A New Image Coding Technique with Low Entropy

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Abstract We introduce a new zerotree scheme that effectively exploits the inter-scale self-similarities found in the octave decomposition by a wavelet transform. A zerotree is useful to efficiently code wavelet coefficients and its efficiency was proved by Shapiro's EZW. In the coding scheme, wavelet coefficients are symbolized and entropy-coded for more compression. The entropy per symbol is determined from the produced symbols and the final coded size is calculated by multiplying the entropy and the total number of symbols. In this paper, we analyze produced symbols from the EZW and discuss the entropy per symbol. Since the entropy depends on the produced symbols, we modify the procedure of symbolic streaming out for the purpose. First, we extend the relation between a parent and children used in the EZW to raise a probability that a significant parent has significant children. The proposed relation is flexibly extended according to the fact that a significant coefficient is highly addressed to have significant coefficients in its neighborhood. The extension way is reasonable because an image is decomposed by convolutions with a wavelet filter and thus neighboring coefficients are not independent with each other.

1 Introduction

In the wavelet-based zerotree image coding[1-4], dependencies among subbands were exploited in EZW (Embedded Zerotree Wavelet)[1] by using quadtrees. The coder is implemented by two procedures: symbol generation (model transformation) and entropy coding of the symbol stream. The EZW owes its performance to zerotrees that efficiently represent many insignificant coefficients. Zerotreeing consists of two passes: a dominant pass and a subordinate pass. Since most of budget is spent for the dominant pass, we focus our attention to this. The symbol stream generated by the dominant pass informs a decoder of the significance map that is described by four symbols. A ZTR symbol is used for a zerotree root that is insignificant and has no significant descendants. An isolated zero symbol (named IZ) is generated, when a coefficient is insignificant but has some significant descendants. Besides these symbols, two symbols are used for significant coefficients; POS and NEG according to their sign. After all, the use of ZTR and IZ symbols is to inform locations of significant coefficients. A symbol stream is produced by combining these four symbols and we modify the symbol stream generation for more compression. Since the entropy per symbol is determined by the probabilities of produced symbols, we thus modify the procedure of the symbol generation with flexible treeing. The tree is flexibly designed in view of entropy. In the EZW scheme, a node on a tree branches out into four nodes and this relation is referred to as a fixed relation in the sense that the relation is not changed. On the other hand, our proposed relation is referred to as a "flexible tree" in the sense that a node on a tree branches into basic four nodes and flexibly extends its branches to nodes in neighbor. The idea to the flexible tree comes from how to extend more branches.

2 Zerotree Based Compression

2.1 Embedded Zerotree Wavelet coding

Jerome M. Shapiro [1] developed an algorithm that exploits a relation between subbands in image compression. In the algorithm, zerotrees have been combined with bit plane coding and demonstrate the effectiveness of wavelet based coding. The algorithm is based on the zerotrees that efficiently represent many insignificant coefficients. As wavelet coefficients are located having some dependencies in bands, the dependencies are well exploited with a quadtree structure. The compression has three step procedures; 1) wavelet decomposition 2) symbol generation 3) entropy coding. We briefly review the coding algorithm and discuss produced symbols and its entropy. To describe the compression scheme, we quote several definitions - like parent, child, ancestor, descendant, root etc. - from the reference [1].

There are two types of passes performed: 1) a dominant pass 2) and a subordinate pass. The dominant pass finds significant coefficients to a given threshold, and the subordinate pass refines all significant coefficients found in all previous dominant passes. We use four symbols to tell a dominant pass to a decoder. A ZTR symbol is used for a zerotree root that is insignificant and has no significant descendants. One more needed symbol is an Isolated Zero symbol (named IZ) used when a coefficient is insignificant but has some significant descendants. Besides the symbols, two symbols are used for a significant coefficient – POS and NEG according to its sign. After all, the use of ZTR and IZ symbols is to inform locations of significant coefficients (POS and NEG) as efficiently as possible.

After a dominant pass, a subordinate pass is performed in order to refine the coefficients found to be significant in the previous dominant passes and these two passes are entropy-coded with an adaptive arithmetic coder[5].

2.2 The Shannon's entropy

As we reviewed in the previous section, a symbol stream is produced from the alternate passes and then the stream is entropy-coded for more compression. In this sub section, we briefly study the Shannon's entropy theorem to analyze the symbol stream.

Let $S = \{s_1, s_2, \ldots, s_n\}$ be a set of n symbols. Given $D = \{d_1, d_2, \ldots, d_l\}$, a data set of l symbols in a sequence (the number l is also called the data length of D), the probability distribution of the symbol set S in the data D is the collection of positive numbers $P = \{p_1, p_2, \ldots, p_n\}$, one for each symbol, defined by

$$p_i = |\{d_k \in D \mid d_k = s_i\}| / l, \quad for\ i = 1,2,\cdots,n. \quad (1)$$

If the probability distribution is the only assumed redundancy information, the pair (S,P) is called a zero-order Markov source. The data sequence D is called a zero-order Markov sequence.

