

# CONTROL STRATEGIES FOR SHUNT ACTIVE POWER FILTERS IN DISTORTION SOURCE VOLTAGE SITUATION

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**ABSTRACT** — The compensation strategy of shunt active power filters is one of the most important link that determine its compensation characteristics. In this paper, a new interpretation of the instantaneous reactive power theory in three-phase circuits was proposed. A compensation strategy ( $i_p, i_q$  mode) was introduced on the basis of the new interpretation. This compensation strategy was compared with other two compensation strategies ( $p, q$  mode and UPF mode). When source voltage is sinusoidal, the three compensation strategies are equivalent to each other. When source voltage is distorted, a sinusoidal source current may result only by using  $i_p, i_q$  mode. This is the advantage of  $i_p, i_q$  mode. The result is verified by simulation.

## 1. INTRODUCTION

Harmonic pollution generated by nonlinear loads such as static rectifiers becomes increasingly serious as they are widely used in power systems [1]. Harmonic pollution is the source of many problems such as communication interference, excessive losses and heating in motors, capacitors, and transformers, blown capacitor fuses, errors in measurement equipment, nuisance tripping of relays and breakers, unstable operation of zero voltage crossing firing circuits, and interference with motor controllers.

Shunt passive filters are widely used to suppress harmonics in power systems. However, they have many problems [2], such as the high sensitivity to the source impedance, resonance with source impedance, and harmonic amplifying phenomenon. These problems restrict the application of shunt passive filters.

To solve the problems of shunt passive filters, active power filter (APF) was proposed [3][4]. Shunt active power filter is one type of APF that is commonly used. The main circuit of shunt APF is a PWM converter using power electronic devices such as BJTs or IGBTs. A shunt APF is controlled in such a way as to actively shape the source current,  $i_s$ , into sinusoidal by injecting the

compensating current,  $i_c$ . The problems of shunt passive filters are avoided by using APF, but APF has higher power losses and higher cost.

A fundamental problem for shunt APF design is the selection of a compensation strategy, that is, the evaluation of the reference signal of compensating current. The strategy proposed by H. Akagi and A. Nabae [4], which is called  $p, q$  mode in this paper, is considered as a classic one. Referring to evaluation of real and imaginary instantaneous powers, it proposed to maintain the real instantaneous power and imaginary instantaneous power to a constant value. Under the assumption of balanced and sinusoidal three-phase source voltages and ideal power electronic devices, this strategy leads to a sinusoidal source current.

According to surveys performed by A. E. Emanuel et al. on medium and high voltage distribution systems [5, 6], the average value of voltage total harmonic distortion (THD) is often 2~3%, and this value will increase in the future. In this situation, the source voltage may not be considered as sinusoidal, and  $p, q$  mode will lead to a distorted source current.

Recently, a new compensation strategy, UPF mode, was proposed by A. Cavallini and G. C. Montanari[7]. According to this compensation strategy, the ratio of  $e_s$  over  $i_s$  is controlled to be a constant value, so that power factor of source is unity. However, when source voltage is distorted, source current is also distorted.

In this paper, a new interpretation of the instantaneous reactive power theory in three-phase circuits was proposed. A compensation strategy ( $i_p, i_q$  mode) was introduced on the basis of the new interpretation. This compensation strategy was compared with other two strategies,  $p, q$  mode and UPF mode. When source voltage is sinusoidal, these compensation strategies are equivalent to each other. However, When source voltage is distorted, a sinusoidal source current may result only by using  $i_p, i_q$  mode. This is the advantage of  $i_p, i_q$  mode. The result was verified by simulation.

