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Optimization of Reinforced Steel Fibrous Concrete Beam for the Objective Flexural Behavior

이 차 돈*　　　　안 지 현**
Lee, Cha Don　　　　Ahn, Jee Hyun

ABSTRACT

The use of steel fibers in conventional reinforced concrete increases the strength and ductility under various loading conditions. In order to examine the possibility of the use of these combinations achieving required strength and ductility of a reinforced concrete beam, a refined optimization procedure based on nonlinear layered finite element method and nonlinear programming technique is developed in this study. Six design variables-beam width and depth, fiber volume fraction, amounts of tensile and compressive rebars, and stirrup spacing-are considered. The developed model can be used as a tool in determining the economical use of steel fibers in designing the reinforced steel fibrous concrete beam.

1. INTRODUCTION

The use of steel fibers in reinforced concrete has the following benefits in flexural behavior[1]: (1) increases in the flexural strength, ductility, and stiffness, (2) improvements in crack control, and (3) preserving integrity of the beam beyond the failure load of the normally reinforced concrete beam. Since steel fibers also show several potential advantages when used to supplement or replace vertical stirrups Recently, A.Samer Ezeldin and Cheng-Tzu Thomas Hsu[7] has developed the computer algorithm that evaluates the optimum use of steel fibers in achieving the required flexural strength and shear strength. The method, however, is unable to predict overall load-deflection behavior of the beam, which usually improves by the addition of steel fibers.

In this study, in order to assess the beneficial effects of the use of SFRC over the plain concrete, displacement controlled nonlinear layered finite element method is used. Six design variables are considered, namely, beam dimensions (width and depth), fiber content, area of tensile steels, area of compressive steels, and stirrup spacing. The search for the optimum design is conducted with the Powell's algorithm which, by iteratively generating the conjugate directions in the design space, leads to the most economical geometry and amount of materials of the beam that meets predefined structural performance of RSFCB in terms of flexural and shear strength as well as flexural ductility.

2. DEVELOPMENT OF THE MODEL

2. 1. Flexural behavior - layered finite element method ; In the layered finite element methods, strain compatiblitiy is assumed and each cross section of a member is divided into N_l layers, some of which may represent the reinforcing bars in compression or tension. The column vectors of forces($\boldsymbol{F}$) and element displacements($\boldsymbol{d}$) are: $\boldsymbol{F} = (Ni,\ Vi,\ Mi,\ Nj,\ Vj,\ Mj)^T$ and

* 정회원, 중앙대학교 건축공학과 부교수
** 중앙대학교 대학원 석사과정

$\boldsymbol{d} = (ui, vi, \theta i, uj, vj, \theta j)^T$, where T denotes transpose of a corresponding vector, and subscripts i and j represent the adjacent cross sections at the ends of element, respectively. The meanings of nodal values in the components of $\boldsymbol{F}$ and $\boldsymbol{d}$ are shown in Fig. 1. In the layered finite element method, [K_e] are constructed by integrating analytically and summing its results over all layers ;

$$[K_e] = \int_V [B]^T \cdot E_s \cdot [B]\, dV = \sum_{i=1}^{ne} [B]^T \cdot E_s \cdot [B] \cdot b_i \cdot \Delta h_i \cdot L \qquad (1)$$

where ne, b_i, and Δh_i stand for number of elements in an element, width of an i-th layer, and depth of the i-th layer, respectively. In order to find solution, the direct iteration method is used. The algorithm can be summarized as follows:

(1). Given small prescribed displacement increments Δw of conrolled displacement w;
- (1.1) calculate new nodal displacements and corresponding strains at midpoints of all layers in all nodal cross sections;
- (1.2) update the secant modulus of elasticities in each layer;
- (1.3) find new nodal displacements and strains; and
- (1.4) repeat the above process until $\sum_{i=1}^{N} |u_{i,old} - u_{i,new}| <$ tolerance, where $u_{i,old}$, $u_{i,new}$, and N represent previous and new ith displacement components and number of total degrees of freedom of the model, respectively.

