_p(s)\,G_{FC}(s)}{1 + G_p(s)\,G_{FC}(s)} = \frac{N_p(s)\,N_{FC(s)}}{sD_p(s) + N_P(s)\,N_{FC}(s)} \quad (13)$$

Since scaling factors are included in denominator of M(s), the change of their values causes immediately the shift of dominant poles. As you know, the transient response of system is mainly governed by dominant poles. Thus the effect of scaling factors on system performance can be analyzed in detail by using the relation between the dominant poles of closed-loop system and PID gains (Refer to [8]). But it's a considerably complicated and hard work.

We may carry out the analysis more conveniently if we use root locus method. PID-like FLC adds one pole located in origin and two zeros to the open loop transfer function of the system. In general, pole padding to the open loop transfer function pushes the root loci toward the right side of s-plane, while zero padding pushes it toward the opposite side[9]. As scaling factors change, the location of poles and zeros changes and then the root locus becomes different, and the response of overall system changes in consequence.

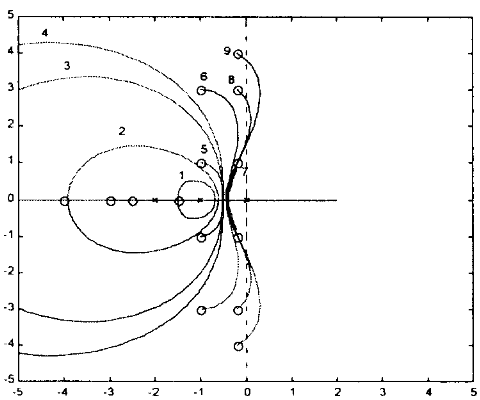
For instance, consider a plant given by

$$G_p(s) = \frac{1}{(s+1)(s+2)} \quad (14)$$

Two zeros of FLC added to open loop transfer function are

$$z_1,\ z_2 = \frac{-\overline{K}_P k_d \pm \sqrt{(\overline{K}_P k_d)^2 - 4\,\overline{K}_D \overline{K}_I k_e k_s}}{2\,\overline{K}_D T k_s} \quad (15)$$

Fig.5 shows root loci according to different values of zeros of FLC.



open loop poles : 0, -1, -2
open loop zeros : ①-1.5,-1.5 ②-1.5,-2.5 ③-3,-3 ④-3,-4 ⑤-1±j1 ⑥-1±j3 ⑦-0.2±j1 ⑧-0.2±j3 ⑨-0.2±j4

Fig. 5 Root Locus of closed-loop System

Thus the following facts are observed from eq.(12) and these loci :

i) as k_u increases, system gain increases
ii) as k_d increases, zeros of FLC become real and their magnitudes increase. Therefore root locus moves from ① toward ④.
iii) as k_e increases, zeros of FLC become complex. When they still remain real, they decrease in magnitude and root locus tends to move from ④ to ①. On the contrary, if they are changed to complex, their imaginary parts increase in magnitude, and root locus moves from ⑤ to ⑨.
iv) as k_s increases, zeros of FLC become complex. When they still remain real, they decrease in magnitude and root locus tends to move from ④ to ①. On the contrary, if they are changed to complex, both imaginary parts and real parts decrease in magnitude, root locus moves from ⑥ to ⑤ or ⑦.

In the same way, similar results can be obtained for PI-like FLC and PD-like FLC.[10]

Although the above observations are not always true with respect to all the possible cases, they represent the general tendency of the behavior of scaling factors related to dominant poles. Moreover, they are still valid by and large even if the plant transfer function is different. Therefore the effect of scaling factor on transient response of system can be extracted from these observations. They are summarized in Table 1.

Table 1. Relation of scaling factors and transient response of system

	k_e	k_d	k_s	k_u
decrease of overshoot/ oscillation	↓	↓	↓	
decrease of response speed(rise time)	↓	↑	↑	
decrease of steady-state error	↑	↓		
increase of system gain				↑

3. On-line Tuning of Scaling Factors

Now, by virtue of the above analysis, we can tune the scaling factors by a technique based on the concepts similar to that for PID gain turning. Maeda and others have been suggested some approaches of this category[4-6]. First, the control performance is evaluated by using overshoot, response speed, and error. Next scaling factors are tuned according to the grade of goodness. However, this procedure is not on-line but repeated learning at the end of control interval.

