

# Controller of the Capacitor Commutated Converter for HvdC transmission

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**Abstract** - A Capacitor Commutated Converter (CCC) has less difficulty of commutation failure in comparison to the conventional line commutated converter. This paper proposes the  $A\gamma'R$  control of the CCC in the inverter operation, which deserves as the  $A\gamma'R$  of the conventional converter. The CCC can be operated in high power factor area by using the proposing  $A\gamma'R$  control. The voltage stability at an AC bus connected the CCC inverter is investigated and estimated its ability of preventing the AC voltage collapse. To estimate the voltage stability, this paper developed the simplified converter mathematical model and led the VSF index. The results shows that the AC voltage stability is guaranteed and enables the interconnection to an weak AC system, when compensation factor of the compensation capacitor is higher than 200%.

**Keyword** - HvdC, CCC,  $A\gamma'R$ , VSF, Compensation Factor

## 1 INTRODUCTION

A conventional HvdC transmission system using line commutated converter has some operation difficulty as follows: the commutation failure occurrence to an insufficient AC voltage, the consumption of large reactive power at the converter, and so on. The commutation of the converter is achieved by distinguishing the conducting thyristor valve with imposing the reverse voltage from the connected AC bus. Therefore, the phase of the AC current flowing through the converter transformer must be delayed inevitably to the ac bus voltage. As a result, the converter consumes a reactive power. This reactive power consumption becomes large especially when the converter is operated in  $A\gamma'R$  mode of the inverter to maintain the commutation margin angle constant. This becomes a serious problem to an HvdC system linked to an weak ac system having small reactive power supply ability and less voltage stability.

The installation of Capacitor Commutated Converter (CCC) as an inverter of an HvdC transmission has been considered to improve these difficulties. Fig.1 shows a basic circuit configuration of 6 pulse CCC. A CCC has a simple circuit topology of having series connected capacitors in between the converter and the sec-

ondary side (converter side) of converter transformer. The capacitors are repeatedly charged and discharged by the alternating converter current. This charged voltage in the capacitor assists the commutation of the valve by superposing on the AC bus voltage. Then, the phase lag of the AC current can be reduced and the amount of the reactive power consumption at the converter decreases. The valve itself slightly differs from the valves of the conventional converter, which is cost effective in contrast to the novel self-commutated converter valve. Because of these reason, the installation of CCC to an HvdC transmission can be considered as one solution of interconnection to an weak AC system.

The conventional HvdC transmission has the difficulty of the voltage instability occurrence caused by the interaction of controllers between Automatic Power Regulator (*APR*) of the rectifier controller and Automatic margin angle Regulator ( $A\gamma'R$ ) of the inverter controller. A CCC also has fear for a commutation failure, because the switching device of the valve does not have self turn off ability and the firing angle must be set to maintain least effective margin angle. This paper describe how to calculate the firing angle to ensure the effective commutation margin angle ( $A\gamma'R$ ). The  $A\gamma'R$  of the CCC corresponds to the  $A\gamma'R$  of the conventional converter. The voltage stability estimation to the CCC inverter connected AC bus hasn't been discussed yet by any authors. This paper investigates the voltage stability by using the index of named Voltage Stability Factor(VSF) and shows that the CCC has preferable characteristics of an interconnection to an weak AC system.

