A New Small Signal Modeling of Average Current Mode Control

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Abstract

A new small signal modeling of an average current mode control is proposed. In order to analyze the characteristics of the control scheme, the discrete and continuous time small signal models are derived. The derivation are mainly come from the analysis of the sampling effect presented in the current control loop. By the mathematical interpretation of a practical sampler representing the sampling effect of a current control loop, the small signal models of an average current mode control can be easily derived. The instability of the current control loop, which gives rise to the subharmonic oscillation, can be identified by the proposed models. To show the usefulness of the proposed models, the simulation and experiment are carried out. The results show that the predicted results by the proposed model are much better agreed with the measured ones than that of the conventional model, even though the high gain of the compensation network of a current control loop is employed.

1. INTRODUCTION

The average current mode control has been recently reported, and the superior characteristics over a peak current mode control such as a good tracking performance of an average current, no slope compensation, and noise immunity have been discussed [7]. In an average current mode control, the compensation network is presented in a current control loop to use the average current as a controlled quantity. The DC information of a sensed inductor current is obtained by the compensation network and compared with the ramp signal to achieve pulse width modulation (PWM). By the introduction of a compensation network in a current control loop, however, the analysis of an average current mode control is more complicated.

In general, the continuous time small signal model is very useful to analyze the characteristics of power supplies[1],[2]. The easiest way of obtaining the

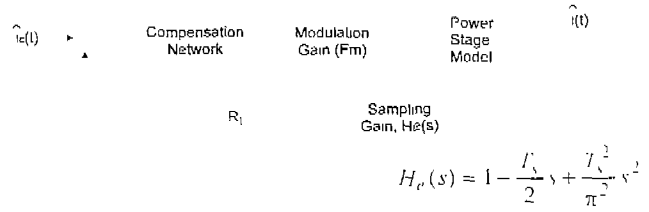

Fig. 1. Conventional small signal model structure of an average current mode control including the sampling gain.

continuous time small signal model is to use the low frequency model [3],[4]. This low frequency model can be simply obtained by using an averaging method. Due to the complexity of the control loop of an average current mode control which is mainly come from the presence of a compensation network, the low frequency model has been a useful tool in a control loop design. However, for a high gain of a compensation network, the dynamic response predicted by this model shows a difference from that of a real system. This is come from the fact that the low frequency model is not accurate in a high frequency region. To remedy this problem, a study to improve the characteristics of a low frequency model of an average current mode control has been reported [8]. In Fig. 1, the small signal model structure of an average current mode control obtained in [8] is shown. The sampling gain He(s) adopted from the modeling of a peak current mode control in [5] is placed at the feedback inductor current loop. By using the similarity of a peak and an average current mode controls, the modeling technique similar to that of a peak current mode control has been applied. By adopting the previous results of a peak current mode control, the continuous time small signal model can be simply obtained. And the derived model can gives the explanation of a subharmonic oscillation which is come from the current loop instability. The sampling effect presented in a current control loop has been accounted and the modification of a low frequency model has been accomplished. In the process of modeling of an average current mode control, the sampling gain obtained in [5] which is for a peak

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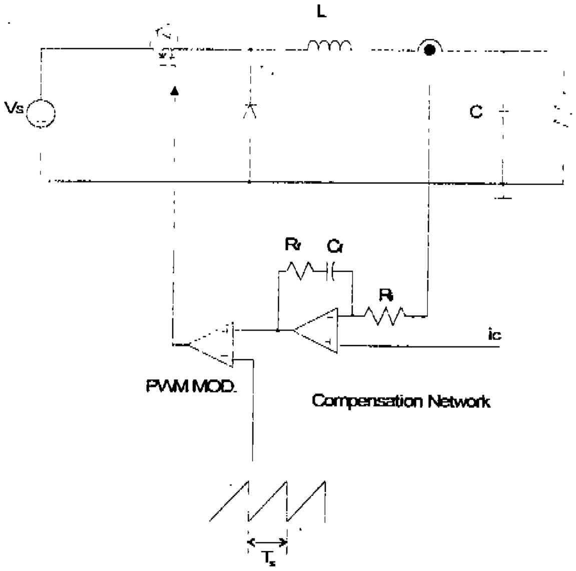


Fig. 2. Circuit diagram of a buck converter employing average current mode control.