## 2. A NEW INTERPRETATION OF THE INSTANTANEOUS REACTIVE POWER THEORY IN THREE-PHASE CIRCUITS

The Theory Proposed by H. Akagi and A. Nabae

The instantaneous value of three-phase voltages and currents are represented by  $e_a, e_b, e_c$  and  $i_a, i_b, i_c$ , respectively. They are transformed to  $\alpha$ - $\beta$  coordinates.

$$\begin{bmatrix} e_\alpha \\ e_\beta \end{bmatrix} = C_{32} \begin{bmatrix} e_a \\ e_b \\ e_c \end{bmatrix} \quad (1)$$

$$\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = C_{32} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} \quad (2)$$

in (1) and (2),

$$C_{32} = \sqrt{2/3} \begin{bmatrix} 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \end{bmatrix}$$

The real instantaneous power  $p$  and imaginary instantaneous power  $q$  are defined as

$$\begin{cases} p = e_\alpha i_\alpha + e_\beta i_\beta \\ q = e_\alpha i_\beta - e_\beta i_\alpha \end{cases} \quad (3)$$

This equation may be represented in matrix form.

$$\begin{bmatrix} p \\ q \end{bmatrix} = \begin{bmatrix} e_\alpha & e_\beta \\ -e_\beta & e_\alpha \end{bmatrix} \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} \quad (4)$$

On the basis of the above definitions of  $p$  and  $q$ , the power quantities of every phase were defined[3].

### A New Interpretation of Instantaneous Reactive Power Theory

Based on the definitions of instantaneous active current and instantaneous reactive current, a new interpretation of instantaneous reactive power theory is proposed.

In  $\alpha$ - $\beta$  coordinate system, voltage vector  $\vec{e}$  and current vector  $\vec{i}$  are composed of  $e_\alpha, e_\beta$  and  $i_\alpha, i_\beta$ , respectively, as shown in figure 1.

$$\vec{e} = e \angle \varphi_e \quad (5)$$

$$\vec{i} = i \angle \varphi_i \quad (6)$$

In (5) and (6),  $e$  and  $i$  are the magnitudes of vectors  $\vec{e}$  and  $\vec{i}$ ,  $\varphi_e$  and  $\varphi_i$  are the arguments of vectors  $\vec{e}$  and  $\vec{i}$  respectively.

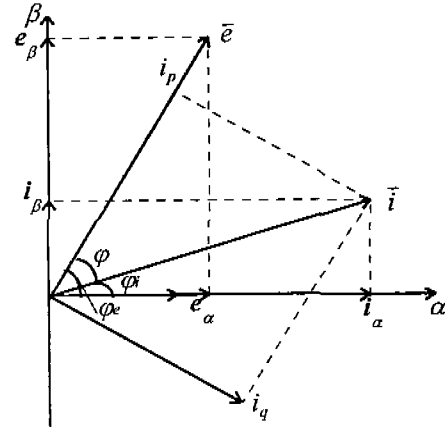


Fig. 1 Vectors in  $\alpha$ - $\beta$  Coordinate System

Instantaneous active current  $i_p$  is defined as the projection of vector  $\vec{i}$  on vector  $\vec{e}$ . Instantaneous reactive current  $i_q$  is defined as the projection of vector  $\vec{i}$  orthogonal to vector  $\vec{e}$ .

$$i_p = i \cdot \cos \varphi \quad (7)$$

$$i_q = i \cdot \sin \varphi \quad (8)$$

Where,  $\varphi = \varphi_e - \varphi_i$ .

The definitions of  $i_p$  and  $i_q$  are equivalent to that proposed by J. L. Willems [8].

Based on the above definitions, instantaneous active power  $p$  and instantaneous reactive power  $q$  in three-phase circuits are defined as

$$p = e \cdot i_p \quad (9)$$

$$q = e \cdot i_q \quad (10)$$

Equations (9) and (10) can be transformed to the following equation.

$$\begin{bmatrix} p \\ q \end{bmatrix} = \begin{bmatrix} e_\alpha & e_\beta \\ e_\beta & -e_\alpha \end{bmatrix} \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} \quad (11)$$

Here, the instantaneous active power  $p$  is equal to the real instantaneous power  $p$  defined by H. Akagi and A. Nabae. The instantaneous reactive power  $q$  is negative to the imaginary instantaneous power  $q$  defined by them, and the mean value of instantaneous reactive power  $q$  is equal to reactive power  $Q$ . Hence, instantaneous reactive power  $q$  defined by (11) is adopted in the following analysis.

