슬라이딩 제어 입력을 이용한 강인 적응 퍼지 제어기

Robust Adaptive Fuzzy Controller Using a Sliding Control Input

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Abstracts In this paper, we propose a robust adaptive fuzzy control scheme using a sliding control input for tracking of a class of MISO nonlinear systems with unknown bounded external disturbances. In the proposed scheme, the nonlinearity is estimated adaptively via a fuzzy inference based on a fuzzy model. A sliding control input is introduced such that boundedness of all signals in the system is guaranteed even though the existence of a fuzzy approximation error and external disturbances. The controller parameters are updated by using a proposed adaptation law, which is similar e_1-modification method. Computer simulation shows the effectiveness of the proposed control scheme.

Keywords : Fuzzy Logic Control, Robust Adaptive Control, Variable Structure Control(VSC)

1 Introduction

Recently, the stability of the adaptive fuzzy control scheme has been studied by many researchers [1, 2]. Most such control schemes cannot avoid the existence of an approximation error between a fuzzy logic system and a nonlinearity of the controlled system. The approximation error may cause the system to be unstable. Therefore, a fuzzy controller should be designed to be robust in the presence of an approximation error and/or an external disturbance. Wang introduced a supervisory control input to resolve the problem [1]. The adaptive fuzzy controller with sliding control input has been proposed in [3]. Lee [4] proposed a control scheme which has a time varying sliding control input to overcome the problem. Kim [2] attacked the problem by modifying an estimation algorithm of the parameters.

In this paper, a robust adaptive fuzzy controller for a class of MISO nonlinear systems. The proposed controller structure is same as the controller proposed in [4], but the estimation algorithm is changed to make the controller to be more robust to the disturbances. The proposed estimation algorithm is originated from the e_1-modification [5]. All signals of overall system is guaranteed in the sense of Lyapunov even though the disturbances. The robustness of the proposed controller is compared to the adaptive fuzzy controller combined with dead zone technique [4] using computer simulations.

2 Robust Adaptive Fuzzy Control

Consider the nth-order nonlinear systems of the form

$$x^{(n)}(t) + f(x(t), \dot{x}(t), \cdots, x^{(n-1)}(t)) + d(t) = u(t) \tag{1}$$

where $f(\cdot)$ is an unknown but bounded continuous function, $u(t)$ is the control input and $d(t)$ is an unknown bounded external disturbance, i.e. $|d(t)| < \epsilon_d, \ \forall t > 0$.

The control objective is to force the plant state vector, $\underline{x} = [x, \dot{x}, \cdots, x^{(n-1)}]^T$ to follow a desired trajectory, $\underline{x}_d = [x_d, \dot{x}_d, \cdots, x_d^{(n-1)}]^T$. Let us define the tracking error vector $\underline{e}(t)$ and the error metric

$s(t)$ as follows:

$$\begin{aligned} \underline{e}(t) &= \underline{x}_d(t) - \underline{x}(t) \\ s(t) &= (\frac{d}{dt} + \lambda)^{n-1} e(t) \ , \quad \lambda > 0 \end{aligned} \tag{2}$$

which, given the canonical form of the plant dynamics (1), can be rewritten as $s(t) = \underline{\lambda}^T \underline{e}(t)$ with $\underline{\lambda}^T = [\lambda^{n-1}, (n-1)\lambda^{n-2}, \cdots, 1]$.

Now, we consider a fuzzy logic system as an estimator of a nonlinear systems. We use the following fuzzy logic system [1] :

$$\begin{aligned} f_{fz}(\underline{x}) &= \sum_{k=1}^{M} \theta_k \xi_k(\underline{x}) \\ &= \underline{\theta}^T \underline{\xi}(\underline{x}) \end{aligned} \tag{3}$$

where M is the number of the fuzzy rules, $\underline{\theta} = [\theta_1, \cdots, \theta_M]^T$, $\underline{\xi}(\underline{x}) = [\xi_i(\underline{x}), \cdots, \xi_M(\underline{x})]^T$, $\xi_k(\underline{x})$ is the fuzzy basis function defined by

$$\xi_k(\underline{x}) = \frac{\prod_{i=1}^{n} \mu_{F_i^k}(x_i))}{\sum_{k=1}^{M} (\prod_{i=1}^{n} \mu_{F_i^k}(x_i))} \tag{4}$$

θ_k are adjustable parameters, and $\mu_{F_i^k}$ are given membership functions, which can be Gaussian, triangular or any other type of membership functions.