Using the above notations, the (zero-order) entropy of the data sequence D is defined to be

$$e(D) = -\sum p_i \cdot \log_2 p_i \qquad (2)$$

3 A flexible relation

As reviewed in the previous section, a dominant pass in the EZW tells where significant coefficients with respect to a given threshold exist and which signs they have. In the pass, we use four symbols – ZTR, POS, NEG and IZ – to inform the locations and signs. Once an image is decomposed using a wavelet, the number of significant coefficients is decided. Therefore, it is the number of ZTR and IZ to decide length of a symbol stream and its entropy. We now consider the occurrence of these symbols. While a ZTR is produced when a coefficient and its descendants are insignificant, an IZ is produced when a coefficient is insignificant but some of descendants are significant.

We first suggest a solution to decrease IZ symbols. An insignificant coefficient is coded as an IZ when some descendants are significant. In other words, the occurrence of IZ is caused from one reason that the significant descendants belong to the insignificant ancestor. Therefore, a simple solution is to suppress the occurrence; let the significant descendants belong to a significant ancestor. It is only possible that the descendants have a power to select their ancestors. In the EZW, the relation does not allow such a selection and always maintains one parent to four children; this is referred as a fixed relation. The relation can be modified with another form so that some children can select their parent. Selecting a parent means that there should be several candidates for the parent and we can imagine that a modified relation must have an overlapped form. It can be made in various forms. One of them is suggested in figure 1 (a). Four parents are displayed in the parent level having their own shapes. These shapes help to understand the relations between parents and their children. Each parent has nine children respectively and some children are shared by several candidates to be their parent; that is, a child can belong to one or more parents. In other words, we can scan a child after a parent among all candidates. This is referred as a modified relation; one parent–nine children.

We give a simple example to explain the modified relation. Assume that there are two parents at parent level – one is significant and the other is insignificant – and six significant children (named as C1 to C6) in child level as shown in the figure 1 (b) and (c). Our goal is to find all significant coefficients according to the scanning order that we do not scan any children before any parents. We will use two relations to find them; the fixed and the modified relations. We have seven coefficients – one parent and six children – to be found as significant coefficients. We first find them with the fixed relation as shown in figure 1 (b). The significant parent P2 has four significant children and they are scanned under P2. However, the insignificant parent P1 has also two significant children; thus the parent should be symbolized with an IZ symbol to find these two significant children. Therefore, we make a symbol stream in this case – IS (at parent level) SZSZ, SSSS (at child level); where S,I,Z mean a significant coefficient, an isolated zero and a zerotree root respectively. The stream has seven S, three Z and one I symbols. On the other hand, when the modified relation is applied to the example as shown in figure 1 (c), all significant children belong to one significant parent and thus we need no IZ symbol. In this case, the symbol stream is output as ZS (at parent level) ZZZSSSSSS (at child level); seven S and four Z symbols. As was shown in the above explanation, two symbol streams were obtained for the same example by using two different relations. We knew that a relation between a parent and its children plays an important role in producing a symbol stream. According to specific relations, the kind and the number of produced symbols are different and thus the resulting entropy is different. In the cases of (b) and (c), entropies are 1.157 bit/symbol and 0.946 bit/symbol respectively. Their entropy coded sizes are 11.57 bits and 10.41 bits. Comparing those two streams, we conclude that one I symbol in the case (b) was replaced with two Z symbols in the case (c). After all, when we change a relation with another, an important thing is how many Z symbols are increased instead of decreasing I symbols. The ratio between the increase and decrease will be an important factor for an entropy coding.

To decrease the ratio, we again change the modified 1-9 relation with a flexible relation. In the previous example, the 1-9 relation was more efficient than the fixed relation as no use of I symbols. However, that is only the special example to explain a relation between I and Z symbols in numbers. If the P1 were also significant in the example, the symbol stream of the case (b) would not have included any I symbol and thus only two Z symbols are needed for the case. This means that the modified relation is not always better than the fixed relation is. Therefore, we need a general relation to compromise these two relations.

We extended the one-to-four parent-child relation with one-to-nine in order to reduce the occurrence of IZ symbols. However, the extension results in the increase of ZTR symbols. The increased ZTRs are reduced by using a flexible relationship defined in figure 2. The flexible extension is designed by a rule such that a parent has four original children and the extended children are determined from the significance of the original children 2,3 and 4. For example, when the child 2 is significant, extension covers additional two children, 5 and 6; in this case, the parent has six children 1 to 6.