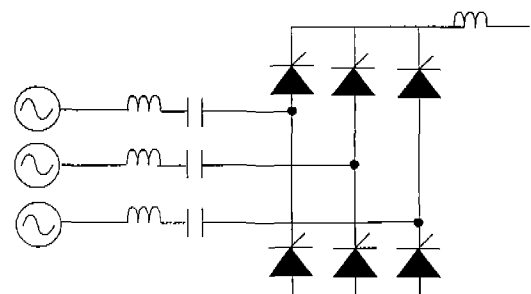


Fig. 1: Capacitor Commutated Converter

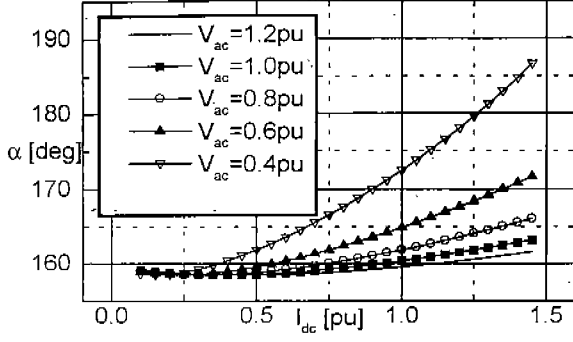


Fig. 2: Constant effective margin angle characteristic

## 2 The model of CCC

The CCC equations given by other author [1] is obtained by solving the circuit equation from Laplace transformation of the system. The equations reveals the output characteristics of the CCC and estimated the commutation process of the converter. However, they did not make it clear the relation among the effective margin angle, the firing angle and the commutation overlap angle. This paper gives additional mathematical model of the CCC, which reveals the relationship among the effective margin angle and the other angles and the state variables.

A CCC repeats commutation as well as the conventional line commutated converter. Commutation capacitors are connected between the valve and the converter transformer. The voltage charged in the commutation capacitor must be considered to calculate the effective commutation margin angle. The equation for calculating the overlap angle from the firing angle is obtained by other author[1]. The effective margin angle is defined as follows; the term from when the valve current becomes zero to when the reverse voltage imposed on the valve becomes zero. By using this boundary conditions, the other equation can be obtained.

Here, we define the leakage inductance of converter transformer : $L$ , capacitance of commutation capacitor: $C$ , AC voltage (induced at the secondary windings of the converter transformer): $V_{ac}$ , DC current: $I_{dc}$ , firing angle: $\alpha$ , commutation overlap angle: $u$  and effective margin angle: $\gamma'$ . The relationship between then can be described as following two non-linear formulas.

$$\begin{aligned}
 f_1(V_{ac}, I_{dc}, \alpha, u) &= \left\{ \frac{k\omega}{\sqrt{2}} V_{ac} [\cos(\alpha) - \cos(\alpha + u)] + \frac{I_{dc}}{C} \right\} \\
 &\quad \left\{ 1 + \cos\left(\frac{u}{\omega\sqrt{LC}}\right) \right\} - \frac{1}{\sqrt{LC}} \sin\left(\frac{u}{\omega\sqrt{LC}}\right) \\
 &\quad \left\{ \frac{k}{\sqrt{2}} V_{ac} [\sin(\alpha) + \sin(\alpha + u)] + \frac{I_{dc}}{\omega C} \left(\frac{2\pi}{3} - \frac{u}{2}\right) \right\} \\
 &= 0 \tag{1}
 \end{aligned}$$

$$\begin{aligned}
 f_2(V_{ac}, I_{dc}, \alpha, u, \gamma') &= \sin\left(\frac{u}{\omega\sqrt{LC}}\right) \left\{ \sqrt{2} V_{ac} \sin(\alpha + u + \gamma') \right. \\
 &\quad \left. - \sqrt{2} k V_{ac} \sin(\alpha + u) - \frac{I_{dc} \gamma'}{\omega C} \right\} \\
 &\quad + \sqrt{LC} \left\{ \sqrt{2} k \omega V_{ac} [\cos(\alpha) - \cos(\alpha + u)] \right. \\
 &\quad \left. \cos\left(\frac{u}{\omega\sqrt{LC}}\right) \right\} + \frac{I_{dc}}{C} \left[ 1 + \cos\left(\frac{u}{\omega\sqrt{LC}}\right) \right] \\
 &= 0 \tag{2} \\
 k &= \frac{1}{1 - LC\omega^2}
 \end{aligned}$$

The obtained model is consisted from two non-linear equations of eq(1) and eq(2). They cannot be solved analytically, but must be solved numerically by using iterative convergence calculation of newton-raphson method.

By using the obtained model, we calculated the firing angle of the CCC to maintain the effective margin angle at the constant value for  $A\gamma'R$  operation. Fig.2 indicates the characteristics of the firing angle to the DC current as a parameter of AC voltage. It shows that the CCC can increase its firing angle when AC voltage is decreased and/or DC current is increased because of commutation assist by the voltage charged in the commutation capacitor. These response is contrary to the conventional line commutated converter.