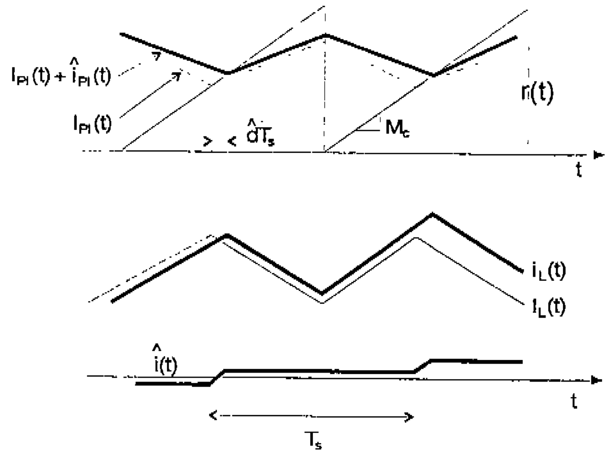


Fig. 3. Modulator waveforms of average current mode control

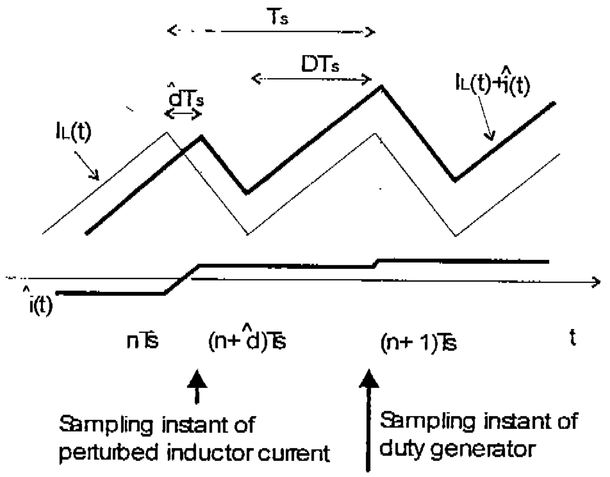


Fig. 4. Waveforms of the inductor current

current mode control is directly used. This leads the continuous time small signal model presented in [8] to be inaccurate if the gain of a compensation network is high.

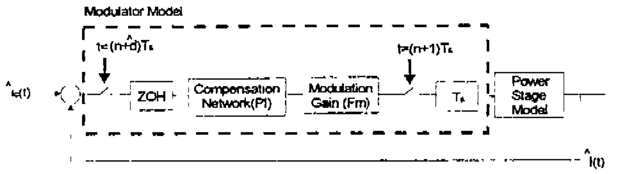
In this paper, the models of an average current mode control are newly derived in discrete and continuous time domains. In order to derive the models, the analysis of the sampling effect presented in a current control loop is carried out. The model of a practical sampler mentioned in a previous paper [6] to denote the difficulty in modeling of the current mode control has been presented in this paper, using two ideal samplers operated on the perturbed current and duty cycle generator with different sampling instants. By the combinations of the practical sampler and the low frequency models, the new model of an average current mode control can be obtained. The instability of the current control loop, which gives rise to the subharmonic oscillation, can also be identified by the proposed model. Furthermore, this proposed model gives more accurate prediction on the behaviors of an average current mode control compared to previous one. To show the validity of the proposed model, the simulation and experiment are carried out.

2. Basic Structure of Proposed Model

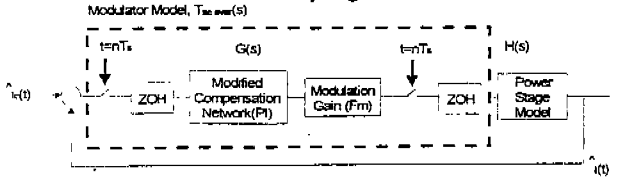
The circuit diagram of an average current mode controlled buck converter which is composed of a power stage, a compensation network, and a modulator employing PWM is shown in Fig. 2. Due to the presence of a compensation network, the characteristics of an average current mode control is considered differently from that of a peak current mode control and the modeling of an average current mode control is complicated.

