# THE DETECTION OF INSTANTANEOUS DISTORTED CURRENT WITH THREE-DIMENSIONAL SPACE VECTOR

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**ABSTRACT** – Active power filter is a kind of device used for compensating instantaneous reactive and harmonic current in three-phase circuits. An essential technology that determines the behavior of an active power filter is the method of detecting the distorted current. Using three-dimensional space vectors, this paper describes a simple method for detecting the distorted current without any coordinate transformation. The effectiveness of the novel method is verified by the theoretical analysis and simulation.

## 1. INTRODUCTION

Because active power filter performs dynamic compensation to harmonics and reactive power, it attracts more and more interests [1]. The main circuit of studied active power filter is shown in Fig.1. A voltage source inverter is used in the main circuit. When APF produces compensation currents that provide the harmonic and reactive currents required by the load, the supply current would be a sinusoidal current, so the distortion is eliminated.

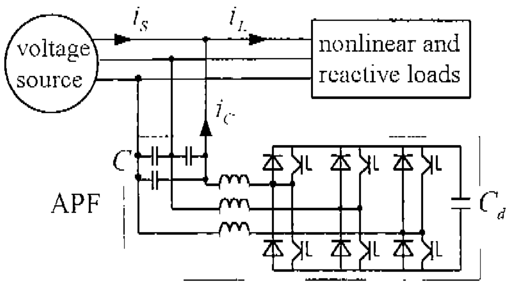


Fig.1 Circuit configuration

In order to obtain good performance, an essential technology that determines the behavior of an APF is the method of detecting the distorted current. The commonly used method is based on the coordinate transformation [2]. Normally, the control strategy of APF based on coordinate transformation is complex. Although some papers have proposed some distorted current detection methods without using coordinate transformation, the accuracy is not satisfied [3]. Using three-dimensional space vector, a new method for detecting the distorted current without coordinate transformation is described in this paper. The effectiveness of this method is verified with the theoretical analysis and simulation.

## 2. THE DETECTION OF INSTANTANEOUS DISTORTED CURRENT USING SPACE VECTOR

The definition of three-dimensional space vector and instantaneous power

Three-dimensional variable $x_a$、$x_b$、$x_c$ can be

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expressed by two-dimensional Park space vector:

$$\boldsymbol{x} = x_a + x_b e^{-j120^\circ} + x_c e^{j120^\circ} \quad (1)$$

Now in three-dimensional space, using three-ax coordinates x, y, and z, which orthogonal with each other, an instantaneous space vector $\boldsymbol{x}$ can be defined as.

$$\boldsymbol{x} = x_a \boldsymbol{x} + x_b \boldsymbol{y} + x_c \boldsymbol{z} \quad (2)$$

So the instantaneous voltage space vector $\boldsymbol{e}$ and instantaneous current space vector $\boldsymbol{i}$ can be described as:

$$\boldsymbol{e} = e_a \boldsymbol{x} + e_b \boldsymbol{y} + e_c \boldsymbol{z} \quad (3)$$

$$\boldsymbol{i} = i_a \boldsymbol{x} + i_b \boldsymbol{y} + i_c \boldsymbol{z} \quad (4)$$

where $\boldsymbol{x}$, $\boldsymbol{y}$, $\boldsymbol{z}$ are the unit vectors of x-axis, b-axis and z-axis respectively.

Suppose that the angle between the instantaneous voltage space vector $\boldsymbol{e}$ and the instantaneous current space vector $\boldsymbol{i}$ is $\theta$, the scalar quantity product of them is defined as the three-phase instantaneous active power, the vector quantity product is defined as the instantaneous reactive power vector.

$$p = \boldsymbol{e} \cdot \boldsymbol{i} = e_a i_a + e_b i_b + e_c i_c \quad (5)$$

$$\begin{aligned} \boldsymbol{q} &= \boldsymbol{e} \times \boldsymbol{i} \\ &= (e_b i_c - e_c i_b)\boldsymbol{x} + (e_c i_a - e_a i_c)\boldsymbol{y} + (e_a i_b - e_b i_a)\boldsymbol{z} \end{aligned} \quad (6)$$