(2). Calculate load P as the reaction at the point of prescribed displacement, w.
(3). Calculate internal forces for each finite element and the strains and stresses of all layers.
(4). Return to step (1) and start for the next step unless the fracture of tensile steels or the concrete crushing at the extreme compressive fiber of the beam occurs.

2. 2. Constitutive models ; A few constitutive models are available for SFRC under compression and tension. They are mostly empirical or semi-empirical models. The models adopted here has been developed by the author[3]. In this study a more conservative strain values are taken based on the study by Hassonun et.al.[4] - linearly increasing maximum usable compressive strain values for the extreme compression fiber in concrete(ε_{cu}) as the fiber volume fractions increase : if $V_f \le 1.0\%$, then $\varepsilon_{cu} = 0.05 V_f + 0.003$; and if $1.0\% \le V_f \le 2.0\%$, then $\varepsilon_{cu} = 0.025 V_f + 0.00325$.

2. 3. Shear strength ; The equations suggested by different researchers [5-8] were evaluated statistically and the equation by Mansur et. al. [5] was chosen for this study :

$$V_c = (0.50\sqrt{f_c} + 176\,\rho\,\frac{V}{M}d)\,bd \le 0.93\sqrt{f'_c}\,bd \qquad (2)$$

$V_{cf} = V_c + \sigma_{ut} \cdot b \cdot d$; and σ_{ut} = pull-out resistance

3. OPTIMIZATION TECHNIQUE

The objective function Z for beams subjected to bending and shear can be formulated as:

$$Z = \rho_{conc} \cdot C_c(b \cdot h) + \rho_{stl} \cdot C'_s A'_s + \rho_{stl} \cdot C_s A_s + \rho_{fr} \cdot C_f V_f (b \cdot h) + C_u(2h + b)$$

$$+ \rho_{stl} \cdot C_v A_v (2h + b)/S \qquad (3)$$

where

ρ_{conc}, ρ_{stl}, and ρ_{fr} = densities of concrete, steel and fibers; C_v, C_s', C_s, C_u, C_f, and C_v = total costs(i.e. sum of material and labor costs) for concrete, compressive steels, tensils steels, steel fibers, forms and stirrups(see Table 2); A_s', A_s, and A_v = areas of compressive steel, tensile steel and stirrup; V_f = volume fraction of fibers; and b and h = width and depth of the beam.

The constraints for the design variables are imposed based on the ACI code. The minimum volume fraction($V_{f,\min}$) is always set to zero. The maximum volume fraction($V_{f,\max}$) is chosen to be 0.015, considering the practical applicability of SFRC in the construction field. Using the nominal shear strength of steel fiber reinforced concrete, V_{cf}, the shear force for stirrup and stirrup spacing are obtained according to the ACI Building Code provisions.

The final objective function(F), thus, is formulated in order to minimize both the cost function(Z) defined above and the differences in required and calculated structural behaviors:

$$\text{Minimize } F = E + Z; \qquad (13)$$

where $E = \omega_P \cdot e_P^2 + \omega_D \cdot e_D^2$;

ω_P, ω_D = weights for flexural strength and ductility, respectively ;

e_P = normalized differences in ultimate flexural strength = $\dfrac{(P_{ult,req} - P_{ult,cal})}{P_{ult,req}}$;

e_D = normalized differences in ductility = $\dfrac{(D_{req} - D_{cal})}{D_{req}}$; and

P_{ult} and D = ultimate flexural strength and ductility, respectively. The subscripts 'req' and 'cal' represent the 'required' and 'calculated', respectively.

The objective function, F, is minimized by the Powell's nonlinear programming technique and one dimensional search is done with the method of Golden Search technique. The Powell's algorithm iteravely generates the conjugate vectors and in a finite number of iterations, local minimum is reached[9].