Back to the previous example, we apply the flexible relation. In the example, the parent P1 is insignificant and has no significant child among its original children; thus the P1 can be symbolized as a ZTR. However, the P2 is significant and has two significant children C3 and C4 among its original children. Therefore, the C7 and C8 are extended from the original child C3 and the C6 and C9 are from C4. Note that we do not need to scan a child twice. The parent P2 has eight children except C5 among nine children. The resulting stream is ZS (at parent level) ZZSS (from the original children of P2) SS (from C3) SS (from C4); seven S and three Z symbols are included. As we can see from the stream, the flexible relation enables to decrease one more Z symbol than the modified relation.

4. Experimental Results

Our flexible tree is designed to reduce the number of IZ symbols and thus let the entropy lower. The decreased IZ symbols induce some increase of ZTR symbols in numbers by defining an extended relation. The ratio between the decrease and increase is efficiently exploited with the flexible treeing. We use two standard images – Lena and Barbara (512 X 512 with a grey scaled level) – from the RPI site, ftp://ipl.rpi.edu/pub/image/still/usc. Our all results are based on 6-scaled octave wavelet transform and we use the 9/7 filter of [6] and mirror extensions at boundaries. Experimentally, the performances are compared with the published results at the reference [1] and they are plotted in figure 3 for Lena image. The performances in PSNR are calculated over the ranges from 256 to 32768 Bytes. Our flexible coder shows 0.2~1.0 dB better performances than the EZW coder. The improvements are based on the symbol replacements by which we use a frequent symbol (ZTR) instead of infrequent symbol (IZ). We know the replacements are well accomplished with the flexible relation as shown in the performance curves.

In addition, we compare the number of symbols between the EZW and our coder. To give an exact comparison, we stop to code right after a threshold becomes 16; that is, the coding is terminated when the dominant and subordinate passes are all coded with respect to the threshold 32. The same condition is applied to the Barbara image and the results are given in table IV-1 (b). As we can see in the table, the numbers of POS and NEG symbols are the same but the compressed sizes are different. In the flexible relation, some IZ symbols are disappeared instead of some increase of ZTR symbols. As we can see in the table 1, we get different symbol streams from those two coders respectively. Comparing with the number of produced symbols in the EZW, our coder produce 2540 less IZ symbols and 8523 more ZTR symbols for the Barbara image. Therefore, the ratio can be calculated by dividing the increase in ZTR by the decrease in IZ; 8523 / 2540 ≒ 3.4. The value 3.4 can be interpreted that one IZ symbol was replaced with 3.4 ZTR symbols. The decreased IZ symbols play a part in lowering entropy and therefore the image can be compressed with a smaller size. We can also calculate each entropy for symbol distribution respectively; 1.274 bits/sym. and 1.138 bits/sym. for the EZW and the proposed coder. For the Lena image, 0.932 and 0.842 bits/symbol are gained respectively and the ratio is 12.2 (4853 / 398). The ratios 3.4 and 12.2 are only obtained after coding but we can use them as criteria for better coding efficiency. By using the low entropy, we can compress more compactly, though total number of symbols is increased.

5. Conclusions

We described a new relation that a parent takes its children with a flexible method. We extend the fixed relation used in the EZW scheme in order to decrease entropy per symbol. The ways to lower the entropy are accomplished by using more symbols that are frequent and less symbols that are infrequent. The infrequent symbol is IZ in the EZW and we can avoid the use of the symbol by extending the relation of parent-child in more largely but compactly; a parent basically has four children and takes more children by the proposed extension rule. With the extended relation, the number of IZ symbol is decreased and the number of ZTR symbols according to the decreased IZs is also decreased. In our simulations, we analyzed the numbers of produced symbols and the entropy per symbol. As the result, we obtained a ratio between increase and decrease of symbols. The ratio is only obtained when a coding has done but it can be considered as criteria in finding the best extension way.

We showed that a symbol stream is coded with less entropy using the flexible relation in parent-child. Experimentally, our flexible coder has 0.2 ~ 1.0 dB better performances than the EZW's. We suppose that the flexible coder can be improved by a more efficient relation in parent-child.