On the basis of the above definitions of  $i_p, i_q$  and  $p, q$ , the current quantities and power quantities of every phase may be defined[9].

The key point of this new interpretation is the introduction of the definitions of  $i_p$  and  $i_q$ .  $i_p$  and  $i_q$  are

defined as projections in  $\alpha$ - $\beta$  coordinate system. These definitions may be considered to fits into a sequence that begins with the classic definitions of the active current and the reactive current in single phase circuits in phase space. Hence, this new interpretation of instantaneous reactive power theory may be considered to fits into a sequence that begins with the classic power theory.

Because  $i_p$  and  $i_q$  are defined as projections, which have clear physical meaning, they may be called instantaneous active and reactive currents. And, since  $p$  and  $q$  are the products of  $e$  and  $i_p$ ,  $i_q$  respectively,  $p$  and  $q$  may be called instantaneous active power and instantaneous reactive power.

### 3. COMPENSATION STRATEGIES FOR APF

The configuration of a shunt APF is shown in figure 2.

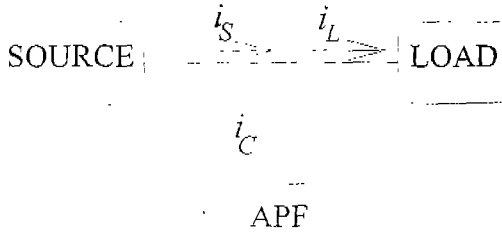


Fig. 2 The Configuration of Shunt APF

Load current is  $i_L$ , and the relationship between  $i_s$ ,  $i_L$ , and  $i_C$  is (12).

$$i_s = i_L + i_C \quad (12)$$

The instantaneous active powers of source, load, APF are  $p_s$ ,  $p_L$ ,  $p_A$ , and the instantaneous reactive powers of source, load, APF are  $q_s$ ,  $q_L$ , and  $q_A$ , respectively.

$$\begin{aligned} p_s &= p_L + p_A \\ q_s &= q_L + q_A \end{aligned}$$

The power main circuit of APF is a PWM converter. To simplify the analysis, it is considered that the converter is ideal, which means that the real compensating current follows exactly the reference signal of compensating current.

$p, q$  mode

This compensation strategy was proposed by H. Akagi and A. Nabae. By controlling  $p$  and  $q$  of APF, the harmonics and reactive power of source can be compensated. This compensation strategy may be represented by the following equation.

$$i_C = \begin{bmatrix} i_{Ca} \\ i_{Cb} \\ i_{Cc} \end{bmatrix} = C_{32}^T \begin{bmatrix} e_\alpha & e_\beta \\ e_\beta & -e_\alpha \end{bmatrix} \begin{bmatrix} p_A \\ q_A \end{bmatrix} \quad (13)$$

To compensate harmonics and reactive power simultaneously, the following must be held

$$\begin{cases} p_A = -\tilde{p}_L \\ q_A = -q_L \end{cases} \quad (14)$$

so that.

$$\begin{cases} p_s = p_L + p_A = \bar{p}_L \\ q_s = q_L + q_A = 0 \end{cases} \quad (15)$$

Where, the upper symbols "—" , "~" represent dc component and ac component respectively, and it's the same in the following paragraphs.

In this situation, the active power of the outputs of power source is constant while the reactive power is zero.

#### UPF mode

By transforming (3), the following equation is obtained

$$q = \frac{1}{\sqrt{3}} [e_a(i_b - i_c) + e_b(i_c - i_a) + e_c(i_a - i_b)] \quad (16)$$

When harmonics and reactive power are compensated simultaneously,  $q=0$ , the following may be achieved:

$$\frac{i_a}{e_a} = \frac{i_b}{e_b} = \frac{i_c}{e_c} = \psi(t) \quad (17)$$

Where,  $\psi(t)$  is controlled as a constant value,  $\psi(t) = \psi_0$ . The waveform of  $i_s$  is the same as that of  $e_s$ , and the power factor of source is 1, so this compensation strategy is called UPF (unity power factor) mode. Here,

$$\psi_0 = \frac{\int_0^T p dt}{\int_0^T (e_a^2 + e_b^2 + e_c^2) dt} \quad (18)$$

