History and Future in Vertical Drain Design A Comparison of Theory and Real Behaviour*

Sven Hansbo

1. INTRODUCTION

A welth of information on the historical background of vertical drain design can be found in a book written by Magnan (1983). In this paper various design methods concerning the effect of preloading in combination with vertical drain installations in soft cohesive soils will be reviewed. Important requirements on drain characteristics are discussed. Monitoring systems for a follow-up of results achieved are commented on. The results of various analytical and numerical design methods are compared. Actual drain behaviour is compared with theoretical predictions.

2. VARIOUS TYPES OF VERTICAL DRAINS

2.1 Circular-Cylindrical Drains

Sand drains which are the most common type of circular-cylindrical drains were first proposed, in 1925, and patented, in 1926, by Daniel D. Moran. He also suggested the first practical application of sand drains as a means of stabilisation of mud soil beneath a roadway approach to the San Francisco Oakland Bay Bridge (Johnson, 1970). This led to some successful laboratory and field experiments followed by the installation of the first drain system in 1934. Porter (1936) described these trials and contributed to the further use and development of the system.

The sand drains originally installed had generally a relatively large diameter, 0.3–0.5 m. Later on small-diameter sand drains have come into use, for example 'sandwicks', 0.05 m in diameter, and 'fabri pack drains', 0.12 m in diameter. The sand in these drains is packed into a synthetic fiber net-type tube which prevents the drains from necking.

Another type of circular-cylindrical drains was developed by, among others, Technique Louis Ménard, the so-called soil-drain. This consists of an open prefabricated tubular plastic core provided with perforations to admit inflow of water.

A range of techniques have been utilised for installation of sand drains. These include so-called non-displacement methods, such as shell and auger drilling, powered auger drilling, water-jetting, flight augering and wash-boring and displacement methods, typically by the use of a driven mandrel.

2.2 Band or Wick Drains

The first type of band-shaped drains, the so-called wick drains, introduced on the market was invented in Sweden by Kjellman and his co-workers at the Swedish Geotechnical Institute. These drains, named Cardboard Wicks (Kjellman, 1948), were made of two cardboard sheets glued together with an external cross-section of 100 mm by 3 mm and including ten longitudinal internal channels, 3 mm in width and 1 mm in thickness.

^{*}This is a preliminary version of a paper to be published in Year 2000 Geotechnics

The efficiency of Cardboard Wicks was first investigated in 1945 in a full-scale test near Upplands Väsby, north of Stockholm, as preliminary measure to the construction of the new Stockholm airport. The results of this investigation has been reported by Chang (1981).

The Cardboard Wick has served as a prototype for all the various band drains now existing on the market. The first of these new types of band drains on the market was named Geodrain, developed at the Swedish Geotechnical Institute. It consists of a core of plastic material surrounded by a filter sleeve with an external cross-section of 95 mm by 4 mm. Both sides of the core are provided with 27 longitudinal grooves whose widths and depths vary with different makes. The filter sleeve was originally made of a special make of paper but was later changed into synthetic material. After some successful applications of Geodrains, a great number of band drains, having more or less similar characteristics, have been developed (see e.g. Hansbo, 1979, 1986, 1993, 1994). A somewhat different type of band drain is the Fibre Drain developed in Singapore (Lee *et al.*, 1995). It consists of one layer of thin, closely knit jute burlap laid inside another layer of thick, but coarsely knit burlap. Four coir strands, 3–6 mm in diameter, pass longitudinally through the inner core formed by the two layers of burlap.

It is interesting to note that Barron (1948), with reference to a contribution by Kjellman (1948), expresses his opinion that "should wick material and installation machines become available in the United States, sand wells may be outmoded". Nowadays very efficient installation machines have come into use and a large number of various band (wick) drains exist on the market. Barron's prophecy has certainly become true.

3. ANALYTICAL APPROACH

3.1 Assumptons based on Darcy's flow law

Regarding the historical development of vertical drain analysis, special interest must be devoted to the contributions given by Barron which form a starting point in the understanding of the result to be expected by vertical drain installations. During the winter 1941–1942 the Providence District incorporated drain wells in plans for reconstruction of a portion of Riverfront Dike, Hartford, Connecticut. This entailed a necessity of having a more exact analysis of the influence of vertical drains on the consolidation process. The analysis published by Barron (1944) was based on existing solutions for one-dimensional vertical consolidation (Terzaghi, 1923; 1925) and radial flow of heat (Glover, 1930). Barron's analysis was based on the following assumptions:

- Darcy's flow law is valid,
- the soil is water saturated,
- displacements due to consolidation take place in the vertical direction only,
- initial excess pore water pressure u_0 is uniform throughout the soil mass when t = 0.
- excess pore water pressure at the drain well surface is zero (no well resistance),
- the cylindrical boundary at external radius R is impervious,
- excess pore water pressure at the upper boundary of the soil mass (z = 0) is zero. The differential equation governing the consolidation process is then given by the expression (Fig. 1):

$$\frac{k_h}{\gamma_w} \left(\frac{1}{\rho} \frac{\partial u}{\partial \rho} + \frac{\partial^2 u}{\partial \rho^2} \right) + \frac{k_v}{\gamma_w} \frac{\partial^2 u}{\partial z^2} = \frac{a_v}{1 + e} \frac{\partial u}{\partial t}$$
 (1)

where k_h and k_v are the permeabilities in the horizontal and vertical directions, γ_w = the unit weight of water,

 ρ and z are the cylindrical coordiates,

u =excess pore water pressure,

 $a_v = -\Delta e/\Delta \sigma' =$ coefficient of theoretical compressibility

e = void ratio

 $a_V/(1+e) = m_V = 1/M = \text{coefficient of volume compressibility } (M = \text{oedometer modulus}),$

t = consolidation time.

Barron proposes that the total degree of consolidation, including the effect of combined radial and vertical outflow of water, be solved according to Carillo (1942) by the expression:

$$u_{\rho z} = \frac{u_{\rho} u_z}{u_0} \tag{2}$$

where u_{0z} = remaining total excess pore water pressure after time t,

 u_p^- remaining excess pore water pressure after time t due to radial drainage, u_z^- eremaining excess pore water pressure after time t due to vertical drainage, u_0^- excess pore water pressure at time t = 0.

Expressed in degree of consolidation U this yields:

$$U_{\rho z} = U_{\rho} + U_{z} - U_{\rho}U_{z} \tag{3}$$

where U_{ρ} = degree of consolidation due to radial outflow of pore water to the drains,

 U_z = degree of consolidation due to vertical outflow of pore water outside the drains.

Defining $U = s/s_p$, where s = settlement at time t and $s_p =$ total primary settlement, the settlement s_{hd} at time t, achieved by the effect of vertical drains only, can be written

$$s_{hd} = \frac{s - s_{vd}}{1 - s_{vd}/s_p} \tag{4}$$

where s_{vd} = settlement caused by one-dimensional vertical consolidation.

Barron assumed two different cases to take place: the case of *free strains* and the case of *equal strains*.

In the *free strain hypothesis* Barron presupposes that no arching takes place and that shearing strains caused by differential settlement do not redistribute the load stresses within the soil at any time during consolidation.

In his original free strain analysis, Barron (1944) assumed that the installation of the drains did not affect the properties of the soil and that the permeability of the drain well was high enough for well resistance to be neglected. He later on included disturbance effects due to installation, a zone of smear (Barron, 1948) with reduced permeability k_s .

In the equal strain hypothesis Barron (1948) presumes arching to redistribute the load so that the vertical strains at a certain depth z become equal irrespective of the radial distance ρ , and, consequently, no differential settlement will take place. This may seem a rather serious condition but is supported by field observations in areas provided with vertical drains. In the equal strain hypothesis he also

includes the effect of well resistance on the consolidation process. Thus, in reality the drains may have a limited capacity of transporting the pore water entering into the drains during the consolidation process. Assuming complete drainage at z = 0 and z = 2l the average degree of consolidation obtained by radial (horizontal) drainage $\overline{U}_{hz} = 1 - \overline{u}_{hz}/u_0$ at depth z is given by the correlation (Fig. 1):

$$\overline{U}_{hz} = 1 - \exp\left\{-\frac{8c_h t}{vD^2} \left[\frac{\exp[\beta(z-2l)] + \exp(-\beta z)}{1 + \exp(-2\beta l)} \right] \right\}$$
 (5)

where

$$v = \frac{n^2}{n^2 - s^2} \ln\left(\frac{n}{s}\right) - \frac{3}{4} + \frac{s^2}{4n^2} + \frac{k_h}{k_s} \left(\frac{n^2 - s^2}{n^2}\right) \ln s$$

 k_h =permeability of the soil in the horizontal direction

 k_s =permeability in the zone of smear

$$c_h = k_h/m_v \gamma_w = k_h M/\gamma_w$$

$$T_h = c_h t/D^2$$

 $d_w = 2r_w$ = diameter of the drain well,

D = 2R = diameter of the dewatered cylinder,

$$n = D/d_w$$

 $s = d_s/d_w$, where d_s = diameter of the zone of smear,

$$\beta = \sqrt{\frac{2k_h(n^2 - s^2)}{k_w R^2 v}} = \sqrt{\frac{2\pi k_h(n^2 - s^2)}{vq_w}}$$

 $q_w = k_w \pi d_w^2 / 4$ = specific discharge capacity of the drain

Barron showed that the difference in results between the assumptions of free strain and equal strain (no well resistance) is practically negligible.

The effect of well resistance was also taken into account by Yoshikuni and Nakanado (1974). Their solution which includes both vertical and horizontal pore water flow (upper and lower boundary surfaces assumed to be drained) but does not include the effect of smear ends up in a rather complex expression. The results obtained in the case of radial drainage and equal vertical displacement of the soil surface (which is not exactly synonymous with the equal strain case), are presented in the form of tables (Yoshikuni, 1979; 1992) for various n and well resistance values, the latter expressed by the parameter:

$$L = \frac{32k_h l^2}{\pi^2 k_w d_w^2} = \frac{8k_h l^2}{\pi q_w}$$

Youshikuni's solution was extended by Onoue (1988a) to include the influence exerted on the consolidation process by multilayered, unisotropic soils and the effect of smear (Onoue, 1988b).