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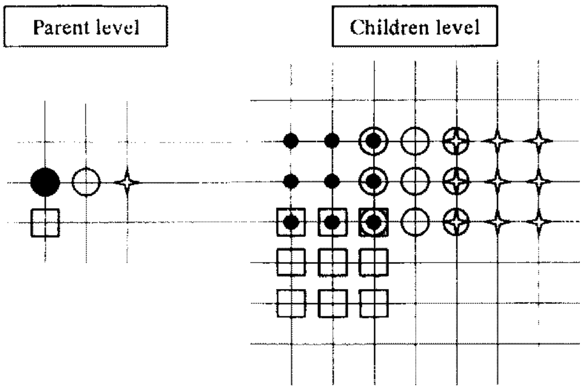
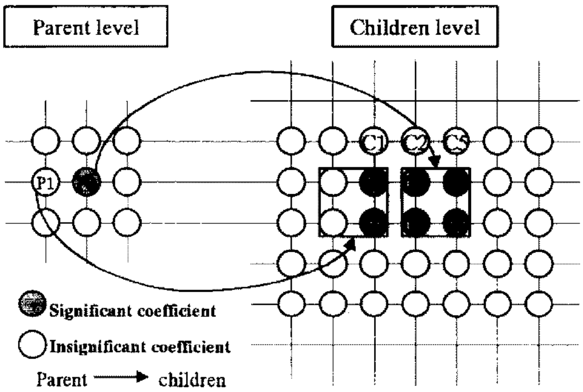
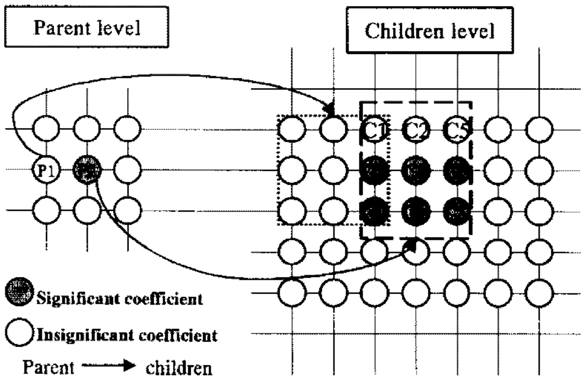


(a) A relation between one parent and its nine children; where four shapes are presented for the purpose of illustrating relations between parents and their children



(b) One parent and four childen; the fixed relation used in EZW.



(c) One parent and nine children; an overlapped relation modified from the fixed relation.

Figure 1. An example to explain a difference between the 1-4 relation and 1-9 relation.

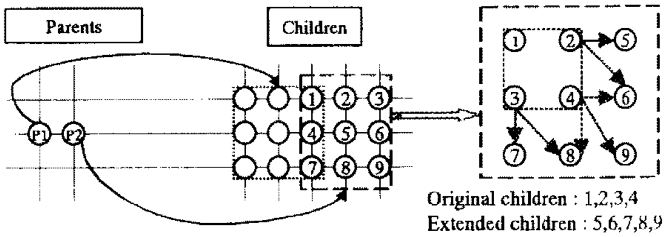


Figure 2. An extension rule for the flexible relation.
Note that a parent has four original children and more children according to the significance of original children.

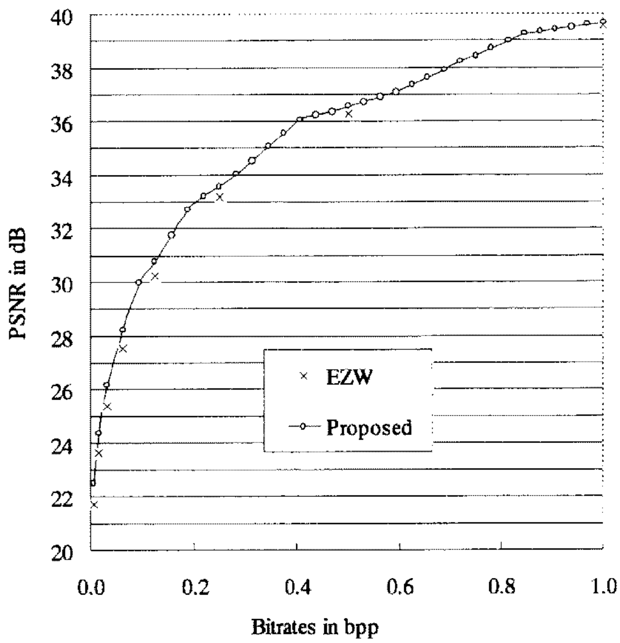


Figure 3. Performance curves for the test images.
Lena (512 X 512, 8 bits grey image)

Table 1. Comparisons of produced symbols in numbers for the same number of significant coefficients.

Used coder	Compressed size (Bytes)	PSNR (dB)	Compression ratio	# of POS	# of NEG	# of ZTR	# of IZ
EZW	6511	32.51	40.26 : 1	3876	3743	43249	1566
Proposed	6288	32.51	41.69 : 1	3876	3743	48102	1168

(a) Lena (512 X 512, 8 bits grey image, original size = 262144 Bytes)

Used coder	Compressed size (Bytes)	PSNR (dB)	Compression ratio	# of POS	# of NEG	# of ZTR	# of IZ
EZW	15878	30.39	16.51 : 1	9741	9586	72003	7603
Proposed	14511	30.39	18.07 : 1	9741	9586	80526	5063

(b) Barbara (512 X 512, 8 bits grey image, original size = 262144 Bytes)