New Mathematical Models with Core Loss Factor for Control of AC Motors

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Abstract—This paper establishes in a new unified manner new mathematical models with core(iron) loss factor for two kinds of AC motors, induction and synchronous motors which are supposed to generate torque precisely or/and efficiently under vector controls. Our new models consist of three basic equations consistent with the others such as differential equation describing electromagnetic dynamics, torque equation describing torque generating mechanism, energy transmission equation describing how injected energy is wasted, saved or transmitted where all vector signals are defined in general frame of arbitrary instant angular velocity. It is clearly shown in our models that equivalent core-loss resistance can express appropriately and separately both eddy-current and hysteresis losses rather than mere vague loss. Proposed model of induction motor is the most compact in sense of the number of employed interior states and parameters. This compact model can also represent eddy-current and hysteresis losses of rotor as well as stator. For synchronous motor, saliency is taken into consideration. As well known model for cylindrical motor can be obtained directly from salient one as its special case.

1. INTRODUCTION

A control system is designed generally based on a mathematical model of plant (object to be controlled). So construction of appropriate mathematical model of plant has especially important meaning for designing control system and seeking excellent control performance. A model for control system must be established from at least two viewpoints of preciseness and compactness. Preciseness is requirement that how precisely model can represent characteristics of plant and compactness is requirement that how compact or simple the model is. Two requirements of preciseness and compactness often conflict to the other and practical mathematical model is constructed under their trade off.

Traditional vector control systems of AC servo motors (induction and synchronous motors) have been designed based on very compact mathematical models without core-loss factor. However core loss especially of stator cannot be neglected for seeking precise and/or efficient torque response in vector control. Although modeling of core loss on single-phase equivalent circuit at steady state of a specific frequency seems to have a century of history[1], recently have appeared a few of papers that point out influence of neglected core loss to the performance of vector control and/or present new dynamic models with core-loss factor for it[2]-[5].

For induction motor, Mizuno et al propose a 4^{th} order model that tends to represent stator eddy-current loss by core-loss resistance equivalently placed in serial with mutual part of stator inductance[2] On the other hand, Levi proposes another model that represents core loss by core-loss resistance in parallel with mutual inductance of stator and rotor[3]. Unfortunately Levi-model as a parallel model seems to be built under unrealistic assumptions that both stator and rotor currents contribute to core loss in the same way and amount, and that leakage inductance of stator and rotor produce no core loss. Levi-model does not consider what kind of core loss it deals with, for example stator or rotor core loss, eddy-current or hysteresis loss etc.

For synchronous motor, a few of serial and parallel models have been reported as well by Uezato, Morimto et al[4],[5]. Uezato-model is a kind of serial models dealing with eddy-current loss constructed by Mizuno approach[4], in other words a synchronous version of Mizuno-model. Morimoto-model is a kind of parallel models[5]. It has core-loss resistance exactly in parallel with stator inductance. Unfortunately its description and analysis are restricted to steady state in a synchronous frame and no attention is paid to motor dynamic response. Morimoto-model does not pay any attention to what kind of core loss it deals with just like Levi-model.

Undoubtedly, mathematical models of AC motors for vector control should consist of at least two basic equations such as differential equation describing electromagnetic dynamics and torque equation describing torque generating mechanism. Of course the composite basic equations should be consistent with the other. It is surprising that previously reported new models with core-loss factor have paid almost no attentions to the mathematical consistency and its examination. Generally speaking, mathematical models as products by approximation should have consistency at least of mathematical level as minimum modeling requirement.

This paper establishes in a new unified manner new mathematical models with core-loss factor for two kinds of AC motors, induction and synchronous motors. Since the AC motors have the same stator and different rotor, it is reasonable and desired to build up a common mathematical model for their stator.

For modeling compactly core-loss phenomena of stator we take the assumptions that stator current is decomposed into two different kinds of equivalent currents such as core-loss current bearing purely core losses and load current bearing purely flux and torque generations, and that core losses occur on an equivalent core-loss resistance. The established models result in parallel models that place stator equivalent core-loss resistance exactly in parallel with stator inductance. It is clearly shown in our models that equivalent core-loss resistance can express appropriately and separately both eddy-current and hysteresis losses rather than mere vague loss or mere core loss.