As shown in Fig.2, $\boldsymbol{q}$ is a vector perpendicular to $e$-$i$ plane, and vector $\boldsymbol{i}$ can be decomposed into two components:

$$\boldsymbol{i} = \boldsymbol{i}_p + \boldsymbol{i}_q \quad (7)$$

where $\boldsymbol{i}_p$ is the instantaneous active current vector, which is parallel to vector $\boldsymbol{e}$. $\boldsymbol{i}_q$ is the instantaneous reactive current vector, which is perpendicular to vector $\boldsymbol{e}$.

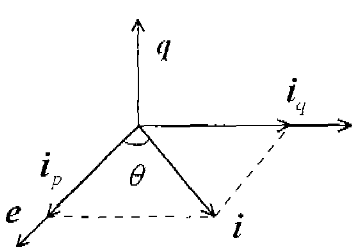


Fig.2 Instantaneous voltage and current space vector

<u>The relation between the conventional definition and the definition of instantaneous power under three-dimensional coordinate</u>

Suppose three-phase voltages are balanced symmetry sinusoidal waves, the currents contain harmonics.

$$\begin{cases} e_a = \sqrt{2}E_1 \sin \omega t \\ e_b = \sqrt{2}E_1 \sin(\omega t - \frac{2}{3}\pi) \\ e_c = \sqrt{2}E_1 \sin(\omega t + \frac{2}{3}\pi) \end{cases} \quad (8)$$

$$\begin{cases} i_a = \sum_n \sqrt{2}I_n \sin(n\omega t - \varphi_n) \\ i_b = \sum_n \sqrt{2}I_n \sin[n(\omega t - \frac{2}{3}\pi) - \varphi_n] \\ i_c = \sum_n \sqrt{2}I_n \sin[n(\omega t + \frac{2}{3}\pi) - \varphi_n] \end{cases} \quad (9)$$

where: $n=3k\pm1$, $k$ is an integer. $\omega$ is the power supply angular frequency, $I_n$, $\varphi_n$ are the rms value and phase angle of the harmonic respectively.

Substitute (8) and (9) into (5), then (10) is attained

$$p = 3E_1 \sum_n I_n \cos[(1 \mp n)\omega t + \varphi_n] \quad (10)$$

Substitute (8) and (9) into (6), then (11) is attained

$$\begin{aligned} \boldsymbol{q} = &\sqrt{3}E_1 \sum_n \pm I_n \sin[(1-n)\omega t + \varphi_n]\boldsymbol{x} \\ &+ \sqrt{3}E_1 \sum_n \pm I_n \sin[(1-n)\omega t + \varphi_n]\boldsymbol{y} \\ &+ \sqrt{3}E_1 \sum_n \pm I_n \sin[(1-n)\omega t + \varphi_n]\boldsymbol{z} \end{aligned} \quad (11)$$

The modulus of vector $\boldsymbol{q}$ is given as (12).

$$\begin{aligned} q = |\boldsymbol{q}| &= \sqrt{3 \times \{\sqrt{3}E_1 \sum_n \pm I_n \sin[(1-n)\omega t + \varphi_n]\}^2} \\ &= 3E_1 \sum_n \pm I_n \sin[(1-n)\omega t + \varphi_n] \end{aligned} \quad (12)$$

From (10) to (12), positive sign is taken when $n=3k+1$, negative sign is taken when $n=3k-1$.

It can be found from (10) and (12), when three-phase voltages are balanced symmetry sinusoidal waves and currents contain harmonics, the instantaneous active power $p$ and reactive power $q$ defined under three-dimensional coordinate can be expressed as:

$$p = \overline{p} + \widetilde{p} \tag{13}$$

$$q = \overline{q} + \widetilde{q} \tag{14}$$

where $\overline{p}$、$\overline{q}$ are direct current components, corresponding to active component and reactive component of fundamental wave in three-phase currents.

$$\overline{p} = 3E_1 I_1 \cos \varphi_1 \tag{15}$$

$$\overline{q} = 3E_1 I_1 \sin \varphi_1 \tag{16}$$

where $\widetilde{p}$、$\widetilde{q}$ are alternating components, corresponding to the harmonics in three-phase currents.