The  $A\gamma'R$  operation of CCC can be realized by using the obtained model, but the iterative convergence calculation is not suitable for the realtime converter control which requires the high speed response. Therefore, this paper intend to approximate its characteristics by a polynomial function of the measurable DC current and AC voltage. The curve in Fig.2 can be approximated by a quadratic function of the DC current. Based on this concept, we design simplified  $A\gamma'R$  calculation as shown in eq.(3).

$$\begin{aligned}
 \alpha &= H_{i0} + H_{i1} I_{dc} + H_{i2} I_{dc}^2 \tag{3} \\
 H_{ij} &= \sum_{k=0}^3 H_{V,jk} V_{ac}^k \quad j = 0, 1, 2
 \end{aligned}$$

This controller needs DC current and ac voltage as inputs. As a first step, it calculates coefficients of a function of DC current as a quadratic function of AC voltage. Then, it calculates the firing angle of  $A\gamma'R$  control as a quadratic function of dc current.  $H_{V,jk}$  is a parameter decided from by approximating the constant effective angle characteristic obtained from Fig.2. The error between the value obtained from iterative convergence calculation and the value calculated by eq.(3) is small enough. Therefore, the simplification of the  $A\gamma'R$  can be put into practical use. Fig.3 shows the schematic diagram of the simplified  $A\gamma'R$  controller.

The fundamental component of the converter AC current can be calculated from the firing angle and the commutation overlap angle. By noting the fundamental component of the AC current as  $I_{ac}$ , the CCC output active and reactive power is given as eq.(4).

$$P_{dc} + jQ_{dc} = \sqrt{3}V_{ac}\bar{I}_{ac} \quad (4)$$

By calculating the output of the CCC under  $A\gamma'R$  control, it becomes clear that the CCC can reduce reactive power consumption to the AC voltage drop. This characteristics is preferable to the AC voltage stability improvement. This paper estimate quantitatively the effect of the CCC under  $A\gamma'R$  control to the AC voltage stability in the following section.

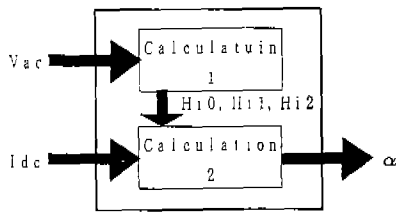
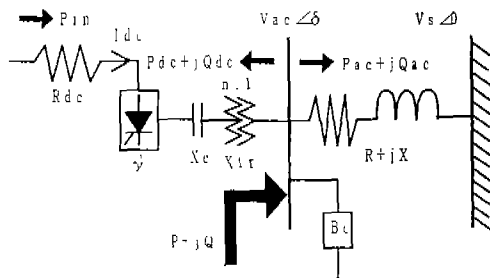


Fig. 3: Automatic effective margin angle regulator



$$P_{in}=1.0\text{pu}, I_{dc}=1.0\text{pu}, \gamma'=20\text{deg}$$

$$V_{ac}=1.0\text{pu}, Q_{ac}=0\text{pu}$$

Fig. 4: studied model

### 3 Voltage stability study by VSF

#### Voltage Stability

The conventional HVdc transmission has a difficulty of unstable interaction between  $APR$  of the rectifier and  $A\gamma'R$  of the inverter. This phenomena is led as followings; On the occurrence of AC voltage drop, the DC voltage drops. The  $APR$  of the rectifier increases the DC current to compensate the DC voltage drop. The DC current increment causes the commutation margin angle shortage. The  $A\gamma'R$  of the inverter decreases the firing angle to maintain the commutation margin angle. This enhances the DC voltage drop and AC voltage drop induced by the increment of the reactive power consumption at the inverter. As a consequence, the positive feedback loop is constituted. This