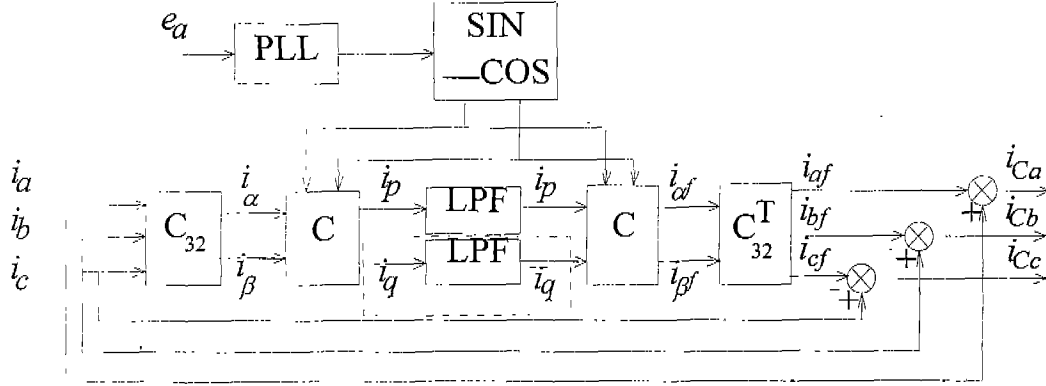


Fig. 3 Block Diagram of  $i_p, i_q$  Mode

$i_p, i_q$  mode

Suppose the source voltage is sinusoidal, and the initial phase of the voltage of phase a is zero. From (7) and (8) the following may be achieved

$$\begin{bmatrix} i_p \\ i_q \end{bmatrix} = \begin{bmatrix} \sin \omega t & -\cos \omega t \\ -\cos \omega t & -\sin \omega t \end{bmatrix} \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} \quad (19)$$

$\omega$  is the angular frequency of source voltage. This compensation strategy may also be represented by the following

$$i_c = C_{32}' \begin{bmatrix} \sin \omega t & -\cos \omega t \\ -\cos \omega t & -\sin \omega t \end{bmatrix} \begin{bmatrix} i_{pA} \\ i_{qA} \end{bmatrix} \quad (20)$$

The block diagram of this mode is shown in Figure 3.

In this figure, PLL is a phase locked loop. The block connected with PLL represents a sine and cosine signals generator, and its outputs are a sine signal  $\sin \omega t$ , which has the same phase angle as the source voltage of phase a, and a cosine signal  $-\cos \omega t$ . LPF is low pass filter. C is a matrix, and

$$C = \begin{bmatrix} \sin \omega t & -\cos \omega t \\ -\cos \omega t & -\sin \omega t \end{bmatrix}$$

When the harmonics and reactive power are to be compensated simultaneously, the following must be satisfied

$$\begin{cases} i_{pA} = -\tilde{i}_{pl} \\ i_{qA} = -\tilde{i}_{ql} \end{cases} \quad (21)$$

so that,

$$\begin{cases} i_{pS} = i_{pl} + i_{pA} = \tilde{i}_{pl} \\ i_{qS} = i_{ql} + i_{qA} = 0 \end{cases} \quad (22)$$

In (21) and (22),  $i_{pS}, i_{pl}, i_{pA}$  are the instantaneous active currents of source, load, APF, and  $i_{qS}, i_{ql}, i_{qA}$  are the

instantaneous reactive currents of source, load, APF respectively. In this situation, the block that used to evaluate  $i_q$ , that is, the part enclosed in the rectangle drawn in dotted line should be removed.