The above equations lead to the following conclusion. When voltage and current are three-phase symmetry sinusoidal waves, the defined instantaneous power $p$ and $q$ using three-dimensional coordinates are constant, as shown in (15) and (16). They equal the values of active and reactive power in the conventional definition. It is thus clear that the new definition applies to not only sinusoidal circuit, but also nonsinusoidal circuit. Therefore, the proposed definition can be considered as an improvement and extension of conventional theory.

<u>Space vector detecting method of instantaneous distorted current</u>

Using the definition of instantaneous voltage, current and power, the instantaneous active current can be detected.

When vector $e$ and vector $\boldsymbol{i}_p$ have same direction, three-phase instantaneous active power is:

$$p = \boldsymbol{e} \cdot \boldsymbol{i} = \boldsymbol{e} \cdot (\boldsymbol{i}_p + \boldsymbol{i}_q) = \boldsymbol{e} \cdot \boldsymbol{i}_p = |e||\boldsymbol{i}_p| \cos 0^\circ = |e||\boldsymbol{i}_p| \tag{17}$$

$$|\boldsymbol{i}_p| = \frac{p}{|e|} = \frac{p|e|}{|e||e|} = \frac{p|e|}{\boldsymbol{e} \cdot \boldsymbol{e}} \tag{18}$$

Because vector $\boldsymbol{e}$ and $\boldsymbol{i}_p$ have same direction, so

$$\boldsymbol{i}_p = \frac{p\boldsymbol{e}}{\boldsymbol{e} \cdot \boldsymbol{e}} \tag{19}$$

When vector $\boldsymbol{e}$ and vector $\boldsymbol{i}_p$ have opposite direction, three-phase instantaneous active power is

$$\begin{aligned} p &= \boldsymbol{e} \cdot \boldsymbol{i} = \boldsymbol{e} \cdot (\boldsymbol{i}_p + \boldsymbol{i}_q) = \boldsymbol{e} \cdot \boldsymbol{i}_p \\ &= |e||\boldsymbol{i}_p| \cos 180^\circ = -|e||\boldsymbol{i}_p| \end{aligned} \tag{20}$$

$$|\boldsymbol{i}_p| = \frac{-p}{|e|} = \frac{-p|e|}{|e||e|} = \frac{-p|e|}{\boldsymbol{e} \cdot \boldsymbol{e}} \tag{21}$$

Because vector $\boldsymbol{e}$ and $\boldsymbol{i}_p$ have opposite direction, so

$$\boldsymbol{i}_p = \frac{-p(-\boldsymbol{e})}{\boldsymbol{e} \cdot \boldsymbol{e}} = \frac{p\boldsymbol{e}}{\boldsymbol{e} \cdot \boldsymbol{e}} \tag{22}$$

From above analysis, the instantaneous active power space vector detecting equation is obtained.

$$\boldsymbol{i}_p = \frac{p\boldsymbol{e}}{\boldsymbol{e} \cdot \boldsymbol{e}} \tag{23}$$

From the above discussion, when three-phase voltage and current are as shown as (8) and (9), the fundamental component produces $\overline{p}$ and harmonic component produces $\widetilde{p}$ in three-phase instantaneous active power. Thus the instantaneous fundamental active current space vector can be derived from (23)

$$\overline{\boldsymbol{i}}_p = \frac{\overline{p}\boldsymbol{e}}{\boldsymbol{e} \cdot \boldsymbol{e}} \tag{24}$$

Substitute (8) and (15) into (24), that is

$$\begin{aligned} \overline{\boldsymbol{i}}_p &= [\sqrt{2} I_1 \cos \varphi_1 \sin \omega t]\boldsymbol{x} \\ &+ [\sqrt{2} I_1 \cos \varphi_1 \sin(\omega t - \tfrac{2}{3}\pi)]\boldsymbol{y} \\ &+ [\sqrt{2} I_1 \cos \varphi_1 \sin(\omega t + \tfrac{2}{3}\pi)]\boldsymbol{z} \end{aligned} \tag{25}$$

It is clear that using (24) the fundamental active component involved in harmonic current is detected accurately.