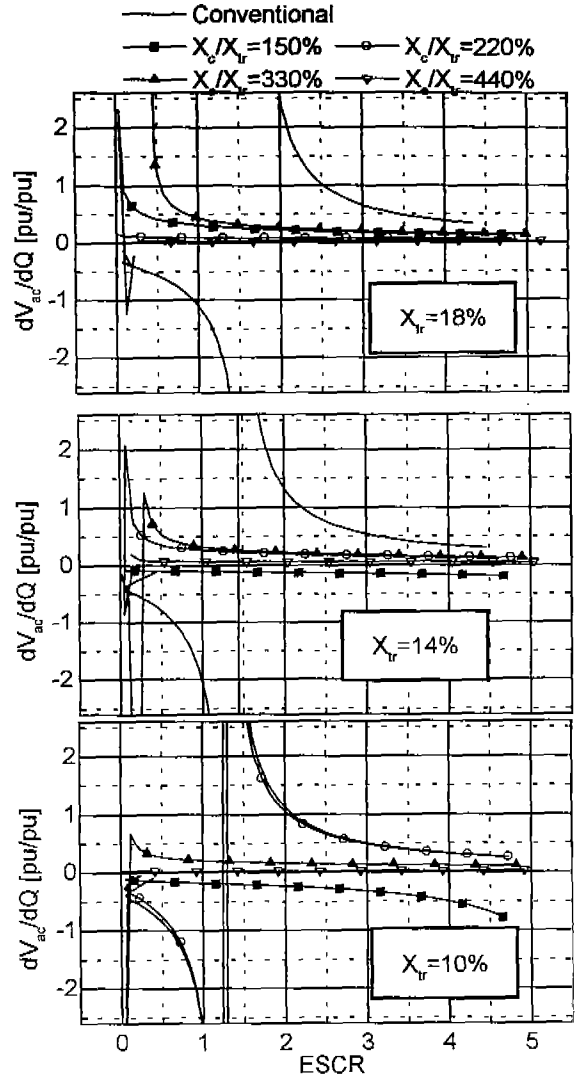


Fig. 5: ESCR- $\frac{dV}{dQ}$  (APR)

causes the AC voltage instability, especially when the inverter is linked to an weak AC system and may induces the voltage collapse in the worst case.

However, the CCC  $A\gamma'R$  of inverter will expected not to constitute the positive feedback loop and will not induces the AC voltage instability. As mentioned in the previous section, the  $A\gamma'R$  control of CCC shows opposite characteristics to the  $A\gamma'R$  control of conventional converter. It increases the firing angle to an AC voltage drop, and it reduces the reactive power consumption at the converter and prevents the AC voltage drop. To prove this qualitative system preferable characteristics, we calculate the Voltage Stability Factor (VSF) as an index of voltage stability and estimate it quantitatively.

The VSF is defined as  $\frac{dV}{dQ}$  and it indicates voltage sensitivity of that point to injected reactive power. If