4. RESULTS OF OPTIMIZATION

Table 1 summarizes the material, labor and total costs for concrete, rebar, forms and steel fibers. The total costs which sum the material cost and the labor cost for each item are represented by appropriate notations(C_c, C_s, C_s', C_u, and C_f) in table 1 and are entered in the objective function Z in eq.(4).

<u>4. 1. Regular beam</u> ; Fig. 2 shows the geometry and loading conditions considered for the optimization of the RSFCB in this study. Constraints for the beam geometry and material quantities are given as : $25cm \le width(b) \le 50cm$, $40cm \le depth(h) \le 90cm$, $0\ cm^2 \le A_s$ (or A_s') $\le 40cm^2$, and $0.0 \le V_f \le 1.5\%$. The cover thickness for the tension steel and compression steel (if any) in a beam is given to be 5.0cm for all beams. The compressive strength of concrete and yield strength of the steel rebars are taken as 270 kg/cm^2 and 4000 kg/cm^2, respectively. U-type of stirrup with yield strength of 4000 kg/cm^2 is used throughout, with cross sectional area of one stirrup being as 0.71 cm^2x2=1.42. cm^2. The

objective was to find optimized values on beam width, depth, reinforcements for tension, compression, and shear as well as volume fraction of steel fibers for the preassigned beam ultimate flexural strength, P_u= 20.0 ton, and different ductilities of 2.0 through 5.0. Table 2 shows the optimized values for these assigned values. The model can adequately obtain the preassigned ultimate flexural strength and ductilities. Although steel fibers are known to increase the flexural ductility of the beam, reinforcements by conventional rebars have shown to be more cost saving solutions for the increase of ductility of the beam from 2.0 to 3.0. As ductility is increased further from 3.0, the area of the compression reinforcement is increased while the area of the tensile reinforcement decreases. For all cases, use of steel fibers in partial replacement of conventional steel rebars is not found to be an economic choice. Total cost for the beam with increased ductility requires higher cost than those with lower ductility. Although the cost for steel fibers is lowered to 950,000 Won/ton as shown in Table 1, assuming that steel fibers were produced domestically and thus readily available, still the optimization process yielded very close outcomes as those in Table 2. This implies that for the use of steel fibers to be realized in a regular rectangular beam, much less expensive steel fibers are needed.

4. 2 Wide beam ; Rectangular beams having relatively large width with restrictions on depth are considered. Depth of a beam is assumed as being restricted in the range between 60cm and 70cm. Except for the wider range of beam width and narrower range of beam depth, same geometry and loading conditions as those of regular rectangular beam are used (Fig.2). Constraints for the beam geometry and material quantities are given as: $50cm \le width(b) \le 200cm$, $60cm \le depth(h) \le 70cm$, $0\ cm^2 \le A_s$ (or A_s') $\le 100cm^2$, and $0.0\% \le V_f \le 1.5\%$. Same cover thicknesses and stirrups are assumed. For given two ultimate flexural strength, P_u= 90.0ton(W-series) and 120.0ton(WH-series), most economical wide beams having different ductilities in the range of 3.0 through 5.5 are sought through the optimization process.

Table 3 shows the selected design values for the optimized wide beams. Except for the beams WH3,WH4 and WH5 upon which a large ductility requirements (D=4.5 through 5,5) are assigned, steel fibers are rarely used as the partial alternative to the conventional reinforcement. The results for the beams WH3,WH4 and WH5 reveal that for some required structural performance the use of steel fibers in increasing the flexural strength, ductility, and shear capacity of the beam may be an economic alternative to the conventional RC beam by properly reducing and increasing the rebar area and stirrup spacing. As in the regular rectangular beam, higher cost is needed in general for more ductile beams.

Like the regular rectangular beam, the use of steel fibers of 950,000Won/ton in rectangular wide beam did not alter the trends shown in Table 3. Table 4 presents the results for those wide beams.

5. CONCLUSION

The following conclusions are derived from this study.