So we suggest an on-line tuning method which uses a tuning index and a sample performance function to determine the changes in scaling factors at each data step.

The tuning index ρ is defined as a ratio of two consecutive values of output error sequence in time[11].

$$\rho = \frac{e(n)}{e(n-1)} \tag{16}$$

It decides whether scaling factors should be updated or not. If $|\rho|$ is greater than a reference ρ_r, then the scaling factors are changed. So as to take both coarse control mode and fine control mode into considerations, we use a variable reference for tuning index defined as follows.

$$\rho_r = \alpha\rho_{r_1} + \beta\rho_{r_2} \tag{17}$$

$$\begin{aligned} &\alpha = 0,\ \beta \neq 0,\ |e| \le E_{th} \\ &\alpha \neq 0,\ \beta = 0,\ |e| > E_{th} \end{aligned} \tag{18}$$

where α, β, and E_{th} are constants.

We cannot use the values of overshoot, rise time, steady state error and amplitude for tuning at each data step because these values are calculated at the end of the control interval. Therefore the sample performance function is proposed in order to measure the system performance at each data step.

The sample performance function is defined by

$$SP(n) = \min\{SPI(n), SPO(n)\} \tag{19}$$

$$SPI(n) = \mu_{e \times \triangle e}(e(n), \triangle e(n)) \tag{20}$$

$$SPO(n) = \mu_{\triangle u}(\triangle u(n)) \tag{21}$$

Since the relation between output error e and change of error △e is as shown in Fig. 6, the fuzzy rule base for SPI is given by Table. 2, and the fuzzy rule base for SPO is presented in Table. 3. The membership functions for the antecedents of these rules are of equi-distant triangular type, and the consequents are fuzzy singletons with BD=0, MM=0.5, and GD=1.

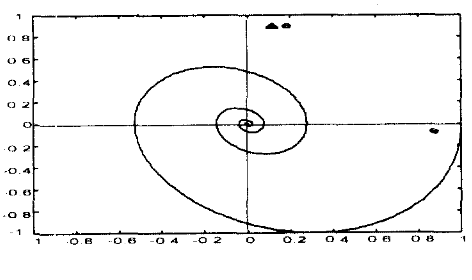


Fig. 6 System Chracteristic in phase plane

Table 2. Rule base for SPI

△e \ e	NB	NS	ZE	PS	PB
PB	GD	MM	MM	BD	BD
PS	MM	GD	GD	MM	BD
ZE	MM	GD	GD	GD	MM
NS	BD	MM	GD	GD	MM
NB	BD	BD	MM	MM	GD

NB:Negative Big, PB:Positive Big, ZE:ZEro
NS:Negative Small, PSpositive Small, GD:GooD
MM:MediuM, BD:BaD

Table 3. Rule Base for SPO

△u	NB	NS	ZE	PS	PB
	BD	MM	GD	MM	BD

Finally scaling factor SF is tuned by

$$SF(n) = SF(n-1) + w \cdot sgn(\triangle SF)(1-SP(n)) \tag{22}$$

where w is the step size of change, and sgn(△SF) is a sign function which indicates the increase or decrease in the quantity of scaling factors.

From eq.(21), if the sample performance is of good grade, then scaling factor variation is lower and vice versa. This on-line tuning procedure is going on until the system reaches to the steady-state, i.e., $|e| < \varepsilon$.

4. Conclusions

Scaling factors act as the mapping operators which map input/output values to the universe of discourse of the fuzzy variables. An appropriate choice of them is necessary and important since they have a great effect on system performance. Therefore, in this paper, we analyze quantitatively the effect of scaling factors via the comparison of QLFM FLC with the conventional PID controller. The result obtained from this analysis may be useful for development of tuning algorithm for them.

Also we suggest an on-line tuning method in which the variable reference for tuning index and the sample performance function play an important role. The proposed method works well, and improves the control performance of FLC.

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