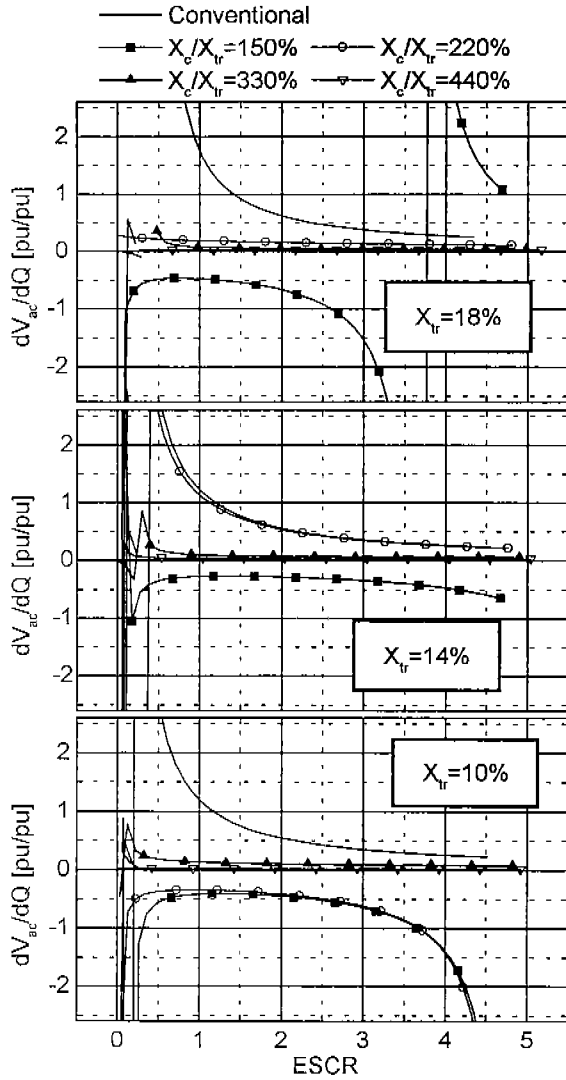


Fig. 6: ESCR- $\frac{dV}{dQ}$  (ACR)

$VSF < 0$  at the AC bus connected to inverter, it means that the AC voltage shows unstable to a reactive power change at that point. If  $VSF > 0$ , it shows that the AC voltage stable to a reactive power fluctuation at that point. It is important that the VSF has a maximum acceptable value even when  $VSF > 0$ , or the VSF must be lower than a finite value to permit the voltage fluctuation by the reactive power deviation in acceptable range. Therefore, the voltage stability by VSF must be estimated not only by its sign but also its absolute value. The VSF is considered as the simple but reliable index for the voltage stability estimation.

To calculate the VSF of the HVdc interconnected system, the model of an HVdc system must be simplified as followings. The rectifier is ideally controlled in *APR* or in *ACR*, the capacitance and inductance of the DC line is ignored, and the inverter is also ideally

controlled in *A $\gamma$ R* or *A $\gamma'$ R*. The studied model is given in Fig.4. When the rectifier is controlled in *ACR*, it can be assumed as a constant current source, and approximated as a constant power source when controlled in *APR*. The *A $\gamma$ R* of conventional converter and the *A $\gamma'$ R* of the CCC are modeled to change the firing angle by its control theorem, and its output is calculated according to the converter model respectively. The calculation process of VSF to the discussing CCC is given in appendix precisely.

As analysis conditions, we set up AC voltage at the inverter connected bus is to 1.0pu and unity power factor at the terminal by compensating the reactive power by the shunt capacitor. The margin angle of the conventional converter or the effective margin angle of the CCC is set to 20°. The converter transformer ratings of  $S_{tr}$  is set by eq.(5).

$$S_{tr} = \sqrt{2}nV_{ac}I_{dc} \quad (5)$$

Here,  $n$ :the transformer tap ratio,  $V_{ac}$ :the AC bus voltage and  $I_{dc}$ : DC current. In HVdc transmission, the rectifier active power and dc current is set 1.0pu ( $\pm 250$ kV, 1,000MW rated operation).

#### VSF to *APR*

Fig.5 shows the calculated VSF values when the rectifier is controlled in *APR*. When ESCR decreases,  $\frac{dV_{ac}}{dQ}$  (VSF) increases to positive infinite, and change the sign from positive to negative at a certain ESCR value, that is called the critical CCC (CESCR). To the case of  $X_{tr}=18\%$ , CESCR of CCC becomes smaller than the conventional converter, if capacitor compensation factor ( $\frac{X_c}{X_{tr}}$ ) is adequately large. This means that the CCC inverter installed HVdc transmission system can be applied from a stiff AC system to an weak AC system. However, because of the non-linearity of the converter output characteristics, the CESCR of CCC with the  $\frac{X_c}{X_{tr}} = 330\%$  case becomes higher than the 150% case and the 220% case.

To the case of  $X_{tr} = 14\%$  and  $X_{tr} = 10\%$ , VSF of  $\frac{X_c}{X_{tr}} = 150\%$  case becomes negative even though ESCR is adequately large. The VSF tendency of case  $X_{tr} = 10\%$  and  $\frac{X_c}{X_{tr}} = 220\%$  is similar to the conventional converter case. The voltage stability also can be improved by the high compensation factor to this case. As a result, it can generally be said that the CCC inverter can improve the voltage stability, but it is important to select proper value of compensation capacitor.