A. Model of Sampling Effect

The sampling effect presented in a current control loop should be considered in order to obtain more accurate models of a current mode controlled converter, and to explain the phenomenon of a subhamonic oscillation. Fig. 3 shows the waveforms of an average current mode control modulator. As shown in this figure, the inductor current $i_L(t)$ consists of a steady-state current $I_L(t)$ and a small perturbed current $\hat{i}(t)$. The pulse width modulation (PWM) is achieved by comparing two signals, i.e., ramp signal, $r(t)$ and a compensator output signal, $i_{PI}(t)$. It is noticed that the sampling effect is included in the waveform of a small perturbed current. As noted in [6], the response of a perturbed current $\hat{i}(t)$ can be considered as that of the low-frequency model connected in series with a practical sampler. The practical sampler is used to indicate that the sampling is done by a series of pulses, not



(a) Model structure with two sampler operating at the different sampling instant



(b) Model structure with two sampler operating at the same sampling instant

Fig. 5. Model structure of average current mode control.

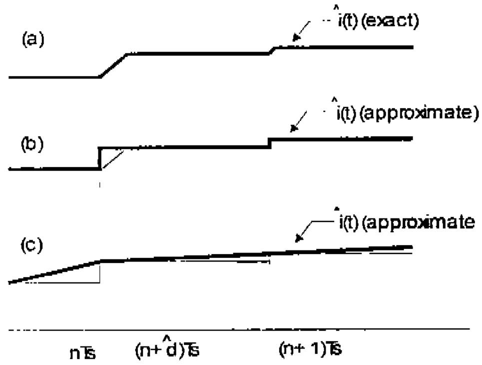


Fig. 6. Approximations of the perturbed inductor current

impulses. This makes it difficult to use the Shannon's sampling theorem. However, this difficulty can be overcome by developing the mathematical expression for a practical sampler as presented in this paper.

To obtain this mathematical expression for an average current mode control, the expanded view of the inductor current is considered as shown in Fig. 4. As can be seen in this figure, there are two slopes in the perturbed inductor current. One is kept zero over the interval between $(n+\hat{d})T_S$ and $(n+1)T_S$, which shows the possibility of existence of an ideal sampler with zero order holder in a current loop. The other slope is a positive or negative one which determines the magnitude variation of a perturbed inductor current. The perturbed current caused by this slope can be rebuilt with the combination of the low frequency model of a power converter and the duty cycle modulator which contains an ideal sampler and a modulator gain. Therefore, the models of a practical sampler can be expressed with two ideal samplers operated at the instants of $(n+\hat{d})T_S$ and $(n+1)T_S$, respectively. Considering the difference between two sampling instants, the model structure of an average current mode control can be drawn as shown in Fig. 5(a). The response of a perturbed inductor current of this model structure is shown in Fig. 6(b). Fig. 5(a) shows that with different sampling instants, one of two samplers is used for the perturbed inductor current and another is for the duty cycle generator. As can be well understood, since two samplers have the different sampling instants, the development of models for an average current mode control is difficult. To unify the sampling instants of two samplers, the model structure is modified by adding another zero order holder as illustrated in Fig. 5(b). This modification is come from the equivalent condition of the perturbed current at the sampling instant, nT_S. The response of a perturbed inductor current of this model structure is shown in Fig. 6(c). At the time of sampling instant, the perturbed inductor current response of the model shown in Fig. 5(a) is the same as that of Fig. 5(b). Fig. 6 shows the relationship between the perturbed current and the sampling instant based on the derived models of Fig. 5.

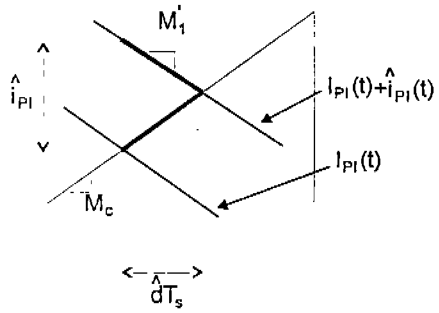


Fig. 7. Expanded view of average current mode modulator waveforms

Consequently, the converter system employing the proposed sampler model is shown in Fig. 5(b), and is useful to analyze the characteristics of an average current mode controlled converter.