Another approach to the equal strain hypothesis in the simple case of no peripheral smear or well resistance, very similar to Barron's approach, was presented already in 1937 (Kjellman, 1948). Kjellman's approach was extended by the author (Hansbo, 1981) to include the effect of smear and well resistance. In this case, the average degree of consolidation is given by the relation:

$$\overline{U}_{hz} = 1 - \exp\left(-\frac{8c_h t}{\mu D^2}\right)$$

$$= 6 - (6)$$

where

$$\mu = \frac{n^2}{n^2 - 1} \left[\ln\left(\frac{n}{s}\right) + \frac{k_h}{k_s} \ln s - \frac{3}{4} \right] + \frac{s^2}{n^2 - 1} \left(1 - \frac{s^2}{4n^2} \right) - \frac{k_h (s^2 - 1)}{k_s (n^2 - 1)} \left(1 - \frac{s^2 + 1}{4n^2} \right) + \frac{k_h \pi z (2l - z) (1 - n^{-2})}{q_{w}}$$

Omitting terms of minor significance we find:

$$\mu = \ln\left(\frac{n}{s}\right) + \frac{k_h}{k_s} \ln s - \frac{3}{4} + \frac{k_h \pi z (2l - z)(1 - n^{-2})}{q_w}$$

The average degree of consolidation \overline{U}_h of the whole layer is obtained by exchanging the value of μ for:

$$\mu_{\text{av}} = \ln\left(\frac{n}{s}\right) + \frac{k_h}{k_s} \ln s - \frac{3}{4} + \frac{2k_h \pi l^2 (1 - n^{-2})}{3q_w}$$

The hydraulic gradient i becomes equal to:

$$i = \frac{u_0}{\mu \gamma_w} \left(1 - \overline{U}_{hz} \right) \left(\frac{1}{\rho} - \frac{4\rho}{D^2} \right) \tag{7}$$

The average degree of consolidation achieved by one-dimensional vertical pore water flow (undrained condition) is generally below 50%. Therefore, the total average degree of consolidation for fully penetrating drains, taking into account both undrained and drained condition can be expressed by the relation:

$$\overline{U}_{\text{av}} = 1 - \left(1 - \frac{2}{l}\sqrt{\frac{c_{\nu}t}{\pi}}\right) \exp\left(-\frac{8c_{h}t}{\mu_{\text{av}}D^{2}}\right)$$
(8)

where 2l = thickness of the clay layer when drained at top and bottom (Fig. 1), corresponding to the length of the drains.

Zeng and Xie (1989)pointed out that the continuity at the drain interface was not satisfied in the solutions given by equations 5-6 and presented a 'correct' solution:

$$\overline{U}_{hz} = 1 - \sum_{m=0}^{\infty} \frac{2}{M} \sin\left(\frac{M}{l}z\right) \exp\left(-\frac{8c_h t}{\eta D^2}\right)$$
 (9)

where

$$M=\frac{2m+1}{2}\pi,$$

$$\eta = \frac{1}{n^2 - 1} \left\{ n^2 \left[\ln \left(\frac{n}{s} \right) + \frac{k_h}{k_s} \ln s - \frac{3}{4} \right] + \frac{2k_h \pi l^2 \left(n^2 - 1 \right)}{M^2 n^2 q_w} \right.$$

With the advances of the finite element and the finite difference methods the consolidation process achieved for any type of loading and drainage condition

can be solved theoretically on the basis of given consolidation and drain parameters (e.g. Onoue, 1988a; Lo, 1991). Among these can be mentioned the finite element program ILLICON developed at University of Illinois at Urbana-Champaign, USA, which is based on the following basic correlations (Lo, 1991):

$$\left(\frac{\partial e}{\partial \sigma_{v}'}\right)_{t} \frac{\partial \sigma_{v}'}{\partial t} + \left(\frac{\partial e}{\partial t}\right)_{\sigma_{v}'} = \frac{\left(1 + e_{0}\right)^{2}}{\gamma_{w}(1 + e)} \left[\frac{\partial k_{v}}{\partial z} \frac{\partial u}{\partial z} + k_{v} \left(\frac{\partial^{2} u}{\partial z^{2}} - \frac{1}{1 + e} \frac{\partial u}{\partial z} \frac{\partial e}{\partial z}\right)\right] + \frac{1 + e}{\gamma_{w}} \left[\frac{\partial k_{h}}{\partial \rho} \frac{\partial u}{\partial \rho} + k_{h} \left(\frac{1}{\rho} \frac{\partial u}{\partial \rho} + \frac{\partial^{2} u}{\partial \rho^{2}}\right)\right] \tag{10a}$$

The excess pore water pressure is computed from the relation:

$$\frac{\partial \sigma_{v}}{\partial t} - \frac{\partial u}{\partial t} = \frac{\partial e/\partial t - (\partial e/\partial t)_{\sigma'_{v}}}{(\partial e/\partial \sigma'_{v})_{t}}$$
(10b)

Here $(\partial e/\partial t)_{\sigma'_{\nu}}$ = change in void ratio with time at a given effective stress, $(\partial e/\partial \sigma'_{\nu})_t$ = change in void ratio owing to change in effective stress.

The left-hand term in equation (10a) is synomonous with the constitutive relation presented by Taylor & Merchant (1940):

$$\frac{de}{dt} = \left(\frac{\partial e}{\partial \sigma'_{v}}\right)_{t} \frac{\partial \sigma'_{v}}{\partial t} + \left(\frac{\partial e}{\partial t}\right)_{\sigma'_{v}}$$

The results obtained by different design methods for drains with well resistance are compared in Fig. 2.

3.2 Assumptions Based on Non-Validity of Darcy's Law

In the course of consolidation, the permeability in particular will be subjected to gradual reduction. However, case studies and experimental evidence have also shown that the coefficient of consolidation increases with increasing magnitude of the load that produces consolidation. Therefore, Terzaghi & Peck (1948) recommended, for the determination of the coefficient of consolidation, that the load increment "applied to the sample after a pressure equal to the overburden pressure has been reached should be of the same order of magnitude as the load per unit area of the base of the structure. A possible explanation for this phenomenon can be an exponential correlation between pore water flow and hydraulic gradient.

Results of permeability tests on clay samples presented by different researchers (e.g. Silfverberg, 1947; Hansbo, 1960; Miller & Low, 1963; Dubin & Moulin, 1986; Zou, 1996) have indicated that the pore water flow v caused by a hydraulic gradient i may deviate from Darcy's law v = ki where k is the coefficient of permeability. Silfverberg and Miller & Low drew the conclusion that there is a threshold gradient i_0 below which no flow will take place, yielding $v = k(i - i_0)$, while the author (Hansbo, 1960) proposed the following relation (the original use of n is exchanged for x not to be mistaken for $n = D/d_w$), Fig. 3:

$$v = \kappa i^x \text{ when } i \le i_l$$
 (11a)

$$v = \kappa x i_l^{x-1} (i - i_0) \text{ when } i \ge i_l$$
 (11b)

where, in the author's opinion, $i_l = i_0 x/(x-1)$ represents the gradient required to overcome the maximum binding energy of mobile pore water [the physical background to non-linear conductivity behaviour is discussed in detail by Hansbo (1960)].

The relations 8a-b proposed by the author were also chosen by Dubin and Moulin (1986) in the analysis of Terzaghi's one-dimensional consolidation theory denoting $\kappa = ak$ where $a = x^{-1}i_l^{1-x}$ and $k = v/(i - i_0)$. The values of i_l have been found to vary from 4–10 (Hansbo, 1960) and 8–35 (Dubin & Moulin, 1986). Using the non-linear flow law given by equation (11a) the consolidation equation, taking both smear and well resistance into account, can be written:

$$\overline{U}_{hz} = 1 - \left[1 + \frac{\lambda t}{\alpha D^2} \left(\frac{\Delta h}{D}\right)^{x-1}\right]^{-\frac{1}{x-1}}$$
(12)

where

 $\Delta h = u_0/\gamma_w$ = the increase in piezometric head caused by the placement of the load.

 $\lambda = \kappa_h M / \gamma_w$ = the coefficient of consolidation,

$$\alpha = \frac{x^{2x}\beta^x}{4(x-1)^{x+1}}$$

with

$$\beta = \frac{1}{3x - 1} - \frac{x - 1}{x(3x - 1)(5x - 1)} - \frac{(x - 1)^2}{2x^2(5x - 1)(7x - 1)} + \frac{1}{2x} \left[\left(\frac{\kappa_h}{\kappa_s} - 1 \right) \left(\frac{n}{s} \right)^{-(1 - 1/x)} - \frac{\kappa_h}{\kappa_s} n^{-(1 - 1/x)} \right] + \frac{(1 - 1/x)n^{-(1 - 1/x)} \left(1 - 1/n^2 \right)^{1/x} \kappa_h \pi z (2l - z)}{2q_w}$$

$$n = D/d_w$$
; $s = d_s/d_w$

The average degree of consolidation \overline{U}_h for the whole layer is obtained by exchanging the last term in the β expression for:

$$\frac{(1-1/x)n^{-(1-1/x)}(1-1/n^2)^{1/x}k_h\pi l^2}{3q_w}$$

When the exponent $x \to 1$, equation (12) yields the same result as equation (6) assuming $\lambda = c_h$ and $\kappa_h = k_h$ and $\kappa_s = k_s$. Thus equation (12) is generally applicable and can, therefore, replace equation (6).