It is pointed out by steel-material researchers Kaido[5] that from standpoints of analysis and modeling of core material produced by today's technology, parallel models can approximate core-loss characteristics over wide frequency range better than serial models. Thus effectiveness of our new models of AC motors are supported together with popularity of variable-frequency motor drives by the resent research results of core material model.

We take the modeling stance that mathematical models of AC motors for vector control should consist of three basic equations consistent mathematically with the others, differential equation, torque equation, and energy transmission equation describing how injected energy is wasted as losses, saved as interior electromagnetic energy, and transmitted as mechanical energy. In energy-efficient control taking core losses into consideration the energy transmission equation with consistency is essential[7]. Proposed new models consist of such three consistent basic equations, where all vector signals are defined in general frame of arbitrary instant angular velocity. The consistency among them is verified of course in general frame. Generality of reference frame gives us more clear or deeper physical meaning of analytical results, and higher degree of freedom of model-based vector control design[8].

Proposed model of induction motor is the most compact in sense of the number of employed interior states and parameters. This compact model can also represent eddy-current and hysteresis losses of rotor as well as stator. For synchronous motor, saliency is taken into consideration. As well known model for cylindrical motor can be obtained directly from salient one as its special case.

2. MODELING OF STATOR WITH CORE LOSSES

A. Circuit Equation

Every vector in the following is defined as a 2x1 vector on d-q coordinates in rotational frame of arbitrary and instant angular velocity ω as shown in Fig.1.

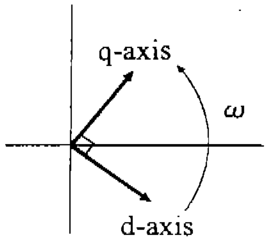

Fig.1 General frame of arbitrary angular velocity ω

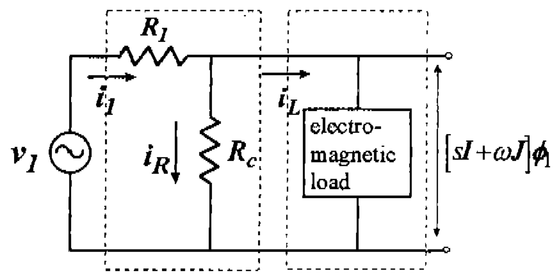

Fig2 Virtual vector circuit in general frame for AC motors with stator core-loss

For modeling compactly core-loss phenomena of stator, it is popular to employ implicitly or explicitly the following assumptions:

1) Harmonic components of current and flux are neglected.
2) Saturation in magnetic circuit is neglected.
3) Core-loss phenomena can be statically modeled.

The third assumption is supported by insistence of Mizuno et al that equivalent time constant of the core(iron) loss phenomena is much smaller than that of dominant dynamics of stator circuit[2] as well as requirement of model compactness. In addition to the above, we employs modeling assumptions such that

4) Stator current is decomposed into two different kinds of equivalent currents such as core-loss current bearing purely core losses and load current bearing purely flux and torque generations and core losses occur on an equivalent core-loss resistance by core-loss current.

Under the assumptions we can establish the following relation of vector signals of stator:

$$\boldsymbol{i}_1 = \boldsymbol{i}_R + \boldsymbol{i}_L \tag{1}$$

$$\boldsymbol{v}_1 = R_1\boldsymbol{i}_1 + R_c\boldsymbol{i}_R \tag{2}$$

$$R_c\boldsymbol{i}_R = [s\boldsymbol{I} + \omega\, \boldsymbol{J}]\boldsymbol{\phi}_1 \tag{3}$$

where $\boldsymbol{v}_1, \boldsymbol{i}_1, \boldsymbol{i}_R, \boldsymbol{i}_L, \boldsymbol{\phi}_1$ are voltage, current, equivalent core-loss current, equivalent load current, flux of stator respectively, and R_1, R_c are wiring(copper) and core(iron)-loss resistances of stator, s is differential operator d/dt and $\boldsymbol{J}$ is a 2x2 skew symmetric matrix such as

$$\boldsymbol{J} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}. \tag{4}$$