Let us consider the approximation error between the fuzzy logic system and the nonlinear function. It is well known that the fuzzy logic system is capable of uniformly approximating any nonlinear function to any degree of accuracy. So we can assume that the approximation error is bounded but unknown as

$$\nu(t) = f(\underline{x}) - f_{fz}(\underline{x}) < \epsilon_A, \ \forall t > 0. \tag{5}$$

The proposed control input is designed as

$$u(t) = x_r(t) + k_d s(t) + \hat{f}_{fz}(\underline{x}|\hat{\underline{\theta}}) + \hat{k}(t) sat(s(t)/\delta) \tag{6}$$

where $x_r(t) = x_d^{(n)}(t) + \underline{\lambda}_v^T \underline{e}(t)$ with $\underline{\lambda}_v^T = [0, \lambda^{n-1}, (n-1)\lambda^{n-2}, \cdots, (n-1)\lambda]$, $x_d^{(n)}(t)$ is the nth derivative of the desired trajectory, k_d is a positive constant feedback gain, $\hat{f}_{fz}(\underline{x}|\hat{\underline{\theta}})$ is the estimate of the continuous nonlinear function which will be obtained adaptively by using the fuzzy logic system, and $\hat{k}(t)$ is a gain of the sliding control input which will be adjusted adaptively to achieve the robustness to the modeling errors and bounded external disturbances.

Using the above control law, the time derivative of the error metric $s(t)$ can be obtained as

$$\dot{s} = -k_d s - \underline{\phi}^T \underline{\xi}(\underline{x}) + \nu + d - \hat{k} sat(s/\delta) \tag{7}$$

where $\underline{\phi} = \hat{\underline{\theta}} - \underline{\theta}$ is defined as the estimation error of the nonlinear function.

The following estimation algorithm is used to estimate the parameters of the fuzzy logic system and the sliding control gain.

$$\dot{\hat{\underline{\theta}}} = \begin{cases} \eta_1 (s\underline{\xi}(\underline{x}) - |s|\hat{\underline{\theta}}) & |s| \le \delta, \\ \eta_1 s \underline{\xi}(\underline{x}) & |s| > \delta \end{cases} \tag{8}$$

$$\dot{\hat{k}} = \begin{cases} -\eta_3 |s| \hat{k} & |s| \le \delta, \\ \eta_2 |s| & |s| > \delta \end{cases} \tag{9}$$

where η_i , $i = 1, 2, 3$ are positive adaptive gains. Then, we can get the following theorem.

Theorem 1 *Consider the closed-loop adaptive control system consisting of the plant (1), the control law (6), (7) and the estimation algorithms (8). Then, all signals in the closed-loop system are bounded*

Proof: Let us define a Lyapunov function candidate as

$$V(t) = \frac{1}{2}[s^2(t) + + \frac{1}{\eta_1} \underline{\phi}^T \underline{\phi} + \frac{1}{\eta_2} \tilde{k}^2(t)) \tag{10}$$

where $\tilde{k}(t) = \hat{k}(t) - k$, k is defined as the summation of the upper bound of the modeling error $\nu(t)$ and the external disturbance $d(t)$ as:

$$k = \epsilon_A + \epsilon_d. \tag{11}$$

When $|s| > \delta$, the time derivative of the Lyapunov function candidate using equations (6) and (8) can be easily derived as

$$\begin{aligned} \dot{V} &= -k_d s^2 + \nu s + ds - \hat{k}|s| + \tilde{k}|s| \\ &\le -k_d s^2 + (\epsilon_A + \epsilon_d)|s| - \hat{k}|s| + \tilde{k}|s| \\ &= -k_d s^2 < 0 \ , \ \forall s \ne 0. \end{aligned} \tag{12}$$

Since, the error metric $s(t)$, and the parameter errors $\tilde{k}, \underline{\phi}$ are uniformly bounded, if $\underline{e}(0)$ is bounded, then $\underline{e}(t)$ is bounded for all $t > 0$. This prove that all signals in the overall system are bounded when $|s| > \delta$.