#### VSF to *ACR*

The VSF curve when *ACR* is applied to rectifier is shown in Fig.6. The VSF of conventional converter becomes more stable than the *APR* case. When  $X_{tr} = 18\%$  and CCC of  $\frac{X_c}{X_{tr}} = 150\%$ , CESCR becomes larger than the *APR* case, hence worse this value is larger than the conventional converter. When  $\frac{X_c}{X_{tr}} = 330\%$  case of *ACR* is more stable than the *APR*

case. When  $X_{tr} = 14\%$  and  $\frac{X_c}{X_{tr}} = 220\%$  the VSF does not differ to the conventional converter. The case of  $X_{tr} = 10\%$ ,  $\frac{X_c}{X_{tr}} = 220\%$  is unstable even though the VSF of conventional converter is stable.  $\frac{X_c}{X_{tr}} = 330\%$  and  $440\%$  are stable at the almost all the range of ESCR in this condition. As obtained from the results, the voltage stability with *ACR* controlled rectifier differs from the voltage stability with *APR* controlled rectifier case.

#### VSF to compensation factor

Fig.7 shows the relation between compensation factor and VSF at ESCR=2.0 and 3.0. In the region of compensation factor is relatively small, the VSF is negative value or unstable. Because the unstable characteristic at a low compensation factor is caused by the mixing with the conventional converter characteristic, it induces the voltage instability. If  $\frac{X_c}{X_{tr}}$  is large to some extent, the VSF becomes positive within finite value that is less than the conventional converter. As to the voltage stability of conventional converter, the *ACR* control of the rectifier gives more voltage stability than *APR* control of the rectifier. On the other hand, when CCC is installed at the inverter, the combination of *ACR* and  $A\gamma'R$  is not always better than the combination of *APR* and  $A\gamma'R$ .

According to these studies, the compensation factor of CCC must be carefully decided to keep voltage stability. Improper compensation loses the voltage stability.

#### 4 CONCLUSIONS

This paper described the relationship among the commutation processes of the CCC firing angle, the commutation overlap angle and the effective commutation margin angle. The relationship can be expressed as two non-linear formulas of the three angles in the steady state. The formulas also defines the static characteristic of the CCC output active/reactive power.

The authors proposed the constant effective margin angle control ( $A\gamma'R$ ) of the CCC to its inverter operation. The firing angle is calculated from the non-linear formulas as a function of the AC bus voltage and the DC current, by performing convergence iterative calculations. Moreover, this paper approximated the firing angle calculation for  $A\gamma'R$  as a polynomial expression of the AC voltage and DC current to implement them to the actual converter control. The calculation error by approximation is negligible under usual operation and affects no harmful influences under critical condition.

The AC voltage stability at the CCC connected bus with inverter operation in  $A\gamma'R$  is studied by using the index named VSF, which is the AC voltage sensitivity to the reactive power change ( $\frac{dV}{dQ}$ ). The unstable interaction between the *APR* of the rectifier control and the  $A\gamma'R$  of the conventional inverter is eliminated by using the  $A\gamma'R$  to the CCC inverter. It reduces the