For the shunt active power filter shown in Fig.1, after $\overline{\boldsymbol{i}}_p$, the fundamental active component of the load current which contain harmonics is obtained, the space vector of instantaneous distorted current which need to be compensated is expressed as follows.

$$\boldsymbol{i}_C = \boldsymbol{i}_L - \overline{\boldsymbol{i}}_p = \boldsymbol{i}_L - \frac{\overline{p}(e_a\boldsymbol{x} + e_b\boldsymbol{y} + e_c\boldsymbol{z})}{e_a^2 + e_b^2 + e_c^2} \tag{26}$$

where, $\boldsymbol{i}_L$ is load current space vector:

$$\boldsymbol{i}_L = i_{La}\boldsymbol{x} + i_{Lb}\boldsymbol{y} + i_{Lc}\boldsymbol{z} \tag{27}$$

In three-phase three-line circuit, we have $e_a + e_b + e_c = 0$, so (26) can be expressed in this way

$$\begin{cases} i_{Ca} = i_{La} - \dfrac{\bar{p}e_a}{2[(e_a+e_b)^2 - e_a e_b]} \\ i_{Cb} = i_{Lb} - \dfrac{\bar{p}e_b}{2[(e_a+e_b)^2 - e_a e_b]} \\ i_{Cc} = i_{Lc} - \dfrac{\bar{p}e_c}{2[(e_a+e_b)^2 - e_a e_b]} \end{cases} \tag{28}$$

Fig.3 shows the diagram of detecting circuit for instantaneous distorted current based on (28). The normally used distorted current detecting method is base on coordinate transformation, for example, $p$, $q$ method and $i_p$,$i_q$ method [4]. When implemented by analogue circuit, $p$,$q$ method needs 12 multipliers, $i_p$,$i_q$ method needs 8 multiplicater, a phase-locked loop and a sinusoidal and cosine generator. The method proposed in this paper only need 8 multiplicater and can be implemented easily. If implemented by software, the calculation need to be done is reduced obviously.

## 3. SIMULATION RESULT

According to (28), the simulation programs can be written, the results are shown in Fig.4. The nonlinear load is simulated by a converter, which consists of a fully controllable bridge of switches. In the Fig.4(a), only the waveforms of phase a, the load current waveform $i_{La}$ when firing angle α is 0°, 30°, 60° respectively are shown. Fig.4(b) and (c) correspondingly present the waveform of the desired compensating current $i_{Ca}$ and the compensated line current $i_{Sa}$, (d) illustrates the power supply voltage waveform $u_a$. The results of simulations indicate that the distorted current can be detected accurately by the proposed method.

## 4. CONCLUSION

In order to obtain an ideal compensation performance, an essential technology for active power filter is the method of detecting the distorted component involved in load current. Using three-dimensional space vector, a new method for detecting