The hydraulic gradient *i* becomes equal to:

$$i = \frac{\Delta h}{D} \left(1 - \overline{U}_{hz} \right) \left[\frac{1}{4\alpha(x-1)} \left(\frac{D}{2\rho} - \frac{2\rho}{D} \right) \right]^{1/x}$$
 (13)

When $x \to 1$, equation (12) yields the same result as that obtained by equation (6). The best agreement between theory and observations has been obtained for x = 1.5 (Hansbo, 1960; 1997a-b) which yields:

$$\overline{U}_{hz} = 1 - \left(1 + \frac{\lambda t}{\alpha D^2} \sqrt{\frac{\Delta h}{D}}\right)^{-2} \tag{14}$$

where

$$\alpha = 4.77 \beta \sqrt{\beta}$$

$$\beta = 0.270 + \frac{1}{3} \left[\left(\frac{\kappa_h}{\kappa_s} - 1 \right) s^{-1/3} - \frac{\kappa_h n^{-1/3}}{\kappa_s} + \frac{n^{-1/3} \left(1 - n^{-2} \right)^{2/3} \kappa_h \pi z (2l - z)}{2q_w} \right]$$

The average degree of consolidation \overline{U}_h for the whole layer is obtained by exchanging β for:

$$\beta_{\text{av.}} = 0.270 + \frac{1}{3} \left[\left(\frac{\kappa_h}{\kappa_s} - 1 \right) s^{-1/3} - \frac{\kappa_h n^{-1/3}}{\kappa_s} + \frac{n^{-1/3} \left(1 - n^{-2} \right)^{2/3} \kappa_h \pi l^2}{3q_w} \right]$$

Inserting β_{av} , the total average degree of consolidation, taking into account both undrained and drained conditions, can be expressed by the approximate relation (approximate since the contribution by one-dimensional vertical consolidation is based on validity of Darcy's law and thus inconsistent with the λ theory):

$$\overline{U}_{av} = 1 - \left(1 - \frac{2}{l}\sqrt{\frac{c_v t}{\pi}}\right) \left(1 + \frac{\lambda t}{\alpha_{av}D^2}\sqrt{\frac{\Delta h}{D}}\right)^{-2}$$
(15)

where 2l = thickness of the clay layer when drained at top and bottom (Fig. 1), corresponding to the length of the drains.

The influence of various magnitudes of well resistance on the results obtained according to equation (14) is exemplified in Fig. 4.

In field conditions the hydraulic gradient i in most cases is very small in comparison with laboratory conditions. Choosing as an example a case with x = 1.5, $\Delta h = 5$ m, D = 1.05 m, $d_w = 0.066$ m and $d_s = 0.15$ m (no well resistance), equation (14) in the initial state $(\overline{U}_{hz} = 0)$ yields i < 10 in 85 % of the total drained cylinder and, at an average degree of consolidation of 50 %, i < 10 in 98 % of the total drained cylinder. If $\Delta h = 8$ m the corresponding figures become 65 % and 91 %, respectively. Thus, if we have to deal with non-Darcian flow with values of i_l about 10, equation (14) can replace equation (6) in most cases, except when the quotient $\Delta h/D$ is very large. In the latter case, one had better apply equation (6).

4. CHOICE OF PARAMETERS

4.1 Equivalent Diameter of Band drains

The first type of wick drains, the so-called cardboard wick, which was invented and introduced on the market by the Swedish Geotechnical Institute, was assumed by Kjellman (1948) to have an equivalent diameter of 50 mm. The author (Hansbo, 1979) showed that the process of consolidation for a circular drain and a wick drain is very nearly the same if the wick drain is assumed to have an equivalent diameter:

$$d_w = 2(b+t)/\pi \tag{16}$$

where b = the width of the drain and t = the thickness of the drain.

According to Atkinson and Eldred (1981), the diameter given by Eq. (9) should be reduced for the effect of convergence of flow lines towards the corners of the wick drain and propose:

$$d_w = (b+t)/2 \tag{17}$$

The n and the s values in the latter case will increase by about 27%. Which of the relations (16) or (17) to be used is an open question, but the difference in result between the two is insignificant in comparison with the influence on the result exerted by the choice of other consolidation parameters to be applied in the design.

4.2 The Zone of Smear

The effect on the consolidation parameters of disturbance caused by the installation of drains, expressed in the terms of zone of smear, depends very much on the method of drain installation, the size and the shape of the mandrel, and the soil structure (Sing & Hattab, 1979; Bergado *et al.*, 1993). Two problems exist: to find the correct diameter value d_s of the zone of smear and to evaluate the effect of smear on the permeability.

The first problem has been the subject of a number of investigations in connection with pile installations which remind of installations by means of closed-end circular mandrels. These investigations indicate that the diameter d_s of the zone of smear can be assumed equal to 2–3 times the diameter d_w of a circular mandrel. Investigations on a laboratory scale (Bergado et al., 1991) indicate that d_s can be put equal to $2d_w$. If the mandrel is non-circular, the diameter d_s should yield an area corresponding to 2–3 times the cross-sectional area of the mandrel, or in the case of band drains, that of the folded anchor utilised for anchoring the drains in the soil.

The other problem, the choice of permeability in the zone of smear, has been treated by several authors. The permeability, of course, will vary in the zone of smear from a minimum nearest to the drain to a maximum at the outer border of the zone. The solution to the problem proposed by the author (Hansbo, 1987a), namely that horizontal layers in the undisturbed soil are turned more or less vertical in the zone of smear, will result in the quotient k_h/k_s being equal to the quotient $k_h/k_v = c_h/c_v$ (see also Bergado *et al.*, 1991).

Onoue et al. (1991) and Madhav et al. (1993) divide the zone of smear into two subzones: an inner, highly disturbed zone", and an outer transition zone in which the disturbance decreases with increaing distance from the drain. Madhav et al. conclude that the author's solution based on axi-symmetric smear conditions and the assumption of only one smear zone is "reasonably accurate for all practical purposes". Chai et al. (1997) use a linear variation of the permeability in the zone of smear on one hand and a bilinear variation on the other and conclude that the assumption of one single average value of permeability will underevaluate the effect of smear.

4.3 Requirements on Band (Wick) Drains

Most of the band drains are made up of a central core with longitudinal channels surrounded by a filter of synthetic material. A first important requirement on these materials is that they must be strong enough to resist the tension and the wear and tear which takes place during drain installation.

Much concern has been devoted to filter criteria. Among the problems mentioned the risk of siltation and the strength of the filters have to be taken into account. The risk of blinding owing to too low a permeability of the filter has almost negligible effect on the consolidation behaviour. The filter and the low-permeable cake of soil particles which may be formed outside the filter and cause so-called blinding will have a fairly small thickness. The consequence of this blinding is easily recognised if the filter is considered as a zone of smear. Assuming, for example, that the filter/filter cake has a thickness of as much as 2 mm (corresponding to $d_s = d_w + 0.004$ m) and that its permeability becomes only 20% of the permeability of the surrounding soil $(k_s/k_h = 0.2)$ the average degree of consolidation, using wick drains with an equivalent diameter $d_w = 0.066$ m, will differ from the ideal case by a maximum of only 2-3%, a negligible difference in result.

The filter material has also been considered important. When the first modern prefabricated wick drain, the Geodrain, was introduced on the market the filter was made of specially prepared paper material. Although the effectiveness of these drains was demonstrated by the results of a large number of drain installations the use of paper as filter material was questioned. The main reason for questioning the use of paper was the risk of filter deterioration caused by fungi or bacteria. This risk has proved by full-scale experiments to be overstated (see Hansbo, 1987). Moreover, there are cases where clogging of the drains would be desireable after that full consolidation under the design load has been attained.

4.4 Well Resistance

Because drains nowadays are frequently installed to great depths well resistance has become a matter of increasing interest. This is understandable since well resistance in such cases can cause a serious delay in the consolidation process. There are several reasons why the discharge capacity of a drain may become low: siltation of the channels in the core of wick drains or of the sand in sand drains; unsatisfactory drain makes with too low a discharge capacity; necking of drains; etc. Back-calculated values of discharge capacity of drains under field conditions have been reported to be quite low for certain makes of band drains without filter (Hansbo, 1986; Chai et al., 1996). However, most of the band drains marketed today have high enough a discharge capacity $(q_w > 150 \, \text{m}^3/\text{year})$ to become negligible in the design (cf. Hansbo, 1994). Moreover, the influence of well resistance decreases with increasing time of consolidation.

Well resistance may be important in the case of small-diameter sand drains. For example, using medium to coarse sand as material in the drains, the permeability can be expected to be $k_w \approx 3000$ m/year (10^{-4} m/sec.). For sandwicks, 0.05 m in diameter, and fabri pack drains, 0.12 m in diameter, this yields $q_w = 6$ m³/year and 34 m³/year respectively.

The effect of well resistance according to equation (6) and equations (12) and (14) is calculated on the assumption that we have to deal with homogeneous soil conditions (k_h and κ_h assumed constant). However, if the soil consists of layers with different characteristics, this can be taken into account in a simple way as suggested by Onoue (1988). The consolidation process is calculated on the assumption that the whole soil profile is homogeneous and has the consolidation properties of each of the respective layers. The distribution of excess pore pressure in the respective layer is extracted and plotted as shown in Fig. 5.

4.5 Correlation between λ and c_h

The ratio of λ to c_h will depend on the hydraulic gradient prevailing in the horizontal direction during the consolidation process. This value can be estimated on the basis of the expression for i, given by equation (7), or more general by equation (13). Since the parameters M and γ_w are independent of the flow conditions, we have $c_h/\lambda = k_h/\kappa_h$. Equalising the areas created by the flow vs. gradient curves in the two cases Darcian and non-Darcian flow, we find the correlation:

$$k_h \approx \frac{2i^{x-1}}{x+1} \kappa_h \text{ when } i \le i_l$$
 (18a)

and

$$k_h \approx \frac{2}{i^2} \left[\frac{i_l^{x+1}}{x+1} + x i_l^{x-1} \left(i - i_l \right) \left(\frac{i - i_l}{2} + \frac{i_l}{x} \right) \right] \kappa_h \text{ when } i \ge i_l$$
 (18b)

Assuming, for example, that the maximum gradients reached during the consolidation process are respectively 2, 5, 15, 25 and 75 and that the exponent x = 1.5 and the limiting gradient $i_l = 8$, we find in due order $\lambda/c_h = \kappa_h/k_h \approx 0.88$, 0.56, 0.34, 0.29 and 0.25. Thus, the higher the value of Δh and the smaller the drain spacing, the lower the ratio of λ to c_h .