Using the concept of virtual vector circuit that can deal with vector signals defied in general frame as signals in the circuit[9], relations of (1) to (4) can be depicted as in Fig.2. It is clearly seen that equivalent core-loss resistance is place exactly in parallel with stator inductance

From (1)-(4), the following simple but important relationships for constructing mathematical model of stator circuit of AC motors are obtained

$$i_L = i_1 - i_R = i_1 - \frac{1}{R_c}\left[v_1 - R_1 i_1\right] \\ = \frac{1}{R_c}\left[-v_1 + \left(R_1 + R_c\right) i_1\right] \tag{5}$$

$$v_1 = R_1 i_L + \left(\frac{R_1 + R_c}{R_c}\right)\left[sI + \omega J\right]\phi_1 \tag{6}$$

B. Torque Equation

Torque τ is generated by interaction of stator flux and current dominated by law of electromagnetic force such as the left-hand law of Fleming. In addition to this law, using the fourth assumption such that torque is generated by load current i_L rather than stator current i_1 itself the following relation is established.

$$\tau = N_p i_L^T J \phi_1 \tag{7}$$

where N_p is the number of pole pairs.

We note that (7) suitably models the often-observed phenomenon that generated torque by conventional vector control technique which neglects core losses and uses stator current i_1 itself for producing torque is generally smaller than associated torque command.

C. Evaluation of Core Losses

It is important to verify how properly the resistance R_c newly introduced in new mathematical model can represent core losses of stator. We show that the losses by the resistance R_c can adequately represent both eddy-current and hysteresis losses caused by stator iron core.

The instant losses by R_c is expressed as product of R_c and squared core-loss current as clearly shown later in conjunction with establishment of energy transmission equation. In the case of $\|\phi_1\| = const$, we can evaluate the losses in terms of stator flux such as

$$R_c \|i_R\|^2 = \frac{1}{R_c}\left\|\left[sI + \omega J\right]\phi_1\right\|^2 = \frac{\omega_{1f}^2}{R_c}\|\phi_1\|^2 \tag{8}$$

where ω_{1f} is angular frequency of stator flux. Let us describe the resistance R_c by two resistances in parallel form shown in Fig. 3. i.e.

$$\frac{1}{R_c} = \frac{1}{R_{c0}} + \frac{1}{R_{c1}\left|\omega_{1f}\right|} \tag{9}$$

Applying (9) to (8) yields

$$R_c \|i_R\|^2 = \frac{\omega_{1f}^2}{R_{c0}}\|\phi_1\|^2 + \frac{\left|\omega_{1f}\right|}{R_{c1}}\|\phi_1\|^2 \,. \tag{10}$$

It is well known as physical phenomena of core material that eddy-current loss of the core is

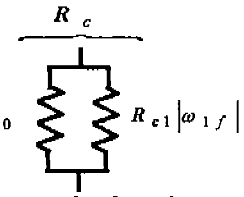

Fig. 3 An example of core loss resistance

proportional to product of squared magnitude and squared frequency of flux, and that hysteresis loss to product of squared magnitude and absolute frequency of flux. (10) implies that our stator equations for AC motors succeeds in modeling clearly and simply such core-loss phenomena appearing in core material.

It should be noted that as clearly indicated in (9) two equivalent resistances can be constant or simply linear over all dynamic range, and that it is especially desirable feature for simple design of vector control system being able to attain good performance over wide dynamic speed range. In serial models such as Mizuno-model and Uezato-model, such desirable characteristics cannot be obtained.

3. MODELING OF INDUCTION MOTOR

A. Differential equation

Let us define new parameters and interior states of induction motor such as[9]

$$M_n \equiv \left(\frac{M}{L_2}\right)M\,, \quad L_{1t} \equiv L_1 - M_n\,,$$

$$W_2 \equiv \frac{R_2}{L_2}\,, \qquad R_{2n} \equiv M_n W_2 = \left(\frac{M}{L_2}\right)^2 R_2$$

$$\phi_{2n} \equiv \left(\frac{M}{L_2}\right)\phi_2 \qquad i_{2n} \equiv \left(\frac{L_2}{M}\right)i_2$$

where right-hand sides of equations are conventionally defined parameters or states.