#### 4. COMPARISON OF THE COMPENSATION STRATEGIES

##### Sinusoidal Source Voltage Situation

###### a. $p, q$ mode

From (12), (13), (1), (2), and (11), the following may be achieved

$$i_s = \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = C_{32}' \begin{bmatrix} e_\alpha & e_\beta \\ e_\beta & -e_\alpha \end{bmatrix} \begin{bmatrix} \bar{p} \\ 0 \end{bmatrix} = \frac{\bar{p}_L}{e_\alpha^2 + e_\beta^2} \begin{bmatrix} e_a \\ e_b \\ e_c \end{bmatrix} \quad (23)$$

In the above equation,  $e_\alpha^2 + e_\beta^2 = 3E^2$  ( $E$  is the rms. value of source line-to-neutral voltage). Because  $e_s$  is sinusoidal,  $i_s$  is sinusoidal, and the power factor of the source is 1.

###### b. UPF mode

According to (17) we obtain

$$i_s = \psi_0 \cdot e_s$$

$i_s$  is sinusoidal, and the power factor of the source is 1.

###### c. $i_p, i_q$ mode

From (12), (20), (21), (1), (2), the following may be achieved

$$i_s = C_{32}' C \begin{bmatrix} \tilde{i}_{pl} \\ 0 \end{bmatrix} = \sqrt{2/3} \cdot \tilde{i}_{pl} \begin{bmatrix} \sin \omega t \\ \sin(\omega t - 2\pi/3) \\ \sin(\omega t + 2\pi/3) \end{bmatrix} \quad (24)$$

$i_s$  is sinusoidal, and the power factor of the source is 1. In fact,  $i_s$  is equal to the active component of the load current.

### Nonsinusoidal source voltage situation

#### a. $p, q$ mode

According to (23) we know that the source current is distorted since the source voltage is distorted.

#### b. UPF mode

According to (17),  $i_s$  is distorted, and its distortion is the same as that of the source voltage.

#### c. $i_p, i_q$ mode

When the compensation strategy shown in (20) is applied,  $i_s$  has no distortion according to (24).

### Discussions

a. If source voltages are sinusoidal and balanced, the results of three compensation strategies are the same.

b. If source voltages are distorted,  $i_s$  is distorted by using  $p, q$  mode, since

- i. source voltages  $e_a, e_b, e_c$  are nonsinusoidal
- ii there are components created by each harmonic voltage and harmonic current with same frequency in  $\bar{p}_L$ , besides the component created by the fundamental voltage and the fundamental current
- iii.  $e_a^2 + e_b^2$  is not constant any more, and it contains harmonic components Just because 3, the distortions of  $i_s$  does not coincide with that of  $e_s$ .

In  $p, q$  mode, the direct quantities used for the evaluation of compensating current are power quantities, while are not current quantities. In distorted source voltage situation, there is not any simple corresponding relation between power quantities and current quantities, and this is the virtual cause of the distortion of source currents.

$i_s$  is distorted but power factor is 1 when UPF mode is adopted.

In  $i_p, i_q$  mode, only fundamental component of source voltage is used, so that the harmonic components of source voltage do not influence the compensation result. Hence, the compensation result is the same as that obtained when source voltage is sinusoidal. Thus,  $i_s$  is sinusoidal when  $i_p, i_q$  mode is used.

The harmonic current injected to power system is the smallest when  $i_p, i_q$  mode is applied, so  $i_p, i_q$  mode is the best compensating strategy.

c. APF may be used to compensate harmonics without compensating reactive power as well as to compensate harmonics and reactive power simultaneously, but we know a larger APF rating is needed for compensating

reactive power. When APF is used only to compensate harmonics, only  $\tilde{p}_L, \tilde{q}_L$  or  $\tilde{i}_{pL}, \tilde{i}_{qL}$  need to be compensated. If  $p, q$  mode is adopted, it should be held that  $p_A = -\tilde{p}_L$ , and  $q_A = -\tilde{q}_L$ . If  $i_p, i_q$  mode is adopted, it should be satisfied that  $i_{pA} = -\tilde{i}_{pL}$ , and  $i_{qA} = -\tilde{i}_{qL}$ .