Provided x = 1.5 and $i_{max} \le 2.5i_h$, the correlation between λ and c_h can be determined approximately from the relation:

$$\lambda = 1.25c_h / \sqrt{i_{\text{max}}}$$
, where $i_{\text{max}} \approx \frac{\Delta h}{D} \left[\frac{1}{2\alpha} \left(\frac{D}{d_s} - \frac{d_s}{D} \right) \right]^{2/3}$ (19)

5. MONITORING OF VERTICAL DRAIN PROJECTS

Monitoring of vertical drain projects is more or less a must since the consolidation characteristics determined by oedometer tests may be misleading. An early follow-up of the results obtained will form the basis for a correct estimate of the result to be expected, so-called active design.

The monitoring systems utilised for control of vertical drain projects usually consists of vertical settlement meters of various types and of piezometers placed at different depths in the soil. In the case of pilot tests the size of the test area is often limited in relation to the thickness of the soil layer subjected to consolidation. Therefore, in such cases the influence on the vertical settlement of horizontal displacements has to be taken into account. This purpose is usually achieved by the installation of inclinometers along the border of the test area.

Considering the derivation of the consolidation theory, the follow-up of the consolidation process nearest at hand is to check the course of excess pore pressure dissipation. However, the interpretation of the results of pore pressure observations can be quite intricate. Thus, the position of the piezometer in relation to the surrounding drains is not known with any certainty, the piezometer tip will penetrate underlying soil in the course of consolidation which may create additional excess pore pressure, gas evolution may take place in the filter tip which will distort the results, back pressure from the surroundings may influence the readings, etc. Therefore, the most reliable and practical approach to the problem is to study the course of vertical and horizontal movements at various depths in the soil.

The aim of preloading in combination with vertical drainage is usually to eliminate unacceptable settlement under future loading conditions. One must then consider the layer in the soil having the lowest coefficient of consolidation. The preconsolidation pressure in the soil has to be increased up to, or preferably above, the effective stress level induced by the future load. This must be realised in all cases where the consolidation process is followed-up only by settlement observations of the soil surface. These observations may be strongly influenced by vertical consolidation U_{ν} and thus lead to the impression that the acceleration of the consolidation process caused by the drains is faster than in reality. In active design, this can be checked theoretically by inserting the values of c_{ν} and $c_{h}(\lambda)$, found by trial and error, that yield acceptable agreement with the course of surface settlement. Then the degree of consolidation obtained by the aid of the drains can be checked by inserting $c_{\nu} = 0$ and the value of $c_{h}(\lambda)$ found. However, if there are layers with more unfavourable consolidation characteristics than on the whole these layers will be decisive for the interpretation of the results obtained.

5.1 Vertical settlement gauges

The settlement of the ground surface is usually measured by means of settlement gauges consisting of rods welded to rectangular plates placed directly on the ground below the fill constituting the overload. Alternatively, particularly in the case of embankments, hoses are placed below and across the fill and the settlement is measured continuously by means of special hose settlement gauges (see e.g. Bergdahl,1996). The settlement at different depths in the interior of the soil can be measured by gauges consisting of rods welded to earth screws, about 200–250 mm in diameter. The rods should be surrounded by a protecting tube in order to prevent the influence of negative friction along the rod and contribution to settlement caused by compression of the soil above the earth screw.

In normal soil conditions so-called bellows hoses can be used instead of earth screws. These hoses are provided with metal rings, usually with an internal spacing of 1.0 m. The underlying idea is that the contact between the bellows hose and the surrounding soil will make each metallic ring follow the settlement taking place at the respective levels of the rings. The level of each ring is measured by means of a sensor which closes an electric circuit when passing the ring. However, this measuring system has proven not to be trusted in cases of very soft compressible soils.

Still another type of settlement observations at different depths can be carried out by means of earth screws provided with magnetic rings. The earth screws, which are placed at the intended depth in the soil around a guiding plastic tube, follow the settlement at the depth in question. Their position can be measured by a special sensor lowered inside the guiding tube. These settlement gauges are advantageous to the bellows hoses in that they can be used irrespective of soil conditions.

5.2 Pore pressure measurements

Although many sophisticated and reliable piezometers exist on the market today, experience shows that the interpretation of the consolidation process on the basis of pore pressure measurements may be quite intricate. The main difficulty in the case of vertically drained areas consists in the uncertainty about the exact position of the ceramic filter tip of the piezometer in relation to the surrounding drains. Since the distance between the filter tip and the surrounding drains is not known with certainty, the observations can give a misleading conception of the average excess pore water pressure dissipation in the course of consolidation. Another

difficulty arises from the fact that the piezometer tip, owing to frictional forces against the piezometer tube by settlement of overlying soil layers, will be penetrating the underlying soil, thereby creating additional excess pore pressure. Gas evolution in the filter tip may also lead to erroneous results.

Pore pressure measurements carried out for the purpose of studying the consolidation process can be performed with less sophisticated piezometers, such as piezometers with open small-diameter standpipes. There is no need of piezometers with very short time lag since the gradual change of pore pressure during the consolidation process is slow.

5.3 Inclinometers

The placement of an overload on water-saturated clay subsoil will give rise to immediate elastic settlements, caused simply by shear (Poisson's ratio v = 0.5), followed by shear creep deformations. This entails longterm horizontal displacements along the border of the overload which have to be taken into account wherever the width of the load is limited in comparison with the depth of the consolidating clay layer. The influence of horizontal displacements on the interpretation of the consolidation process are visualised in this paper by the results obtained in the Bangkok Test Area.

The horizontal displacements are generally studied by means of inclinometers, consisting of a flexible tube installed more or less vertically through the soil layer. The horizontal movement at a certain depth is obtained by successive observations of the inclination of the tube from the ground surface downwards. Sophisticated equipments exist for the measurement of the inclination (see e.g. Bergdahl, 1996)

6. CASE RECORDS

It is interesting to check by case records whether the theory based on non-Darcian flow gives results in better agreement with real behaviour than the theory based on validity of Darcy's law. As already mentioned, the author in the study of the full-scale tests at Skå-Edeby (Hansbo, 1960) found that the best agreement between theory and practice was obtained by assuming non-Darcian flow with the exponent x = 1.5-1.6. The findings then presented, assuming x = 1.5, i.e. the flow law $v = \kappa i \sqrt{i}$, have been confirmed in later studies (Robertson *et al.*, 1988; Hansbo, 1994; Hansbo, 1997a-b; Hansbo, 1998).

In a case where the monitoring system is based on settlement observations of the soil surface (which is most common) the settlement observed vs. total settlement refers to the average degree of consolidation. Assuming that Darcy's law is valid, the settlement due to one-dimensional vertical consolidation can be obtained utilising the diagrams of average one-dimensional consolidation vs. time factor T_v found in most textbooks on soil mechanics. If, on the other hand, the stress increase varies non-linearly with depth below the ground surface, or if a limited layer at a certain depth is considered, then the effect of one-dimensional vertical consolidation can be calculated by finite difference methods as suggested by Helenelund (see Hansbo, 1994). The effect of one-dimensional consolidation assuming non-Darcian flow can be studied in the paper presented by Dubin and Moulin (1986). The difference in result as compared to the Terzaghi solution will be most pronounced at the end of the consolidation process. Therefore, and in consideration of all the uncertainties involved in the choice of consolidation

parameters, Terzaghi's solution can be applied when judging the contribution in the consolidation process caused by one-dimensional vertical outflow of water. The best thing, of course, is to study the settlement of a dummy area without drains and with similar loading conditions.

In practice, when choosing the consolidation parameters from the results of full-scale tests with vertical drains, the influence on the consolidation process of one-dimensional vertical consolidation can generally be neglected. The contribution to the settlement by one-dimensional consolidation will only be reflected through a higher value of the coefficient of consolidation c_h (or λ) found by trial and error. This type of analysis can be utilised in cases where dummy areas without drains are missing.

5.1 The Skå-Edeby Test Field

The test field arranged at Skå-Edeby, situated some 25 km west of Stockholm, is one of the oldest and best documented test fields throughout the world. It was established by the Swedish Government in 1957 for the purpose of examinig the effectiveness of vertical sand drains in a planned soil improvement project at the Stockholm International Airport to be. The test field comprised originally four test areas. Test Area I, 70 m in diameter, is divided into three equal sectors with drain spacings 0.9 m, 1.5 m and 2.2 m. Test Areas II–IV are 35 m in diameter. The drain spacing in Test Areas II and III is 1.5 m. Test Area IV is undrained. The overload on Test Areas I, II and IV consisted of 1.5 m gravel (27 kN/m²) while the overload on Test Area III was 2.2 m gravel (40 kN/m²) which necessitaded a loading berm of gravel, 0.7 m thick and 12 m wide. Test Area III was unloaded by 0.7 m gravel (12 kN/m²) after 3 1/2 years in order to study whether secondary consolidation could be eliminated by means of preloading. The drains were installed in equilateral triangular pattern by means of a closed-ended mandrel, 0.18 m in outer diameter.

Later on, in the early 1970s, Test Area V, 31 m in diameter, was established in order to study the effectiveness of the then newly invented Geodrain. The drain spacing in this case was 0.9 m. The overload was originally 1.5 m gravel (27 kN/m²) but was increased after 3 1/2 years by 1.1 m gravel (20 kN/m²) in order to examine the longterm effectiveness of the drains.

The soil conditions below the test areas have been described in detail by Hansbo (1960) and can be summarised as follows. Below a 1.5 m thick dry crust, the soil consists of normally consolidated, highly plastic clay to a depth of 9–15 m (average about 12 m). From the results of oedometer tests the following consolidation characteristics were found: coefficients of consolidation $c_{\nu} = 0.17$ m²/year (standard deviation 0.03 m²/year) and $c_h = 0.7$ m²/year (only one test); compression ratio $CR = C_c/(1+e_0) = \varepsilon_2/\log 2 = 0.37$ (standard deviation 0.06). A corrected interpretation of the results of the oedometer tests with regard to disturbance of the soil samples resulted in CR = 0.52 (standard deviation 0.18).