For modeling rotor dynamics we try to use basically the same model as conventional one of the minimum number of parameters[9]. Then stator flux ϕ_1 can be decomposed into leakage flux and rotor normalized flux ϕ_{2n} such as

$$\phi_1 = L_{1t} i_L + \phi_{2n} \tag{11}$$

and dynamics of rotor can be modeled as

$$\omega_{2n} J \phi_{2n} = R_{2n} i_{2n} + \left[sI + \omega J\right]\phi_{2n} \tag{12}$$

where ω_{2n} is electrical velocity of rotor. Currents of load and rotor, and flux are inductively linked such as

$$i_f = i_L + i_{2n} \tag{13}$$

$$\phi_{2n} = M_n i_f\,. \tag{14}$$

Rotor equations (11)-(14) together with stator equations yield a virtual vector circuit shown in Fig.4.

From both stator equations (5), (6) and rotor equations (11)-(14), we can establish directly the following dynamic equations using the minimum number of states and parameters for induction motor with core losses.

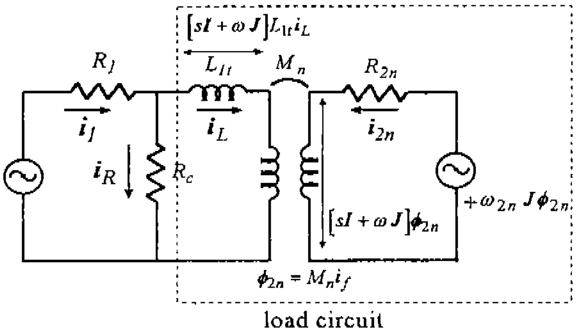

Fig.4 Virtual vector circuit in general frame for induction motor with core-loss

(i) Circuit Equation Form

$$v_1 = (R_1 + R_c)i_1 - R_c i_L \tag{15}$$

$$v_1 = R_1 i_L + \left(\frac{R_1 + R_c}{R_c}\right)[sI + \omega J][L_{1t} i_L + \phi_{2n}] \tag{16}$$

$$R_{2n} i_L = \left[(s + W_2)I + (\omega - \omega_{2n})J\right]\phi_{2n} \tag{17}$$

(ii) State Space Equation Form

$$\begin{aligned} s i_L &= -\left[\frac{1}{L_{1t}}\left(\frac{R_1 R_c}{R_1 + R_c} + R_{2n}\right)I + \omega J\right] i_L \\ &+ \frac{1}{L_{1t}}[W_2 I - \omega_{2n} J]\phi_{2n} + \frac{R_c}{L_{1t}(R_1 + R_c)} v_1 \end{aligned} \tag{18}$$

$$s\phi_{2n} = R_{2n} i_L - \left[W_2 I + (\omega - \omega_{2n})J\right]\phi_{2n} \tag{19}$$

$$i_1 = \frac{R_c}{R_1 + R_c} i_L + \frac{1}{R_1 + R_c} v_1 \tag{20}$$

In circuit equation form, (15), (16) indicate stator dynamics and (17) rotor dynamics respectively. State space form no longer indicates such simple structure, but shows another structure. Note that only five parameters including core-loss resistance R_c are used. In both dynamic equations, load current i_L and normalized rotor flux ϕ_{2n} are employed as interior states. It is of course possible to employ stator flux ϕ_1 instead of rotor flux ϕ_{2n}. Dynamic equations using load current and stator flux can be immediately constructed by simply applying (11) to (15)-(20).

B. Torque Equation

By simply applying (11) to (7) we can get the following torque equation in terms of the employed interior states

$$\tau = N_p i_L^T J\left[L_{1t} i_L + \phi_{2n}\right] = N_p i_L^T J\phi_{2n} \tag{21}$$

The effectiveness of (21) can be verified as well by observing that it can result in the well-known torque relation from viewpoint of rotor. i.e.

$$\begin{aligned} \tau &= N_p i_L^T J\phi_{2n} = N_p\left[i_f - i_{2n}\right]^T J\phi_{2n} \\ &= N_p \phi_{2n}^T J i_{2n} = N_p \phi_2^T J i_2 \end{aligned} \tag{22}$$

Note that we make no modification to conventional rotor model at all so far.