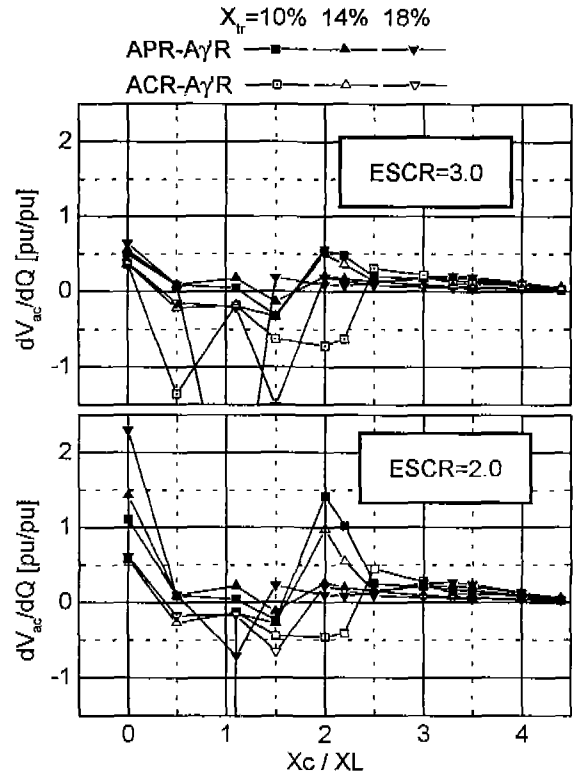


Fig. 7:  $\frac{X_c}{X_{tr}} - \frac{dV}{dQ}$

ESCR and enables an HVdc interconnection to an weak AC system.

The VSF improvement depends on the compensation capacitor of the CCC and is largely affected by the relative value, which is expressed by compensation factor. The result shows that the compensation factor larger than 200% guarantees voltage stability improvement. However, suitable value of the compensation factor must be carefully selected.

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## 6 APPENDIX

### Voltage Stability Factor (VSF) Method

Define as state variables  $V_{ac}$ ,  $\delta$  and  $I_{dc}$  and as control variables  $P_I$ ,  $\gamma'$ ,  $n$ ,  $B_C$ ,  $P$ ,  $Q$  and  $V_s$ .

$$\begin{aligned}\bar{X} &= (V_{ac}, \delta, I_{dc}) \\ \bar{U} &= (P_I, \gamma', n, B_C, P, Q, V_s)\end{aligned}\quad (6)$$

The active and reactive powers,  $P_{dc}$ ,  $Q_{dc}$ ,  $P_{ac}$  and  $Q_{ac}$ , are functions of the state and control variables.

$$\begin{aligned}P_{dc} &= P_{dc}(V_{ac}, I_{dc}, \gamma', n) \\ Q_{dc} &= Q_{dc}(V_{ac}, I_{dc}, \gamma', n) \\ P_{ac} &= P_{ac}(V_{ac}, \delta, V_s) \\ Q_{ac} &= Q_{ac}(V_{ac}, \delta, V_s)\end{aligned}\quad (7)$$

The net injection of active power into the dc bus and the injection of active and reactive power into the ac bus are given by the functions  $g_1$ ,  $g_2$  and  $g_3$ .

$$\begin{aligned}g_1(\bar{X}, \bar{U}) &= P_I - P_{dc} \\ g_2(\bar{X}, \bar{U}) &= P - P_{ac} + P_{dc} \\ g_3(\bar{X}, \bar{U}) &= Q - Q_{ac} - Q_{dc}\end{aligned}\quad (8)$$

These functions can be put into vector form and the stationary points, or the equilibrium points, are solutions to the equation.

$$\begin{aligned}\bar{g}(\bar{X}, \bar{U}) &= (g_1, g_2, g_3)^t \\ &= 0\end{aligned}\quad (9)$$

A certain operating point of the system is thus a solution to equation (9) and is denoted by  $(\bar{X}_0, \bar{U}_0)$ . Small variations,  $(\Delta\bar{X}, \Delta\bar{U})$  around this operating point must also be solutions to equation (9), i.e.

$$\bar{g}(\bar{X}_0 + \Delta\bar{X}, \bar{U}_0 + \Delta\bar{U}) = 0 \quad (10)$$

If this equation is linearized we get

$$G_{\bar{X}} \cdot \Delta\bar{X} + G_{\bar{U}} \cdot \Delta\bar{U} = 0 \quad (11)$$

$$(G_{\bar{X}})_{ij} = \frac{\partial g_i}{\partial X_j} \quad (12)$$

$$(G_{\bar{U}})_{ij} = \frac{\partial g_i}{\partial U_j} \quad (13)$$

Where  $G_{\bar{X}}$  and  $G_{\bar{U}}$  are Jacobian. The matrix element of these matrices;  $\partial\bar{g}/\partial\bar{X}$  and  $\partial\bar{g}/\partial\bar{U}$  are defined. In equation (12) and (13) the matrix-elements of  $G_X$  and  $G_U$  are calculated in the point  $(\bar{X}_0, \bar{U}_0)$ .  $\gamma'$ ,  $n$ ,  $B_C$  and  $V_s$  are kept constant. The quantities  $P$  and  $Q$  are the power injections into the ac bus connected to

the inverter and are both zero at the steady state. In order to calculate  $\frac{dV_{ac}}{dQ}$  in APR, the following vectors are used.