1. The optimization model developed by the nonlinear layered finite element method and nonlinear programming technique can successfully find the optimum values for the beam sectional dimensions and the amount of reinforcements for the given ultimate flexural strength and ductility.
2. Although there are beneficial effects from the use of steel fibers on the increase in flexural strength, ductility and shear capacity of the beam in addition to the improved serviceability, it does not lead to an economical solution both in the regular rectangular beams and in most wide rectangular beams for the given loading conditions and material properties considered in this study.
3. For the wide beams having the required ultimate flexural strength of 120ton with relatively high ductility ratio of 4.5 through 5.5, the amount of steel fibers close to 1.5% volume fraction is needed to realize the economic construction of the beam. Incorporation of steel fibers reduced the

area of rebars in tension and increased the spacing of stirrups. This could be a positive indicative to the use of steel fibers for more general and popular structural components (such as beam which is in special need of high ductility requirement).

4. The developed model can be also used for various structures governed mainly by flexure. Further research seems to be needed for these structures using the developed model.

REFERENCES

1. P.Soroushian, C.D.Lee, and Z.Bayasi, "Fiber Reinforced Concrete : Theoretical Concepts and Structural Design," Seminar Proceedings, Michigan State Univ., E.Lansing, U.S.A, 1987, pp.8.1-8.30
2. A.Samer Ezeldin and C.T.Hsu,"Optimization of Reinforced Concrete Beams," ACI Structural Journal / Jan.-Feb., 1992, pp.106-114.
3. P.soroushian and C.D.Lee, "Constitutive Modeling of Steel fiber Reinforced Concrete under Direct Tension and Compression," Report No. Msu-ENGR-89-002, Michigan State University, E.Lansing, USA, 1989.
4. M.N.Hassoum and K.Shahebjam,"Plastic Hinge in Two-Span Reinforced Concrete Beams Containing Steel Fibers," Proceedings, Canadian Society of Civil Engineering(Montreal), May 1985, pp.119-139.
5. M.A.Mansur and P.Paramasivam, "Shear Strength of Fibrous Concrete Beams without Stirrups," J. of Structural Engineering, Vol. 112, No. 9, September 1986, pp. 206-207.
6. A.K.Sharma, "Shear Strength of Steel Fiber Reinforced Concrete Beams," ACI Journal. July-August 1986, pp. 624-627.
7. R.L.Jindal, "Shear and Moment Capacities of Steel Fiber Reinforced Concrete Beams," SP81-1, pp. 1-16.
8. R.Narayanan and I.Y.S.Darwish, "Use of Steel Fibers as Shear Reinforcement," ACI Structural Journal, May-June 1987, pp. 216-227.
9. M.J.D.Powell,"An Efficient Method for Finding the Minimum of a Function of Several Variables Without Calculating Derivatives," Computer Journal, Vol.7, 1964.

Table 1 - Material, labor and total costs for concrete, rebar, forms and steel fibers.

items		material costs	labor costs	total costs
name	notation	(Won)	(Won)	(Won)
concrete / $m3$	C_C	38,990	82,668	121,658
rebar / ton	C_S, C_S', C_V	332,740	340,991	673,731
forms / m^3	C_U	5,945	13,254	19,199
steel fibers/ ton	C_f	1,500,000 (KOR)	200,000	1,700,000
		950,000 (USA)	200,000	1,150,000

Table 2 - Optimized values for the regular rectangular beam.

beam	load-diplacement curve					optimized results						
	objective		optimized			beam section		reinforcements			fiber	cost
	P_U (ton)	D	P_U (ton)	D	displ. (cm)	width (cm)	depth (cm)	A_S (cm^2)	A'_S (cm^2)	S (cm)	V_f (%)	won (x1000)
R1	20.0	2.0	20.0	2.00	6.8	30.7	40.6	23.0	9.2	17.2	0.0	75.50
R2	20.0	3.0	20.0	3.05	5.8	25.0	60.2	16.4	0.0	27.0	0.0	80.75
R3	20.0	4.0	19.9	3.94	7.1	25.5	61.0	14.4	3.2	27.4	0.0	82.85
R4	20.0	5.0	20.0	5.00	8.0	26.1	62.8	12.9	7.1	28.3	0.0	87.38