In the analysis of the consolidation process achieved by preloading and installation of vertical drains, one will have to consider the effect of disturbance on the consolidation characteristics caused by drain installation. Thus the coefficient of consolidation c_v will decrease and the compression ratio CR will increase in relation to undisturbed condition. Therefore, the results obtained from dummy areas in undrained conditions will generally lead to an overestimation of the contribution to the settlements observed of one-dimensional consolidation in undrained

condition. In order to reduce the discrepancy between drained and undrained consolidation parameters can be done by inserting the values of s_p into equation (4) obtained in the drained instead of the undrained condition (Hansbo, 1998). Previously, the results obtained in Test Area II at Skå-Edeby were analysed in this way (Hansbo, 1998). The results obtained in Test Areas I, 0.9 and 1.5 m drain spacing, and the results obtained in Test Area V have been analysed previously by the author (Hansbo, 1997b). In this paper the sector in Test Area I with 2.2 m drain spacing will be chosen as an example

On the basis of the compression characteristics of the clay, the primary compression of the layer between depths 2.5 and 7.5 m to be expected in the sector with 2.2 m drain spacing in Test Area I was estimated at 0.5 m and in Test Area IV at 0.47–0.54 m (Hansbo, 1960). A follow-up of the course of settlement according to Asaoka (1978), based on settlement observations at equal time intervals $(s_{i-1}, s_i, s_{i+1},$ etc.) results in the following relation: $s_i = 0.1453 + 0.7882s_{i-1}$ in test area I which yields the primary compression $s_p = 0.69$ m $(s_{i-1} \rightarrow s_i)$ and $s_i = 0.086 + 0.811s_{i-1}$ in test area IV which yields the primary compression $s_p = 0.46$ m. The total primary settlement in test area I is estimated by Asaoka's method to be 1.26 m and in Test Area IV 1.12 m.

The influence of undrained condition on the consolidation process in the drained Test Area I, obtained according to equation (4) from the measurements carried out in the undrained Test Area IV, is shown in Fig. 6, top. The result obtained when the influence of undrained condition is neglected is shown in Fig. 6, bottom.

The theoretical compression of the 5 m thick layer between depths 2.5 and 7.5 m according to equations (6) and (14) is obtained for $\lambda = 0.25$ m²/year and $c_h = 0.35$ m²/year (undrained condition considered according to equation (4)) and for $\lambda =$ 0.4 m²/year and c_h =0.6 m²/year (undrained condition neglected). The surface settlement curves are obtained for $\lambda = 0.3 \,\mathrm{m}^2/\mathrm{year}$ and $c_h = 0.45 \,\mathrm{m}^2/\mathrm{year}$ (undrained condition considered) and for $\lambda = 0.5 \text{ m}^2/\text{year}$ and $c_h = 0.75 \text{ m}^2/\text{year}$ (undrained condition neglected). As can be seen, the difference in result obtained when neglecting the effect of undrained condition is only reflected through the values of coefficient of consolidation found by trial and error to give the best fit to field data. The results obtained from the settlement observations in the undrained area shows that the drainage conditions are considerably more favourable than would be the case according to one-dimensional consolidation theory based on the total thickness of the clay layer. Most probably, pervious layers in the clay deposit and fissures in the dry crust contribute to shortening the drainage paths. As was shown by the author (Hansbo, 1960), escape of pore water in a horizontal outward direction also has an appreciable influence on the settlements observed in this case. Theoretically, assuming one-dimensional consolidation, the course of settlement observed in the undrained area corresponds to a distance between horizontal drainage surfaces of about 6 m whereas the total thickness of the clay layer is about 10-12 m.

An alternative method of determining the course of settlement of the ground surface can be made according to equation (8). Inserting the values l=6 m (total thickness of the clay layer, drained at top and bottom, is about 12 m), $c_{\nu}=0.16$ m²/year and $c_h=4c_{\nu}=0.64$ m²/year, the settlement curve shown in Fig. 7 is obtained. The agreement between theory and observations according to the classical theory has now improved but is not as good as the agreement obtained on the basis of equation (15), inserting $c_{\nu}=0.16$ m²/year and $\lambda=0.45$ m²/year. In Fig. 7, a similar comparison is made between the settlement

curves obtained according to equations (8) and (15) and observations of the ground surface settlement in Test Area II (Hansbo, 1998), with the same loading conditions as in Test Area I but with 1.5 m drain spacing instead of 2.2 m. Inserting in this case l=6 m, $c_{\nu}=0.22$ m²/year, $c_{h}=4c_{\nu}=0.88$ m²/year and $\lambda=0.58$ m²/year, a much better agreement is obtained on the assumption of non-Darcian flow, equation (15), than on the assumption of Darcian flow, equation (8).

Assuming that $i_l = 8$, the values $c_h = 0.64$ m²/year and $\lambda = 0.45$ m²/year for Test Area I with 2.2 m drain spacing, equation (18a) yields a maximum hydraulic gradient of 3.2 during the consolidation process, while the values $c_h = 0.88$ m²/year and $\lambda = 0.58$ m²/year for Test Area II with 1.5 m drain spacing correspond to a maximum hydraulic gradient of 3.6. These values correspond with the maximum values of i ($\overline{U}_h = 0$) at the outer border of the zone of smear in both the Test Areas if $\Delta h = 2.3$ m (Skempton's pore pressure coefficient B = 1 and A = 0.77, cf. Hansbo, 1959).

In practice, the influence of one-dimensional vertical consolidation exerted on the consolidation process at normal drain spacing and thickness of the drained layer is unimportant for the evaluation of the drainage project. It may have an important influence, however, if the drain spacing relatively speaking is large as compared to the thickness of the drained layer.

5.2 The Bangkok Test Field

In connection with the planning of a new international airfieldin Bangkok, Thailand, three test areas were arranged in order to form a basis for the design of soil improvement by preloading in combination with vertical drains. The results of the settlement observations in two of these test areas, TS 1 and TS 3, showed a better agreement with equation (12) than with equation (6) (Hansbo, 1997b). In this paper, the results obtained in test area TS 3 will be examined more detailed than in the reference given. The crest width of TS 3 is 14.8 m (square) and the bottom width 40 m. It is provided with an approximately 10 m wide loading berm, 1.5 m thick. The fill placed on the area amounts to maximum of about 4.2 m, corresponding to a load of about 80 kN/m^2 . Owing to submergence of the fill during the course of settlement the load will be reduced successively by about 9s, where s = settlement of the soil surface.

The drains, type Mebradrain, in the test area were installed in a square pattern with a spacing of 1.0 m which yields D=1.13 m. The equivalent drain diameter determined according to equation () becomes $d_w=0.066$ m. The smear zone is estimated at $d_s=0.20$ m. The permeability ratios k_h/k_s and κ_h/κ_s are assumed equal to the ratio c_h/c_v .

The consolidation characteristics of the clay deposit, determined by oedometer tests, can be summarised as follows: average coefficient above the preconsolidation pressure $c_v = 1.06 \text{ m}^2/\text{year}$ (standard deviation = 0.061 m²/year), $c_h = 1.37 \text{ m}^2/\text{year}$ (standard deviation = 0.050 m²/year), average virgin compression ratio CR = $C_c(1 + e_0) = 0.43$ (standard deviation = 0.07) and an average recompression ratio RR = 0.028 (standard deviation = 0.005). This yields $k_h/k_s = \kappa_h/\kappa_s = 1.3$ (in a paper previously published by the author (Hansbo, 1997b) this ratio was assumed equal to 2). The clay penetrated by the vertical drains is slightly overconsolidated with a preconsolidation pressure about 15–50 kPa higher than

the effective overburden pressure. The clay below the tip of the drains is heavily overconsolidated.

The monitoring system consisted of vertical settlement meters placed on the soil surface and at different depths and of inclinometers to study the horizontal displacements. Unfortunately, the results of the settlement observations at various depths are contradictory and, therefore, only the surface settlement observations can be trusted. The contribution to the vertical settlement of horizontal deformations is analysed on the basis of the inclinometers placed 7.8 m from the centre of the test area. Denoting the area created by horizontal deformation versus depth by A, the vertical settlement s owing to the horizontal deformations is calculated as the mean of the two values 4A/14.8 and $\pi \cdot A/14.8$. The total settlement observed, including the settlement caused by horizontal deformations, and the thus corrected settlement curve, representing merely the effect of consolidation, are shown in Fig. 8. A follow-up of the course of consolidation settlement according to Asaoka's method, based on settlement observations at equal time intervals $(s_{i-1}, s_i, s_{i+1}, \text{ etc.})$ results in the following relation: $s_i = 0.2625 + 0.8195s_{i-1}$ which yields the primary settlement $s_p = 1.45$ $m(s_{i-1} \rightarrow s_i)$.