$$\begin{aligned}\Delta\bar{X} &= (\Delta V_{ac}, \Delta\delta, \Delta I_{dc})^t \\ \Delta\bar{U} &= (0, 0, 0, 0, 0, \Delta Q, 0)^t\end{aligned}\quad (14)$$

In order to calculate  $\frac{dV_{ac}}{dQ}$  in ACR, the following vectors are used.

$$\begin{aligned}\Delta\bar{X} &= (\Delta V_{ac}, \Delta\delta, 0)^t \\ \Delta\bar{U} &= (\Delta P_I, 0, 0, 0, 0, \Delta Q, 0)^t\end{aligned}\quad (15)$$

Small variation  $\Delta\alpha$  of CCC around operating point is led from equation (3).  $\Delta\alpha$  is denoted as a function of small variations  $(\Delta V_{ac}, \Delta I_{dc})$ .

$$\begin{aligned}\Delta\alpha &= (H_{i1} + 2H_{i2}I_{dc})\Delta I_{dc} + \sum_{j=0}^2 I_{dc}^j \Delta H_{ij} \\ \Delta H_{i,jk} &= \sum_{k=1}^3 k \cdot H_{Vjk} V_{ac}^{k-1} \Delta V_{ac}^k\end{aligned}\quad (16)$$

Equation (1) and (16)  $\Delta u$  is appeared by  $\Delta V_{ac}$  and  $\Delta I_{dc}$ . Hence small variation of inverter power flow  $(\Delta P_{dc}, \Delta Q_{dc})$  are denoted by  $\Delta V_{ac}$  and  $\Delta I_{dc}$ .

If the rectifier controller is APR, equation (11) and (14) gives as follows.

$$\Delta\bar{X} = G_{\bar{X}}^{-1} \cdot \begin{bmatrix} 0 \\ 0 \\ \Delta Q \end{bmatrix} \quad (17)$$

From equation (17),  $\frac{dV_{ac}}{dQ}$  can be obtained.

$$\begin{aligned}\frac{dV_{ac}}{dQ} &= \frac{\Delta V_{ac}}{\Delta Q} \\ &= -G_{\bar{X}}^{-1}{}_{13} \\ &= \frac{-\frac{\partial P_{ac}}{\partial \delta} (-2R_{dc}I_{dc} + \frac{\partial P_{dc}}{\partial I_{dc}})}{\det(G_{\bar{X}})}\end{aligned}\quad (18)$$

If the rectifier controller is ACR, equation (11) and (15) gives as follows.

$$\begin{aligned}\begin{bmatrix} \Delta V_{ac} \\ \Delta\delta \end{bmatrix} &= -G_{\bar{X}}'^{-1} \begin{bmatrix} 0 \\ \Delta Q \end{bmatrix} \\ G_{\bar{X}}' &= \begin{bmatrix} \frac{\partial g_2}{\partial V_{ac}} & \frac{\partial g_2}{\partial \delta} \\ \frac{\partial g_3}{\partial V_{ac}} & \frac{\partial g_3}{\partial \delta} \end{bmatrix}\end{aligned}\quad (19)$$

From equation (19),  $\frac{dV_{ac}}{dQ}$  can be obtained.

$$\begin{aligned}\frac{dV_{ac}}{dQ} &= \frac{\Delta V_{ac}}{\Delta Q} \\ &= -G_{\bar{X}}'^{-1}{}_{12} \\ &= -\frac{1}{\det(G_{\bar{X}}')} \frac{\partial P_{ac}}{\partial \delta}\end{aligned}\quad (20)$$