C. Energy transmission equation and consistency

Instant effective power injected to the motor is evaluated by multiplying both sides of (2) by stator current i_1^T and using (3) as follows:

$$\begin{aligned} i_1^T v_1 &= R_1\|i_1\|^2 + i_R^T[sI + \omega J]\phi_1 + i_L^T[sI + \omega J]\phi_1 \\ &= R_1\|i_1\|^2 + R_c\|i_R\|^2 + i_L^T[sI + \omega J]\phi_1 \\ &= R_1\|i_1\|^2 + R_c\|i_R\|^2 + i_L^T[sI + \omega J]\left[L_{1t} i_L + \phi_{2n}\right] \\ &= R_1\|i_1\|^2 + R_c\|i_R\|^2 + L_{1t} i_L^T[s i_L] + i_L^T[sI + \omega J]\phi_{2n} \\ &= R_1\|i_1\|^2 + R_c\|i_R\|^2 + \frac{s}{2L_{1t}}\|L_{1t} i_L\|^2 + i_L^T[sI + \omega J]\phi_{2n} \end{aligned} \tag{23}$$

On the other hand, instant energy associated with rotor is evaluated by multiplying both sides of (12) by rotor normalized current i_{2n}^T and using torque equation in (21) as follows:

$$\begin{aligned} 0 &= i_{2n}^T[sI + \omega J]\phi_{2n} + R_{2n}\|i_{2n}\|^2 - \omega_{2n}\, i_{2n}^T J\phi_{2n} \\ &= i_{2n}^T[sI + \omega J]\phi_{2n} + R_{2n}\|i_{2n}\|^2 + \omega_{2m}\tau \end{aligned} \tag{24}$$

where ω_{2m} is mechanical angular velocity of rotor such as

$$\omega_{2m} = \frac{\omega_{2n}}{N_p} \tag{25}$$

Summing up (23) and (24) and applying (13), (14) yields

$$\begin{aligned} i_1^T v_1 &= R_1\|i_1\|^2 + R_c\|i_R\|^2 + s\left(\frac{1}{2L_{1t}}\|L_{1t} i_L\|^2 + \frac{1}{2M_n}\|\phi_{2n}\|^2\right) \\ &+ R_{2n}\|i_{2n}\|^2 + \omega_{2m}\tau \end{aligned} \tag{26}$$

Equations (23), (24), (26) describe relations of energy transmission of stator, rotor and total of them respectively. Positive terms $R_1\|i_1\|^2$, $R_c\|i_R\|^2$ indicate losses of stator, more precisely copper and iron(core) losses respectively. The third and fourth terms in (23), the first term in (24), and the third term in (26) indicate instant change of electromagnetic energy saved in inductances L_{1t}, M_n. Another positive term $R_{2n}\|i_{2n}\|^2$ indicates electromagnetic losses in rotor as shown below.

$$R_{2n}\|i_{2n}\|^2 = R_2\|i_2\|^2 \tag{27}$$

The last term $\omega_{2m}\tau$ is instant mechanical energy as output.

In order to show simply physical meaning of each term in the energy transmission equations we use stator core-loss current i_R, and normalized current i_{2n}. It is of course possible to evaluate (23), (24), (26) in terms of employed interior states only such as stator load current i_L and rotor normalized flux ϕ_{2n}.

Equation (26) is our seeking energy transmission equation as the third basic equation for mathematical model, It clearly shows that injected energy is converted into physically meaningful energies only such as loss, interior saved and output mechanical energies in closed form. In other words, (26) does not have any suspicious factors of physically vague meaning. In addition, It is obtained from other basic equations such as dynamic equation and torque equation as derivation process shows. This means that a set of three basic equations constructing motor model holds complete consistency.

D. Rotor core losses

For induction motor, rotor has core losses as well as stator. We try to evaluate the losses represented by rotor normalized resistance.

Let us describe the rotor normalized resistance R_{2n} in more precise form similar to (9) such as

$$\frac{1}{R_{2n}}=\frac{1}{R_{2n0}}+\frac{1}{R_{2n1}|\omega_s|} \tag{28}$$

where ω_s is slip angular frequency, and two new parameters R_{2n0}, R_{2n1} are supposed to be constant.