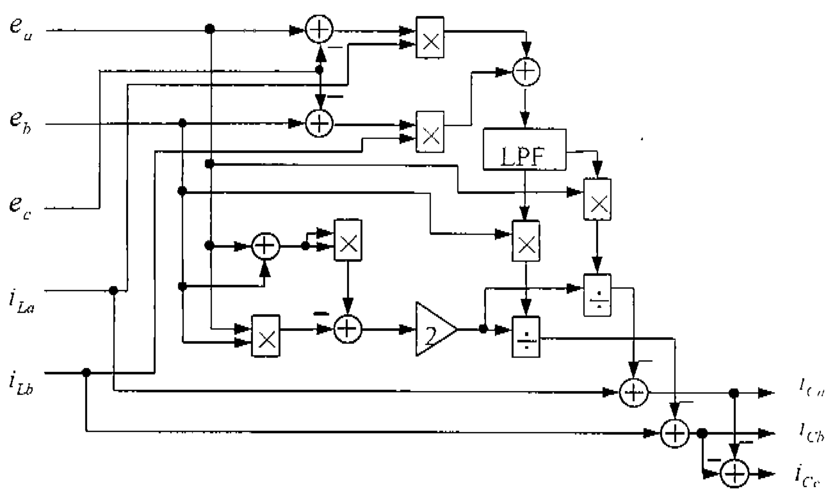


Fig.3 Diagram of detecting circuit for instantaneous distorted current

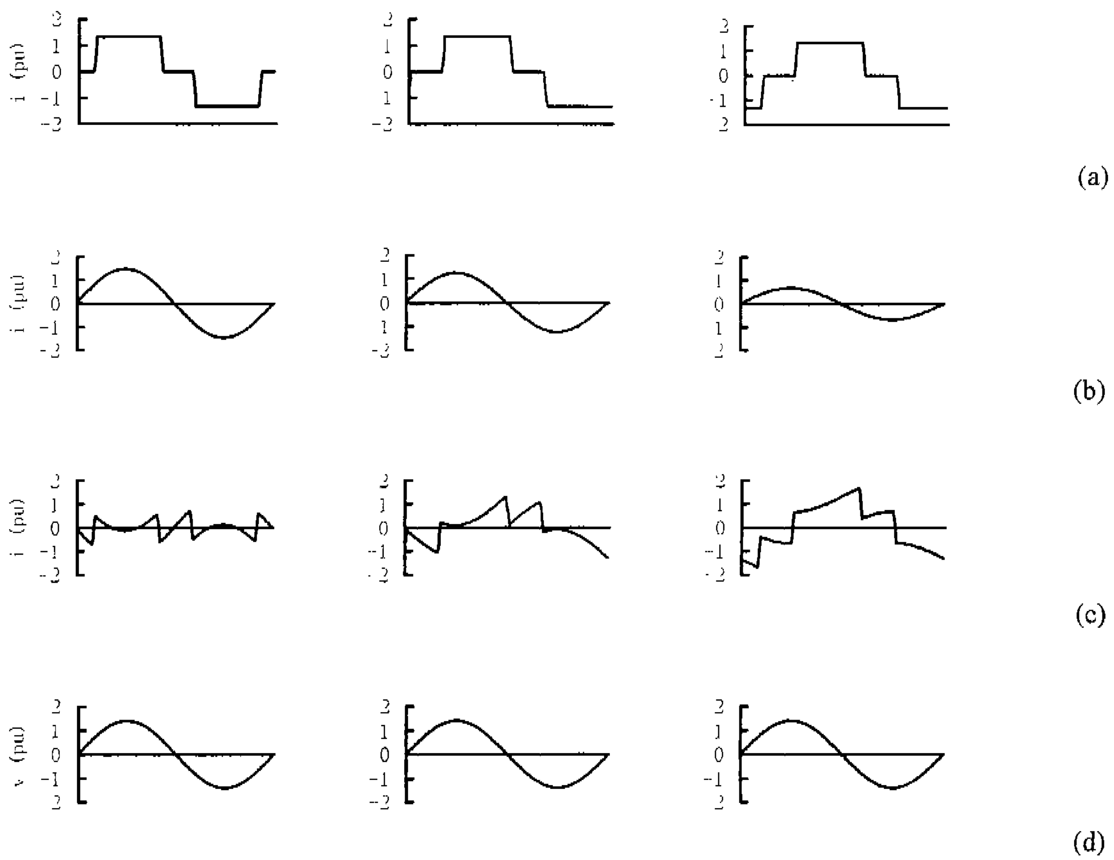


Fig.4 Simulation results

the distorted current is described in this paper. The proposed method does not need complex coordinate transformation, has high accuracy and is easy to implement. The effectiveness of the novel method is verified with the theoretical analysis and simulation.

## 5. REFERENCES

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