The settlement analysis based on the λ method is carried out in 4 successive steps: loadstep 1 with load $\Delta q_1 = 20 \text{ kN/m}^2$; loadstep 2 with load $\Delta q_2 = 30 \text{ kN/m}^2$; loadstep 3 with $\Delta q_3 = 10 \text{ kN/m}^2$ and loadstep 4 with $\Delta q_4 = 20 \text{ kN/m}^2$. The primary consolidation settlements caused by these loadsteps determined on the basis of the consolidation characteristics are $\Delta s_1 = 0.15$ m; $\Delta s_2 = 0.6$ m; $\Delta s_3 = 0.2$ m and $\Delta s_4 = 0.5$ m, in total 1.45 m. Now, 50 days after the start point of loading (consolidation time t = 50 - 15 = 35 days; $\Delta h = 2$ m; $\lambda = 0.37$ m²/year) we find \overline{U} = 0.21 which yields s = 0.03 m. In loadstep 2 the load $\Delta q_2 = 30$ kN/m² has to be increased by $0.79 \cdot 20 = 16 \text{ kN/m}^2$ corresponding to $\Delta h_{2,\text{corr}} = 4.6 \text{ m}$ and $\Delta s_{2,\text{corr}}$ = 0.6 + 0.12 = 0.72 m. After 75 days when loadstep 2 is completed we find [t =(75-50)/2 $\overline{U} = 0.12$ which yields $\Delta s = 0.09$ m and s = 0.12 m. After 140 days when loadstep 3 is being applied we have $(t = 12.5 + 65 = 77.5 \text{ days}) \ \overline{U} = 0.50$ from which $\Delta s = 0.36$ m and s = 0.39 m. This yields $\Delta h_{3,corr} = 3.3$ m and $\Delta s_{3,corr}$ = 0.2 + 0.36 = 0.56 m. After 220 days when loadstep 4 is being applied we find $(t = 80 \text{ days}) \ \overline{U} = 0.46 \text{ from which } \Delta s = 0.26 \text{ m} \text{ and } s = 0.26 + 0.39 = 0.65 \text{ m}.$ This yields $\Delta h_{4,\text{corr}} = 1.8 + 2.0 = 3.8 \text{ m}$ and $\Delta s_{4,\text{corr}} = 0.5 + 0.3 = 0.8 \text{ m}$. After 250 days when loadstep 4 is completed we have (t = 15 days) $\overline{U} = 0.13$ which yields $\Delta s =$ 0.10 m and s = 0.75 m. 100, 200 and 400 days later we have $\overline{U} = 0.59$, $\overline{U} = 0.74$ and $\overline{U} = 0.89$ from which $\Delta s = 0.47$ (s = 0.47 + 0.65 = 1.12 m), 0.67 (s = 1.24m) and 0.71 m (s = 1.36 m), respectively.

The theoretical course of settlement determined in the conventional way is less complicated in that the total consolidation curve can be determined for each loadstep separately and added to each other. Assuming $c_h = 0.93 \text{ m}^2/\text{year}$ (in the paper previously mentioned (Hansbo, 1997b) the coefficient c_h was assumed equal to 1.2 m²/year owing to the fact that the ratio k_h/k_s was put equal to 2 instead of 1.3 now applied) we find, to give an example, 400 days after the start of the loading process ($t_1 = 385 \text{ days}$, $\overline{U} = 0.92$; $t_2 = 340 \text{ days}$, $\overline{U} = 0.89$; $t_3 = 260 \text{ days}$, $\overline{U} = 0.82$; $t_4 = 170 \text{ days}$, $\overline{U} = 0.67$) the settlement $s = 0.92 \cdot 0.15 + 0.89 \cdot 0.6 + 0.82 \cdot 0.2 + 0.67 \cdot 0.5 = 1.17 \text{ m}$.

Assuming i_l =10, the values c_h =0.93 m²/year and λ =0.37 m²/year yields according to equation (18a) a predominant hydraulic gradient of about 10 during the consolidation process.

The results obtained by the two methods of analysis are shown in Fig. 8.

5.3 The Vagnhärad Vacuum Test

An interesting full-scale test in which consolidation of the clay was achieved by the vacuum method was reported by Torstensson (1984). The subsoil at the test site consists of postglacial clay to a depth of 3 m and below this of varved glacial clay to a depth of 9 m underlain by silt. The clay is slightly overconsolidated with a preconsolidation pressure about 5–20 kPa higher than the effective overburden pressure. The coefficient of consolidation c_h and the average virgin compression ratio CR according to Torstensson were found equal to 0.95 m²/year and 0.7 (max. 1.0), respectively.

The vacuum area, 12 m square, was first covered by a sand/gravel layer 0.2 m in thickness, and then by a Baracuda membrane which was buried to 1.5 m depth along the border of the test area and sealed by means of a mixture of bentonite and silt. Mebradrains were installed in a square pattern with 1.0 m spacing to a depth of 10 m. The average underpressure achieved by the vaccum pump was -85 kPa. After 67 days the vacuum process was stopped and then resumed after 6 months of rest. From the shape of the settlement curve (Fig. 9) Asaoka's method yields the correlation $s_i = 0.0756 + 0.9075s_{i-1}$ from which $s_p = 0.82$ m. This value is low with regard to the loading conditions and the compression characteristics. The main reason for this seems to be that the applied vacuum effect is not fully achieved in the drains. Thus, the primary settlement 0.82 m corresponds to a vacuum effect of about -35 kPa. Another reason may be that the test area is too small as compared to the thickness of the clay layer.

The theoretical settlement curve in this case has to be determined in two steps, the first one up to a loading time of 67 days leading to a then settlement $s_1 = \overline{U}_{h1}s_p$. In the next loadstep, starting again from the time of resumption of the application of vacuum, the remaining primary settlement is obtained from the relation $\Delta s = \overline{U}_h(s_p - s_1)$. i.e. the settlement $s_t = s_1 + \overline{U}_{ht}(s_p - s_1)$ where t starts from the time of resumption of the application of vacuum.

Introducing the values D = 1.13 m, $d_w = 0.066$ m, $d_s = 0.15$ m, $k_h/k_s = \kappa_h/\kappa_s = 4$ and $\Delta h = 3.5$ m, the best agreement between theory and observations is found for $\lambda = 0.75$ m²/year and $c_h = 2.0$ m²/year (Fig. 9). Even in this case the λ theory agrees better with observations than the classical theory.

Assuming i_l =10, the values c_h =2.0 m²/year and λ =0.75 m²/year yields according to equation (18b) a predominant hydraulic gradient of about 11 during the consolidation process.

5.4 The Arlanda Project

The extension of the international airfield at Arlanda some 30 km north of Stockholm entails, among other things, the construction of a new runway at a site with very bad soil conditions. The soil at the site consists of up to 5 m peat underlain by high-plasticity, very soft normally consolidated clay with a maximum thickness of about 10 m. Preloading has been undertaken both in undrained

condition and in combination with vertical drain installations, the latter wherever the thickness of the clay layer exceeds 5 m. The consolidation process is monitored by settlement and pore pressure observations. Mebradrains were installed in equilateral triangular pattern with a drain spacing of 0.9 m. The core utilised in the drains is equal to the core once used only in Geodrains (see Hansbo, 1979, 1981). Two cases of observations will be presented: one (site K) where the overload consists of 19.5 m sand and gravel ($\Delta q = 390 \text{ kN/m}^2$) and the other (site L) where the overload consists of 16.2 m sand and gravel ($\Delta q = 325 \text{ kN/m}^2$). These loads are about 10 times higher than the overloads applied at Skå-Edeby.

The soil consists of clay with silt and sand seams, at site K to a depth of 9.7 m (with a sand layer from 1.6 to 1.8 m) and at site L to a depth of 7.8 m. The undrained shear strength of the clay is fairly constant, about 5–10 kPa, irrespective of depth. In order to cope with the influence on the course of settlement of time-consuming stepwise placement of the load, each break in the rate of loading has been analysed separately on the basis of a direct use of the oedometer curves (which was deemed more reliable than an evaluation via the compression ratio CR, or the oedometer modulus M). At site K the following settlement values Δs were obtained: loadstep $0-80 \text{ kN/m}^2 \Delta s_1 = 1.75 \text{ m}$; loadstep $80-215 \text{ kN/m}^2 \Delta s_2 = 0.50 \text{ m}$; loadstep 215-390 kN/m² $\Delta s_1 = 0.30$ m (total primary settlement 2.55 m). At site L the following settlement values Δs were obtained: loadstep 0–80 kN/m² Δs_1 = 0.95 m; loadstep 80–325 kN/m² Δs_2 = 0.55 m (total primary settlement 1.5 m). These settlement values have been checked and adjusted in accordance to Asaoka's method, based om settlement observations at equal time intervals (s_{i-} s_i , s_{i+1} , etc.). This results in the following relations: $s_i = 0.5265 + 0.7948s_{i-1}$ at site K which yields the primary settlement $s_p = 2.57$ m and $s_i = 0.3487 + 0.7669$ s_{i-} ₁ at site L which yields the primary settlement $s_p = 1.50$ m. The coefficient of consolidation according to the oedometer tests varies from about 0.2–0.3 m²/year at preconsolidation pressure to about 0.5–1.0 m²/year (maximum 2.5 m²/year) at the end of primary consolidation under the applied overload.

The loading conditions and the settlement observations in the two cases are shown in Fig. 8. Assuming $d_w = 0.066$ m and $d_s = 0.15$ m and $k_h/k_s = \kappa_h/\kappa_s = 2$, the best fit between theory and observations is obtained for $\lambda = 0.5$ m²/year and $c_h = 2$ m²/year.

In a case like this where a great deal of the consolidation process takes place during the loading period the analysis of the consolidation process according to equation (14) is carried out in the following way. The degree of consolidation U_1 , inserting $\Delta h_1 = \Delta q_1/\gamma_w$, determines the course of settlement in the first loadstep. The value of Δh to be inserted in the next loadstep is chosen as $\Delta h_2 = \left(1 - \overline{U}_1\right)\Delta q_1/\gamma_w + \Delta q_2/\gamma_w$, and so on. The settlement curves are then adjusted for the rate of loading according to the well-known graphical procedure suggested by Terzaghi.

As can be seen the correlation between theory and observations in this case where the hydraulic gradient is considerably higher than the limiting i_l value is equally good, or even better, according to the classical theory, equation (6), than according to equation (14).

CONCLUSIONS

Permeability tests on clay have indicated that there can be a deviation from Darcy's flow law at small hydraulic gradients. The consolidation theory developed accordingly yields a better agreement with case records than the classical consolidation theory based on validity of Darcy's flow law, except in cases where the hydraulic gradients created by the overload are excessive in relation to the limiting gradients where the flow increases linearly with increasing hydraulic gradient.

In a vertical drain project, the effect on the consolidation process of onedimensional consolidation in undrained condition is relatively difficult to predict but can generally be neglected. Its contribution can be included by an increase in the coefficient of consolidation to be applied in the vertical drain analysis.

The vertical drain analysis based on a curved correlation between flow and hydraulic gradient is general and can be utilised also when the correlation is linear. Thus the correlation deduced can replace the classical consolidation theory for vertical drains.