Using (12) and (28), the losses represented by the rotor normalized resistance can be evaluated under condition of $\|\phi_{2n}\|=const$ as follows:

$$\begin{aligned} R_{2n}\|i_{2n}\|^2 &= \frac{1}{R_{2n}}\left\|\left[sI+(\omega-\omega_{2n})J\right]\phi_{2n}\right\|^2 \\ &= \frac{\omega_s^2}{R_{2n0}}\|\phi_{2n}\|^2+\frac{|\omega_s|}{R_{2n1}}\|\phi_{2n}\|^2 \end{aligned} \tag{29}$$

The first term of right side of (29) indicates combined losses of copper and eddy-current, and the second one hysteresis loss. Thus it is insisted that the rotor normalized resistance of our model can represent properly rotor core-loss phenomena as well as core-loss resistance of stator.

4. MODELING OF SALIENT SYNCHRONOUS MOTOR

A. Differential equation

As shown in Fig.5, suppose that N-S axis of rotor magnet takes angle θ to the principal axis of d-q coordinates in rotational frame of arbitrary and instant angular velocity ω.

As already explained in connection with Fig.2, our model takes the modeling stance that only load component of stator current i_1 contributes to generations of flux and torque. Then model of stator flux of salient synchronous motor on our stance can be directly established by replacing simply stator current i_1 of flux model having no core losses[10] with stator load current i_L. i.e.

$$\phi_1=\phi_i+\phi_m \tag{30}$$

$$\phi_i=\left[L_aI+L_bQ(\theta)\right]i_L \tag{31}$$

$$Q(\theta)=\begin{bmatrix} cos(2\theta) & sin(2\theta) \\ sin(2\theta) & -cos(2\theta) \end{bmatrix} \tag{32}$$

$$\phi_m=\Phi\begin{bmatrix} cos(\theta) \\ sin(\theta) \end{bmatrix}\ ;\ \Phi=const \tag{33}$$

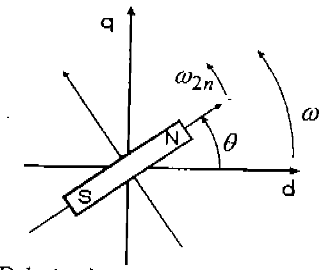

Fig.5 Relation between orthogonal coordinates and magnet flux.

where $\Phi(=const)$ is maximum flux due to magnet, ar L_a, L_b are stator inductances. Existence of non-ze inductance L_b implies that of saliency. L_a, L_b ha such relation with so-called d-q inductances L_d, L_q as

$$\begin{bmatrix} L_d \\ L_q \end{bmatrix}=\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} L_a \\ L_b \end{bmatrix}.$$

Simply applying (30) to stator equations (5), (6) yiel the objective differential equation describi electromagnetic dynamics of the motor where stat load current is used as a interior states.

(i) Circuit Equation Form

$$v_1=(R_1+R_c)i_1-R_ci_L \tag{34}$$

$$\begin{aligned} v_1 &= R_1i_L+\left(\frac{R_1+R_c}{R_c}\right) \\ &\quad *\left[\left[sI+\omega J\right]\left[L_aI+L_bQ(\theta)\right]i_L+\omega_{2n}J\phi_m\right] \end{aligned} \tag{35}$$

(ii) State Space Equation Form

$$\begin{aligned} si_L &= -\frac{R_1R_c}{R_1+R_c}\left[\frac{L_aI-L_bQ(\theta)}{L_a^2-L_b^2}\right]i_L \\ &+\left[-\omega I+\frac{2L_b\,\omega_{2n}}{L_a^2-L_b^2}\left[L_aQ(\theta)-L_bI\right]\right]Ji_L \\ &+\left[\frac{L_aI-L_bQ(\theta)}{L_a^2-L_b^2}\right]\left[-\omega_{2n}J\phi_m+\frac{R_c}{(R_1+R_c)}v_1\right] \end{aligned} \tag{36}$$

$$i_1=\frac{R_c}{R_1+R_c}i_L+\frac{1}{R_1+R_c}v_1 \tag{37}$$

B. Torque Equation

By simply applying (30) to torque equation (7) we ca get the following in terms of load current.

$$\begin{aligned} \tau &= N_pi_L^TJ\left[L_bQ(\theta)i_L+\phi_m\right] \\ &= N_p\left[L_bi_L^TJQ(\theta)i_L+i_L^TJ\phi_m\right] \end{aligned} \tag{38}$$

The first term of right-hand side indicates so-calle reluctance torque due to saliency.