Table 3 - Optimized values for the wide beam

wide beam	load-diplacement curve					optimized results						
	objective		optimized			beam section		reinforcements			fiber	cost
	P_U (ton)	D	P_U (ton)	D	displ. (cm)	width (cm)	depth (cm)	A_S (cm^2)	A'_S (cm^2)	S (cm)	V_f (%)	won (x1000)
W1	90.0	3.0	90.0	3.00	5.7	81.1	62.8	69.5	15.2	20.0	0.07	234.7
W2	90.0	4.0	90.0	4.00	6.4	88.5	69.0	54.5	9.6	18.3	0.14	263.1
W3	90.0	5.0	89.9	5.00	7.5	90.3	69.7	49.3	25.0	18.0	0.13	273.9
WH1	120.0	3.0	120.1	3.00	5.1	104.2	69.6	81.8	5.5	15.6	0.05	305.5
WH2	120.0	4.0	119.9	4.00	6.4	132.3	68.3	74.5	54.3	12.3	0.01	356.4
WH3	120.0	4.5	119.4	4.47	7.6	93.1	69.6	59.2	27.7	17.4	1.40	397.1
WH4	120.0	5.0	117.9	4.94	8.4	73.1	67.9	60.2	52.8	22.2	1.47	339.3
WH5	120.0	5.5	120.0	5.44	8.7	99.5	68.9	55.7	41.5	16.3	1.48	429.1

Table 4 - Optimized values for the wide beam (WH series with 950,000Won/ton worth steel fibres)

wide beam	load-diplacement curve					optimized results						
	objective		optimized			beam section		reinforcements			fiber	cost
	P_U (ton)	D	P_U (ton)	D	displ. (cm)	width (cm)	depth (cm)	A_S (cm^2)	A'_S (cm^2)	S (cm)	V_f (%)	won (x1000)
WH1	120.0	3.0	119.9	3.00	5.1	104.8	69.5	82.0	5.1	15.5	0.05	305.2
WH2	120.0	4.0	120.0	4.00	6.4	131.6	68.4	74.7	5.6	12.3	0.02	355.1
WH3	120.0	4.5	120.2	4.47	7.6	95.5	67.3	63.3	28.4	17.0	1.45	361.4
WH4	120.0	5.0	120.5	5.06	8.0	84.8	69.6	58.4	39.3	19.1	1.47	342.4
WH5	120.0	5.5	120.0	5.50	8.8	92.7	69.5	54.9	49.5	17.5	1.49	371.7

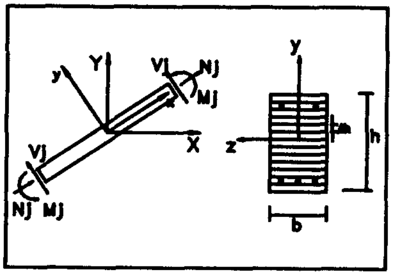

Fig.1. Notations in the layered finite element

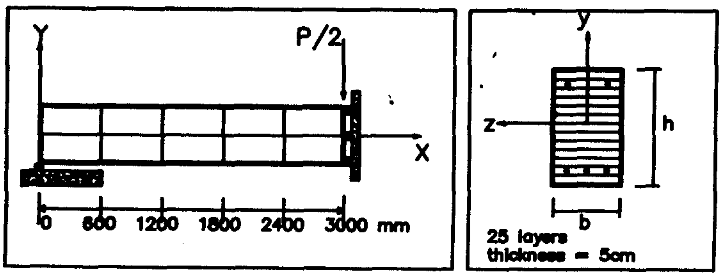

Fig. 2. Loading Conditions and Layered Beam Section for Modeling