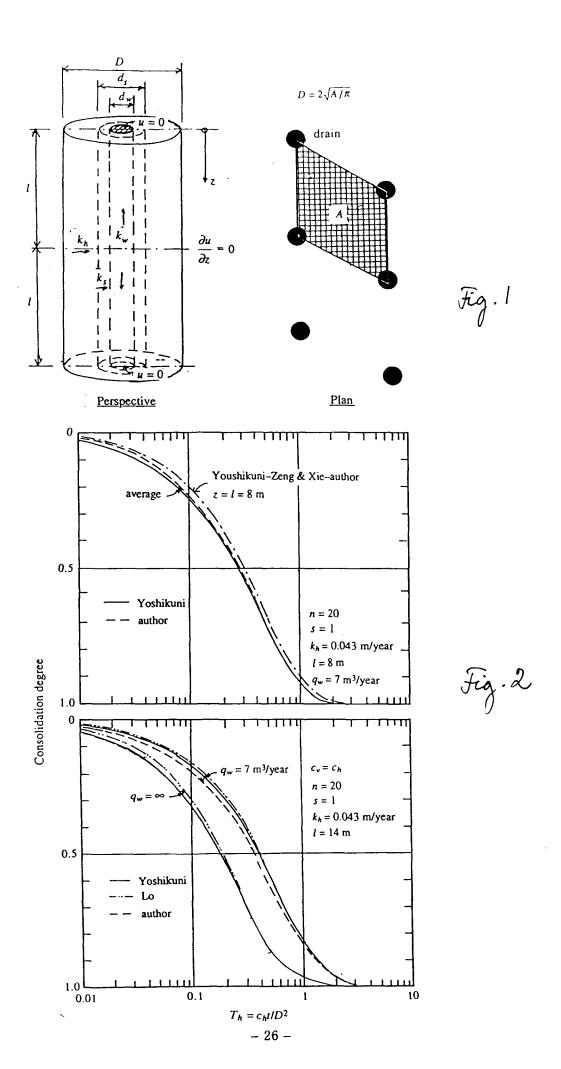
REFERENCES

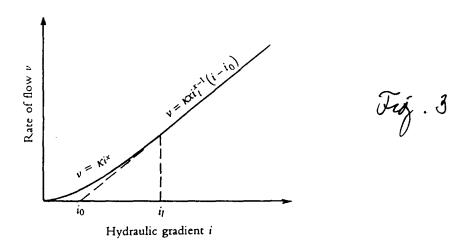
- Asaoka, A. (1978). Observational procedure of settlement prediction. *Soils and Foundations*, Jap. Soc. Soil Mech. Found. Eng., Vol. 18, No. 4, pp. 87–101.
- Atkinson, M. S. & Eldred, P. J. L. (1981). Consolidation of soil using vertical drains. *Géotechnique*, Vol. 31, No. 1, pp. 33–43.
- Barron, R. A. (1944). The influence of drain wells on the consolidation of fine-grained soils. Diss., Providence, U S Eng. Office.
- Barron, R. A. (1948). Consolidation of fine-grained soils by drain wells. *Transactions ASCE*, Vol. 113, Paper No. 2346, pp. 718–742.
- Bergado, D. T., Akasami, H., Alfaro, M. C. & Balasubramaniam, A. S. (1992). Smear effects of vertical drains on soft Bangkok clay. *J. Geot. Eng.*, Vol. 117, No. 10, pp. 1509–1530.
- Bergado, D. T., Alfaro, M. C. & Balasubramaniam, A.S. (1993). Improvement of soft Bangkok clay using vertical drains. *Geotextiles and Geomembranes* 12, Elsevier Science Publishers Ltd., pp. 615–663.
- Bergdahl, U. (1996). Site investigations. In Hartlén, J. & Wolski, W. (Editors). *Embankments on organic soils*. Elsevier, Developments in Geotechnical Engineering, 80, Ch. 2, pp. 31–82.
- Carillo, N. (1942). Simple two and three dimensional cases in the theory of consolidation of soils. *J. Math. Phys.*, Vol. 21, No. 1.
- Chai, J., Bergado, D. T., Miura, N. & Sakajo, S. (1996). Back calculated field effect of vertical drain. 2nd Int. Conf. on Soft Soil Engineering, Nanjing, pp.270–275.
- Chai, J. C., Miura, N. & Sakajo, S. (1997). A theoretical study on smear effect around vertical drain. *Proc. 14th Int. Conf. Soil Mech. Found. Eng.*, Hamburg, Vol. 3, pp. 1581–1584.
- Chang, Y. C. E. (1981). Long-term consolidation beneath the test fills at Väsby. Swedish Geotechnical Institute, Report No. 13.
- Dubin, B. & Moulin, G. (1986). Influence of critical gradient on the consolidation of clay. In: Yong/Townsend (Editors). *Consolidation of Soils. Testing and Evaluation. ASTM* STP 892, pp. 354–377.
- Glover, R. E. (1930). Technical Memorandum No. 158, *Bureau of Reclamation*, Denver, Colorado.
- Hansbo, S. (1960). Consolidation of clay, with special reference to the influence

- of vertical sand drains. Diss. Chalmers Univ. of Technology, Swedish Geotechnical Institute, Proc. No. 18.
- Hansbo, S. (1979). Consolidation of clay by band-shaped prefabricated drains. *Ground Engineering*, Vol. 12, No. 5, pp. 16–25.
- Hansbo, S. (1981). Consolidation of fine-grained soils by prefabricated drains. *Proc. 10th Int. Conf. Soil Mech. Found. Eng.*, Stockholm, Vol. 3, Paper 12/22, pp. 677–682.
- Hansbo, S. (1983). How to evaluate the properties of prefabricated drains. *Proc.* 8th Eur. Conf. Soil Mech. Found. Eng., Helsinki, Vol. 2, Paper 6.13, pp. 621–626.
- Hansbo, S. (1986). Preconsolidation of soft compressible subsoil by the use of prefabricated vertical drains. *Tijdschrift der openbare werken van België*, No. 6, pp. 553–562.
- Hansbo, S. (1987a). Design aspects of vertical drain and lime column installations. *Proc. 9th South East Asian Geot. Conf.*, Bangkok, Vol. II.
- Hansbo, S. (1987b). Fact and fiction in the field of vertical drainage. In: R. C Joshi & F. J. Griffiths (Editors). *Prediction and Performance in Geotechnical Engineering*, pp. 61–72.
- Hansbo, S. (1993). Band drains. In: Moseley, M. P. (Editor). *Ground Improvement*. Blackie Academic & Professional, CRC Press, Inc.
- Hansbo, S. (1994). Foundation Engineering. Elsevier Science B. V., Developments in Geotechnical Engineering, 75.
- Hansbo, S. (1997a). Practical aspects of vertical drain design. *Proc. 14th Int. Conf. Soil Mech. Found. Eng.*, Hamburg, Vol. 3, pp. 1749–1752.
- Hansbo, S. (1997b). Aspects of vertical drain design—Darcian or non-Darcian flow. *Géotechnique* 47, No. 5, pp. 983–992.
- Hansbo, S. (1998). Design of vertical drain installations—A comparison of real behaviour and theory. *Canadian Getot. J.*
- Johnson, S. J. (1970). Foundation precompression with vertical sand drains. J. Soil Mech. Found. Div., ASCE, Vol. 96, SM 1, pp. 145–175.
- Kjellman, W. (1948). Consolidation of fine-grained soils by drain wells. *Trans. ASCE*, Vol. 113, pp. 748–751, (Contribution to the discussion on Paper 2346).
- Lee, S. L., Karunaratne, G. P., Aziz, M. A. & Inoue, T. (1995). An environmentally friendly prefabricated vertical drain for soil improvement. *Proc. Bengt Broms Symposium on Geotechnical Engineering*, Singapore, pp. 243–261.
- Lo, D. O. K. (1991). Soil improvement by vertical drains. Diss., Univ. of Illinois at Urbana-Champaign.
- Madhav, M. R., Park, Y.-M. & Miura, N. (1993). Modelling and study of smear zones around band shaped drains. *Soils and Foundations*, Vol. 33, No. 4, pp. 133–147.
- Magnan, J. P. (1983). Théorie et pratique des drains verticaux. Lavoisier Tec & Doc, Paris.
- Onoue, A. (1988a). Consolidation of multilayered anisotropic soils by vertical drains with well resistance. *Soils and Foundations*, Jap. Soc. Soil Mech. Found. Eng., Vol. 28, No. 3, pp. 75–90.
- Onoue, A. (1988b). Consolidation by vertical drains taking well resistance and smear into consideration. *Soils and Foundations*, Jap. Soc. Soil Mech. Found. Eng., Vol. 28, No. 4, pp. 165–174.
- Onoue, A. (?). Finite difference analysis of consolidation by vertical drains with well resistance. *Proc. 6th ICONMIG*
- Porter, O. J. (1936). Studies of fill construction over mud flats including a description of experimental construction using vertical sand drains to