When $|s| \le \delta$, the time derivative is

$$\begin{aligned} \dot{V} &= -k_d s^2 + \nu s + ds - \hat{k} s^2 - |s| \underline{\phi}^T \hat{\underline{\theta}} - |s| \tilde{k} \hat{k} \\ &\le -|s|[k_d|s| + k + \tilde{k}(\tilde{k} + (k + s)) + \underline{\phi}^T \underline{\phi} - \underline{\phi}^T \underline{\theta}] \end{aligned} \tag{13}$$

Hence, $\dot{V} < 0$ outside a compact region D as

$$D = \{(s, \underline{\phi}, \tilde{k}) | k_d|s| + k + \tilde{k}(\tilde{k} + (k+s)) + \underline{\phi}^T\underline{\phi} - \underline{\phi}^T\underline{\theta} \leq 0\}. \tag{14}$$

By "*Lagrange stability*" Theorem [5], if follows that all the signals are bounded, when $|s| \leq 0$.

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Remark 1 Let us consider a robustness of the adaptive fuzzy control schemes in practical sense. As commented by Wang [1], we can make the adaptive system to have not only the boundedness of the all states but also the error converge to zero by using a sliding control input with sufficient big gain. However, the large control is undesirable because it may increase implementation cost and excite high frequency unmodeled dynamics. Moreover, the perfect switching is not possible in real world so that it may break the stability of the the adaptation system. The control scheme presented in [4] used a dead zone adaptation technique and a boundary layer method to prevent the problems caused from *chattering*. As shown in the next section, however, the control scheme will be diverge in large disturbance environment. Since the sliding control gain is not decreased, if error cannot be reduced sufficiently the gain will be very high. The high gain may cause the instability. Therefore, in this paper, the e_1 modification technique was introduced to adjust the control gain.

3 Computer Simulation

In this section, we show performance of the proposed control scheme by computer simulation applied to a chaotic system.

Let us consider the Duffing forced oscillation system:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -0.1x_2 - x_1^3 + 12cos(t) + d(t) + u(t). \end{aligned} \tag{15}$$

Without control, i.e., $u(t) = 0$, the system is chaotic. The external disturbance $d(t)$ is defined as

$$d(t) = \begin{cases} 0.5sin(6t) & 0 \geq t \leq 250 \\ sin(6t) & t > 250. \end{cases}$$

Let the initial state $\underline{x}(0) = [1.5\ 0]^T$, the desired trajectory $x_d(t) = sin(t)$.

We define three fuzzy sets over the interval [-2,2], and membership functions for x_1 and x_2 as $\mu_{F_1^i}(x_i) = exp(-(x_i+2)^2)$, $\mu_{F_2^i}(x_i) = exp(-(x_i/0.5)^2)$ and $\mu_{F_3^i}(x_i) = exp(-(x_i-2)^2)$. Hence, there are 9 rules in rule base for fuzzy inferences, that is, $M = 9$. The initial values of the parameters of fuzzy logic system are selected to zero, i.e. $\hat{\underline{\theta}}(0) = \underline{0}$. We choose $k_d = \lambda = 2$, $\eta_1 = 50$, $\eta_2 = 1$, $\eta_3 = 20$, $\delta = 0.005$. The initial values of the sliding control gain $\hat{k}(0)$ is set to be zero. The Euler's approximation method is used for integration with a sampling period of 0.01 sec.

As commented in Remarks 1, if the gain of the sliding control input will be too high, which cause the overall system to be unstable. Figure 1 shows the instability of the control system with dead zone estimation algorithm. The response of the proposed control scheme is shown in Figure 2. It shows that the sliding control gain $\hat{k}$ and the parameters of the fuzzy logic system $\hat{\underline{\theta}}$ are bounded. Figure 3 shows the trajectory of output error $e(t)$. From the figure, the controller with dead zone estimation algorithm shows fast and better tracking performance initially than that with the proposed controller, because of the high sliding control gain.