Because  $q_s = \bar{q}_L \neq 0$ , (17) is not true any more. Evidently UPF mode can only be used when harmonics and reactive power are compensated simultaneously. This is a limitation of UPF mode.

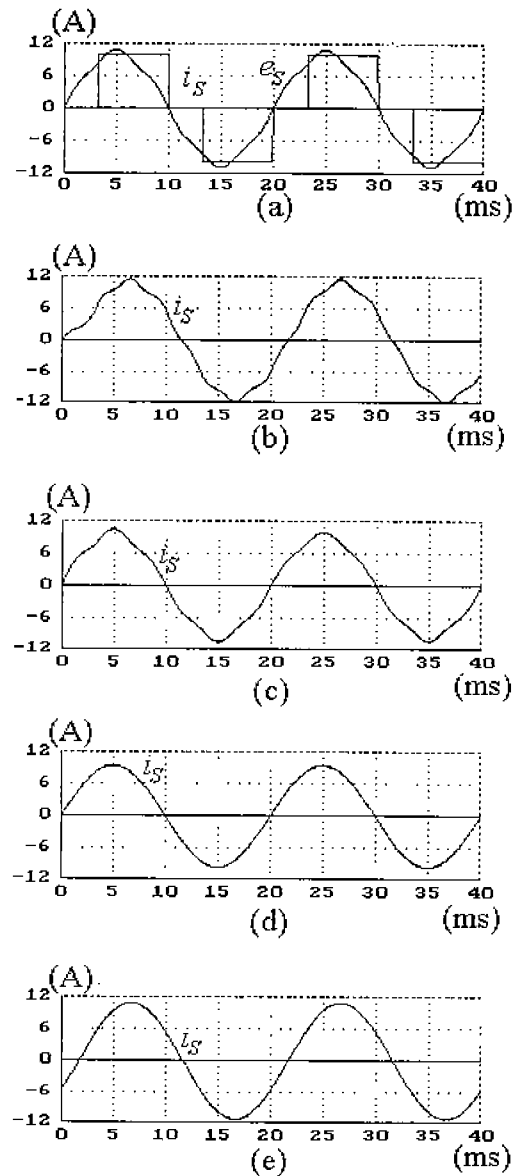


Fig. 4 Simulation Results

## 5. SIMULATION

The simulation results of an illustrative example with distorted source voltage (5% 5th harmonic), and a three phase full-bridge rectifier load with inductor on dc side, are shown in Figure 4. Waveforms of source voltage and load current are shown in Figure 4a. The source current using  $p, q$  mode is shown in Figure 4b. Figure 4c and Figure 4d show the source currents when UPF mode and  $i_p, i_q$  mode are applied respectively. Figure 4e shows the source current when  $i_p, i_q$  mode is applied only to compensate harmonics.

## 6. CONCLUSION

In this paper, a new interpretation of instantaneous reactive power theory was proposed, and especially the definitions of instantaneous active current  $i_p$  and instantaneous reactive current  $i_q$  were proposed. These definitions may be considered to fit into a sequence which begins with the classic definitions of the active and reactive currents in single phase circuits in phase space. Hence, this new interpretation of instantaneous reactive power theory may be considered to fit into a sequence that begins with the classic power theory. And because  $i_p, i_q$  are defined as projections, they may be called instantaneous active and reactive currents. Furthermore, since  $p, q$  are the products of  $e$  and  $i_p, i_q$  respectively,  $p, q$  may be called instantaneous active power and instantaneous reactive power.

On the basis of the new interpretation,  $i_p, i_q$  mode was introduced, and it was compared with other two compensation strategies. When source voltages are sinusoidal and balanced, they are equivalent to each other. However, when source voltages are not sinusoidal, the results are quite different. When  $i_p, i_q$  mode is used, the source currents are sinusoidal. If  $p, q$  mode is applied, the source currents are distorted. When UPF mode is adopted, the source currents are distorted but power factor is 1. It's evident that, according to the value of harmonic current injected into power system,  $i_p, i_q$  mode is the best compensation strategy.

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