- hasten stabilization. *Proc. 1st Int. Conf. Soil Mech. Found. Eng.*, Vol. I, Cambridge, USA, Paper L1, pp. 229–235.
- Robertson, P. K., Campanella, R. G., Brown, P. T. & Robinson, K. E. (1988). Prediction of wick drain performance using piezocone data. *Canadian Geotech. J.* 25, pp. 56-61.
- Silverberg, L. (1947). In Statens Geotekniska Institut, 1949. Redogörelse för Statens geotekniska instituts verksamhet under åren 1944–1948. (Report on the activities of the Swedish Geotechnical Institute during the years 1944–1948). Swedish Geotechnical Institute, Meddelande No. 2.
- Sing & Hattab (1979). Sand drains. Civil Engineering, June, pp. 65–67.
- Taylor, D. W. & Merchant, W. (1940). A theory of clay consolidation accounting for secondary compression. J. Mathematics and Physics, No. 1.
- Terzaghi, K. (1923). Die Berechnung der Durchlässigkeitsziffer des Tones aus dem Verlauf der hydrodynamischen Spannungsberechnungen. *Akad. Wissensch.*, *Wien, Sitzungsber.*, Bd. 132, H 1, Mat.-Naturwissensch. Klasse.
- Terzaghi, K. (1925). Erdbaumechanik. Leipzig u. Wien.
- Terzaghi, K. & Peck, R. (1948). Soil Mechanics in Engineering Practice. John Wiley and Sons, Inc.
- Torstensson, B.-A. (1984). Konsolidaering av lera medelst vakuummetoden och/ eller konstgjord grundvattensänkning i kombination med vertikaldränering. Resultat av fullskaleförsök i Vagnhärad. (Consolidation of clay by means of the vacuum method and/or artificial groundwater lowering in combination with vertical drainage. Results of full-scale tests at Vagnhärad). *Internal Report*.
- Yoshikuni, H. & Nakanado, H. (1974). Consolidation of soils by vertical drain wells with finite permeability. *Soils and Foundations*, Vol. 14, No. 2, pp. 35-45
- Yoshikuni, H., 1979. Design and control of construction in the vertical drain methods, Doctoral Thesis, Gihoudi, Tokyo. (In Japanese).
- Yoshikuni, H. (1992). Basic consolidation theory of vertical drain method. Faculty of Engineering, Hiroshima Unviersity.
- Zeng, G. X. & Xie, K. H. (1989). New development of the vertical drain theories. Proc. 12th Int. Conf. Soil Mech. Found. Eng., Rio de Janeiro, Vol. 2, Paper 18/28, pp. 1435–1438.
- Zou, Y. (1996). A non-linear permeability relation depending on the activation energy of pore liquid. *Géotechnique* 46, No. 4, pp. 769–774.
- Fig. 1. Terms used in the analysis of vertical drains: D = diameter of soil cylinder dewatered by a drain, $d_s =$ diameter of the zone of smear, $d_w =$ drain diameter, z = depth coordinate, l = length of drain when closed at bottom (2l = length of drain when open at bottom), $q_w =$ specific discharge capacity of the drain (vertical hydraulic gradient inside the drain i = 1).
- Fig. 2. Top: Example showing the effect of well resistance on radial drainage (average and for z = l) according to Yoshikuni (L = 1), Zeng & Xie, equation (9), and the author, equation (6). Bottom: Example showing the result of combined vertical and radial drainage without and with well resistance according to Yoshikuni (L = 0 and L = 3), Lo (ILLICON) and the author, equation (8).
- Fig. 3. Observed deviation from Darcy's law at low hydraulic gradients in clay.

- Fig. 4. Example showing the effect of smear and well resistance according to equation (14) for n = 20, z = l = 10 m, $\kappa_h = 0.03$ m/year, $\kappa_h/\kappa_s = 4$ and $\Delta h = 4$ m.
- Fig. 5. Consolidation of multi-layered anisotropic soil by vertical drains with well resistance. Simplified solution according to Onoue.
- Fig. 6. Results of settlement observations at Skå-Edeby, Sweden. Test Area I: 2.2 mdrain spacing (D=2.31 m). Top: observations corrected with regard to influence of undrained contribution to settlement (compression) according to equation (4). Bottom: uncorrected observations. EOP=end of primary consolidation settlement (compression) estimated according to Asaoka's method. Full lines: analytical results according to equation (14). Broken lines: analytical results according to equation (6). Horizontal displacements negligible (cf. Hansbo, 1960). Settlement corrected with regard to immediate elastic settlement.
- Fig. 7. Settlement of ground surface obtained at Skå-Edeby in Test Area I: 2.2 m drain spacing (D = 2.31 m) and in Test Area II: 1.5 m drain spacing (D = 1.58 m) EOP = end of primary consolidation settlement estimated according to Asaoka's method. Full lines: analytical results according to equation (15). Broken lines: analytical results according to equation (8).
- Fig. 8. Settlement of ground surface in the Bangkok test field, Thailand. Test area TS 3: 1.0 m drain spacing (D = 1.13 m). Settlement corrected with regard to immediate and longterm horizontal displacements. EOP = end of primary consolidation settlement estimated according to Asaoka's method. Full lines: analytical results according to equation (14). Broken lines: analytical results according to equation (6).
- Fig. 9. Results of settlement observations at Vagnhärad, Sweden. Consolidation by vacuum. 1.0 m drain spacing (D = 1.13 m). EOP = end of primary consolidation settlement estimated according to Asaoka's method. Full lines: analytical results according to equation (14). Broken lines: analytical results according to equation (6).
- Fig. 10. Results obtained at Arlanda Airport, sites K and L. Full lines: analytical results according to equation (14). Broken lines: analytical results according to equation (6).





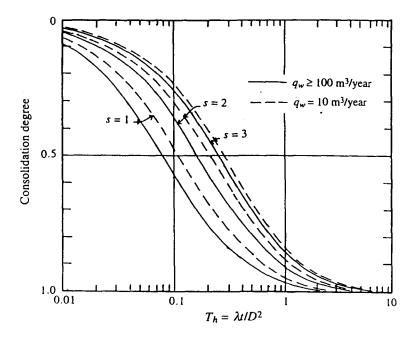
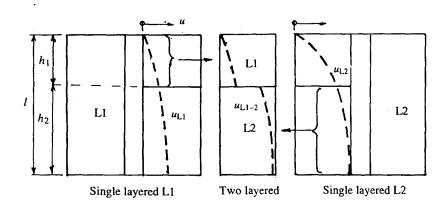
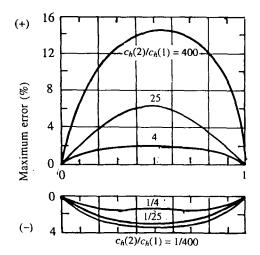
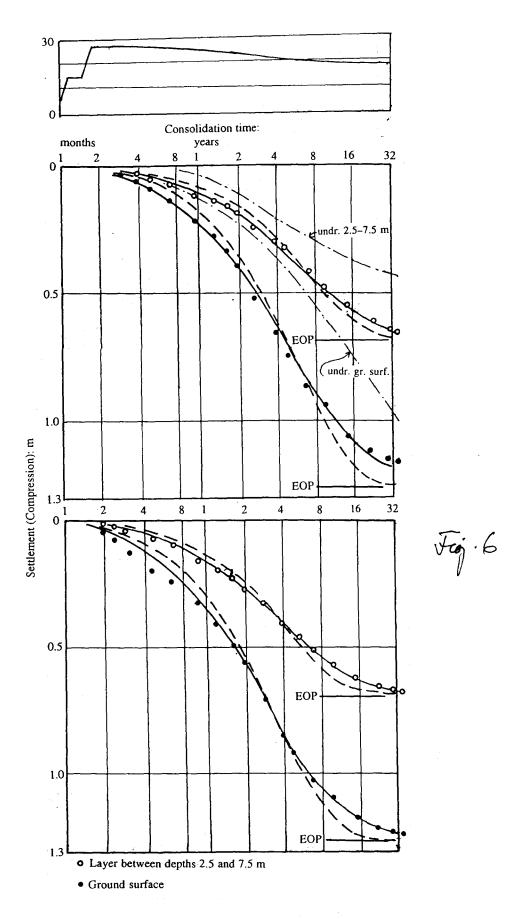


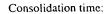
Fig. 4





Tig. 5





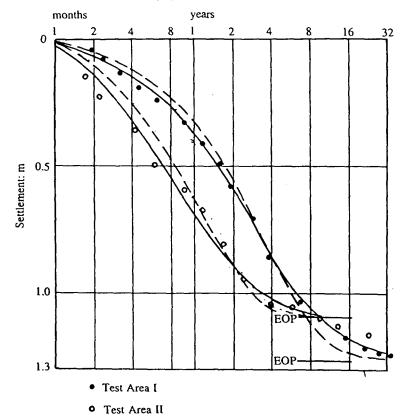
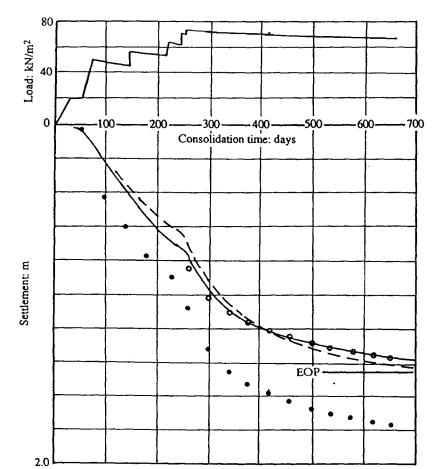
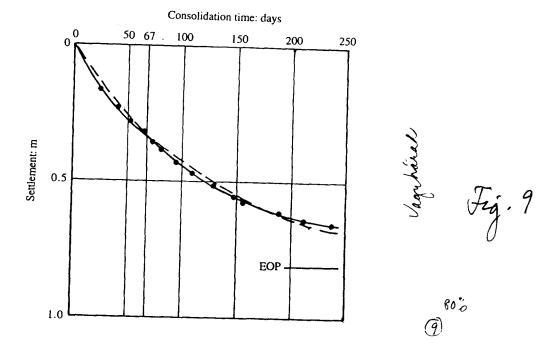


Fig. 7



Jig. 8

Total settlementConsolidation settlement



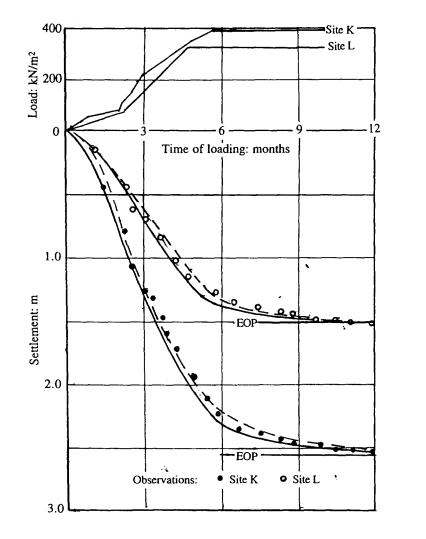


Fig. 10