B. Modification of Compensation Network

The sampling instant of an ideal sampler placed on a duty cycle generator is changed to that of the perturbed inductor current, and the modification of a compensator gain should be considered. For the model structure shown in Fig. 5(a), the output value of a compensator is varied during the time intervals of $t=(n+\hat{d})T_S$ and $t=(n+1)T_S$. The variations can be expressed as

$$\begin{aligned}\Delta\hat{i}_{PI} &= k_i(1-\hat{d})T_s\hat{e} \\ &= k_iT_s\hat{e} - k_iT_s\hat{d}\hat{e}\end{aligned} \qquad (1)$$

where $\hat{e}$ denotes the error signal between the perturbed reference and perturbed inductor currents. By assuming that the perturbed quantities are small, the second term of

TABLE I

The parameters of a buck converter

Switching Frequency (F_S)	Source Voltage (V_S)	Output Voltage (V_O)	Inductance (L)	Capacitance (C)	Load (R)
70 kHz	25V	8.5	70 uH	470 uF	8 Ω

right hand side of (1) can be reduced to zero. In order to guarantee the equivalent condition of the perturbed inductor current at $t=(n+1)T$ for the model structure shown in Fig. 5(b), the compensation network should be modified from (1) as follows:

$$G'_{comp}(s) = k'_p + \frac{k_i}{s} \tag{2}$$

where

$$k'_p = k_p + k_i T_s . \tag{3}$$

It is noticed that by the unification of two sampling instants of a practical sampler model, the modification of a compensation network should be carried out according to the equivalent condition of the perturbed inductor current at $t=(n+1)T$ for the model structure shown in Fig. 5(b).

3. SMALL SIGNAL MODELING

A. Discrete Time Small Signal Model

Although the design insights can not be provided, the discrete time model is useful for understanding the behaviors of the power converter. For the case of an average current mode controlled converter, it is difficult to achieve the discrete time model due to the presence of the compensating network in a current control loop. However, by approximating the perturbed inductor current as shown in Fig. 6(c) which is come from the proposed approach, the discrete time model can be easily obtained. As an example, the buck converter is considered as a power stage model. From the averaged model of a buck converter [7], the gain of inductor current to duty cycle can be expressed as

$$F_i(s) = \frac{\hat{i}(s)}{\hat{d}(s)} = \frac{V_s}{Ls} \tag{4}$$

where V_s is the source voltage. The expanded view of average current mode modulator waveforms is shown in Fig. 7. From this figure, the modulator gain can be obtained as

$$F_m = \frac{\hat{d}}{\hat{i}_{pi}} = \frac{1}{M_e T_s + \left(k_p M_1 + k_i M_1 D T_s\right) T_s} \tag{5}$$

where M_1 and M_e are the on-time and external ramp slopes, respectively. With the transfer function of a zero order holder as

$$H_{zoh}(s) = \frac{1-e^{-sT_s}}{s} \tag{6}$$

the discrete time expression of a combined gain of $H_{zoh}(s)$, $G'_{comp}(s)$, and F_m is derived as

$$G(z) = Z\left\{H_{zoh}(s)G'_{comp}(s)F_m\right\} = F_m \frac{k'_p z - k_p}{z-1} \tag{7}$$

And the z-domain expression of a combined gain of $H_{zoh}(s)$ and $F_i(s)$ is obtained as

$$H(z) = Z\left\{H_{zoh}(s)F_i(s)\right\} = \frac{V_s}{L}\frac{T_s}{z-1} \tag{8}$$

From (7) and (8), the discrete time model of a current loop transfer function for an average current-mode controlled buck converter is obtained as follows:

$$T_i(z) = \frac{G(z)H(z)}{1+G(z)H(z)} = \frac{\beta z + \alpha - \beta}{z^2 + (\beta - 2)z + 1 + \alpha - \beta} \tag{9}$$

where

$$\alpha = F_m \frac{V_s}{L} T_s^2 k_i, \qquad \beta = F_m \frac{V_s}{L} T_s k'_p \tag{10}$$

B. Continuous Time Small Signal Model

In analyzing and designing the controlled power converter, it is very useful to employ the continuous time small signal model. From the proposed model structure of an average current mode control shown in Fig. 5(b), the continuous time small signal model including the sampling effect presented in a current control loop can be simply achieved. To obtain the continuous time small signal model, the followings are defined as

$$G(s) = H_{zoh}(s)G'_{comp}(s)F_m, \qquad H(s) = H_{zoh}(s)F_i(s) \tag{11}$$

From the model structure shown in Fig. 5(b), the perturbed inductor current can be obtained as follows:

$$\hat{i}(s) = \frac{G^*(s)H(s)}{1+G^*(s)H^*(s)}\hat{i}_c^*(s) \tag{12}$$

where * denotes the sampled quantity.