C. Energy transmission equation and consistency

Instant effective power injected to the motor i evaluated by multiplying both sides of (2) by stato current i_1^T and using (3) as follows:

$$i_i^Tv_1=R_1\|i_1\|^2+i_R^T[sI+\omega J]\phi_1+i_L^T[sI+\omega J]\phi_1$$

$$= R_1\|i_1\|^2 + R_c\|i_R\|^2 + i_L^T[sI + \omega J]\phi_1$$

$$\begin{aligned} &= R_1\|i_1\|^2 + R_c\|i_R\|^2 \\ &\quad + i_L^T[sI + (\omega - \omega_{2n})J]\phi_1 + \omega_{2n}\, i_L^T J\phi_1 \end{aligned} \tag{39}$$

The third term of (39) can be rearranged using (30)-(32) as

$$\begin{aligned} & i_L^T[sI + (\omega - \omega_{2n})J]\phi_1 \\ &= i_L^T[sI + (\omega - \omega_{2n})J]\phi_i \\ &= i_L^T\left[(\omega_{2n} - \omega)L_b JQ(\theta)i_L + [L_a I + L_b Q(\theta)]si_L\right] \\ &= \frac{L_a}{2}s\|i_L\|^2 + \frac{L_b}{2}i_L^T[2(\omega_{2n} - \omega)JQ(\theta)i_L + 2Q(\theta)si_L] \\ &= \frac{s}{2}\left(L_a\|i_L\|^2 + L_b\left(i_L^T Q(\theta)i_L\right)\right) \\ &= \frac{s}{2}\left(i_L^T\phi_i\right) \end{aligned} \tag{40}$$

The fourth term of (39) can be rearranged using (7) and (25) as

$$\omega_{2n}\, i_L^T J\phi_1 = \omega_{2m}\tau\,. \tag{41}$$

Applying (40) and (41) to (39), we finally get the following energy transmission equation as the third basic equation:

$$\begin{aligned} i_1^T v_1 &= R_1\|i_1\|^2 + R_c\|i_R\|^2 + \frac{s}{2}\left(i_L^T\phi_i\right) + \omega_{2m}\tau \\ &= R_1\|i_1\|^2 + R_c\|i_R\|^2 \\ &\quad + \frac{s}{2}\left(L_a\|i_L\|^2 + L_b\left(i_L^T Q(\theta)i_L\right)\right) + \omega_{2m}\tau \end{aligned} \tag{42}$$

The third term of (42) indicates instant change of electromagnetic energy saved in inductances L_a, L_b. Physical meaning of the remaining terms are clear. It is the same as in case of induction motor.

The energy transmission equation in (42) as the third basic equation of mathematical model is in clearly closed form. In other words, (42) does not have any suspicious factors of physically vague meaning. In addition, It is obtained from other basic equations. This implies that a set of three basic equations constructing salient motor model holds complete consistency.

5. CONCLUDING REMARKS

This paper has proposed new mathematical models with core(iron) loss factor for two kinds of AC motors, induction and synchronous motors which are supposed to generate torque precisely or/and efficiently under vector controls. The new models have the following features distinguished from the previous models:

1) Both models of induction and synchronous motors are established in unified manner, where common or different features of the models can be clearly seen.
2) The models consist of three basic equations with complete consistency such as differential equation, torque equation and energy transmission equation.
3) The models can represent clearly, appropriately and separately both eddy-current and hysteresis losses of stator. The model of induction motor can represent rotor losses as well.
4) The model of induction is the most compact in sense of the number of employed interior states and parameters. The model of synchronous motor take saliency into consideration.
5) Reference frame employed for establishment and analysis of models is general frame of arbitrary instant angular velocity, which can allow easy and deep comprehension of physical meaning of analytical results, and high degree of freedom of model-based vector control design.

The newly developed mathematical models are very useful for solving design and analysis problems for vector control of AC motors with core losses. Indeed we have already gotten some fruitful results on some of them[7],[11]. However a lots of problems remains open where mathematical models such as the proposition are indispensable.

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