4 Conclusion

We have proposed a robust adaptive fuzzy control scheme using a modified estimation technique to guarantee the global stability of the overall system. The proposed estimation algorithm is based on e_1 modification technique. Simulation result shows that the proposed control scheme has more robustness respect to perturbation such as approximation error and external disturbances than a dead zone estimation technique.

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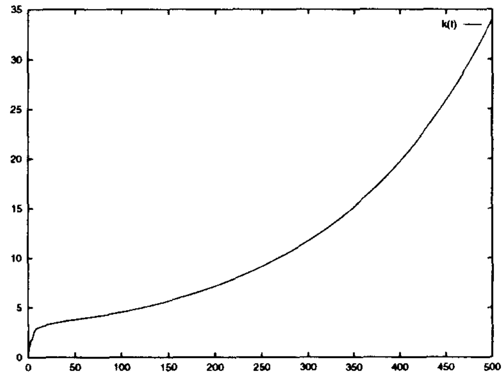
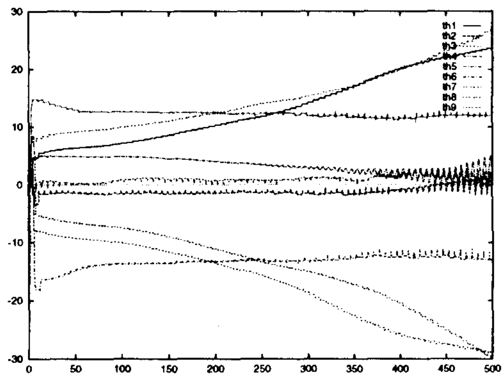
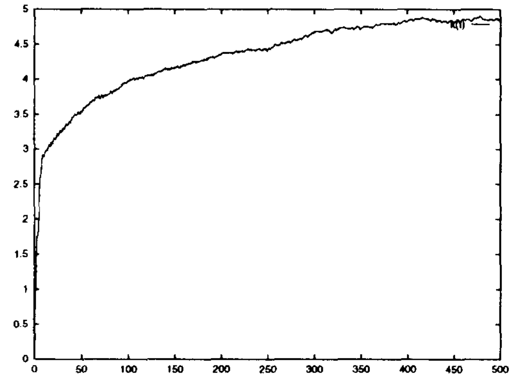


(a) The trajectory of the sliding control input gain, $\hat{k}(t)$

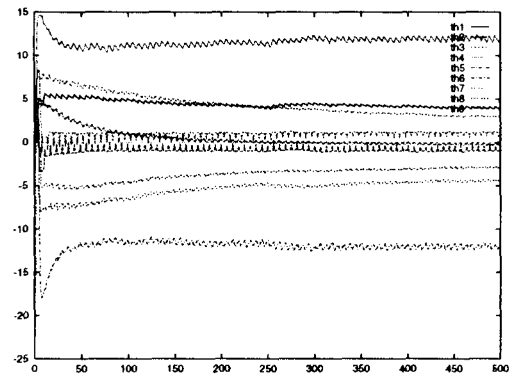


(b) The trajectory of the parameter error vector, $\underline{\hat{\theta}}(t)$

Figure 1: The trajectories of estimated parameters used by the deadzone estimation method



(a) The trajectory of the sliding control input gain, $\hat{k}(t)$



(b) The trajectory of the parameter error vector, $\underline{\hat{\theta}}(t)$

Figure 2: The trajectories of estimated parameters used by the proposed estimation method

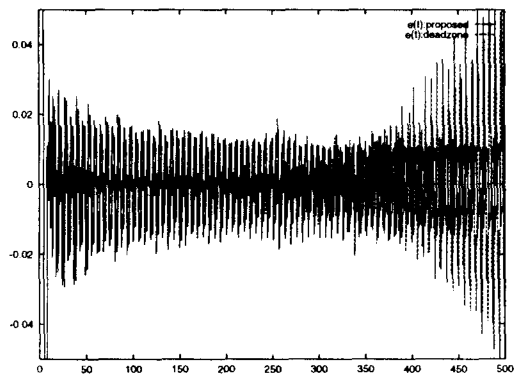


Figure 3: The comparison of the error trajectory