From (12), the current loop transfer function can be obtained as

$$\frac{\hat{i}(s)}{\hat{i}_c(s)} = \frac{G^*(s)F_i(s)}{1+G^*(s)H^*(s)} . \tag{13}$$

4. Simulation and Experimental Results

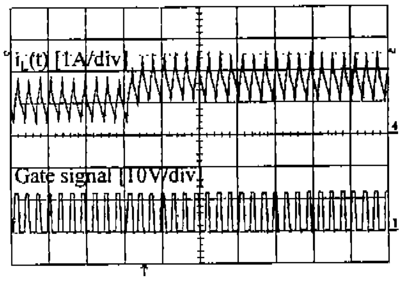
The parameters used are shown in Table 1. To show the accuracy of the proposed continuous time small signal model, the transient responses of the inductor current are examined under several different compensation network gains. Fig. 8 shows the inductor current responses with the compensation network gains of k_p=0.48 and k_i=0.0253/T_S for the step change of a reference signal. The experimental results for an inductor current and a gate signal are shown in Fig. 8(a). To verify the usefulness of the proposed model, simulation results are obtained and compared with the proposed and conventional models, i.e., low frequency and Tang's models in Fig. 8(b). For the convenient comparison, the results of a circuit level simulation using Psim is also presented in this figure, which has the same waveshape as the experimental result of Fig. 8(a). The low frequency model reveals the incorrectness compared to the experimental result. The inductor current response of simulation and experimental results, however, is well agreed with that of a proposed continuous time small signal and Tang's models for this low gains of a compensation network.

In Figs. 9 and 10, the inductor current responses are shown with the compensation network gains of k_p=0.48, k_i=0.5411/T_S and k_p=0.48, k_i=1.1905/T_S, respectively. The response of the inductor current becomes oscillatory when the integral gain of a compensation network is increased. As can be noticed in these figures, the inductor current response predicted by Tang's model is more oscillatory than that by the proposed model for a large integral gain of a compensation network. And the simulation and experimental results are well agreed with the predicted results by a proposed model. Particularly in Fig. 10, the predicted result by Tang's model shows that the current control loop is unstable even though the experimental result is not. Fig. 11 shows a bode plot for the control-to-inductor current transfer function of the proposed and conventional models with two different values of K_i. As shown in this figure, for the low integral gain of a compensation network, the proposed and conventional models show similar frequency response. For the increased value of an integral gain, the peaking of a frequency response is also increased.

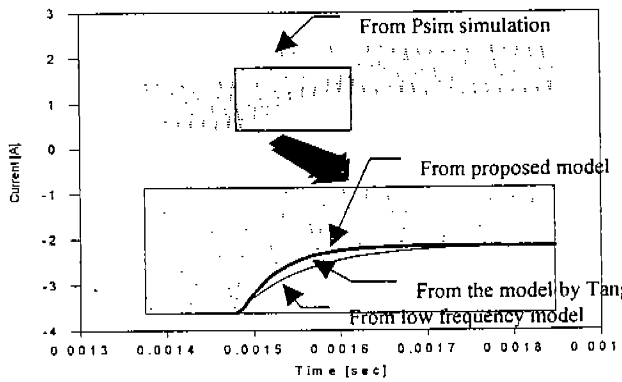
These results show that the newly proposed model is useful to predict the behaviors of the perturbed inductor current of power supplies employing an average current mode control.

5. Conclusions

In this paper, a new small signal modeling method for an average current mode control is proposed. By considering the sampling effect presented in a current control loop, the model structure of an average current mode control is derived. From the derived model structure, the discrete time and the corresponding continuous time models are obtained. From the derived discrete time model, the sampling gain in a continuous time domain is obtained. The proposed continuous time model shows a good performance in predicting the behaviors of a perturbed inductor current regardless of the compensator gains. Simulation and experiment are carried out for an average current mode controlled buck converter to verify the usefulness of the proposed model. With the variations of the compensation network gains, the time response of an inductor current of a proposed model is compared with those of the conventional models. From the simulation and experimental results, it is expected that the proposed continuous time model can be effectively used in analyzing the characteristics of an average current mode controlled converter.

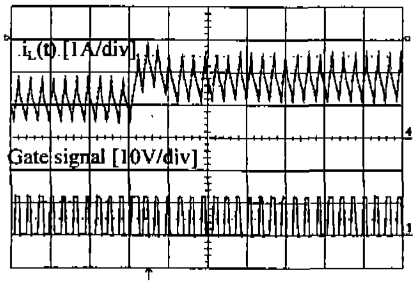


(a) Experimental results

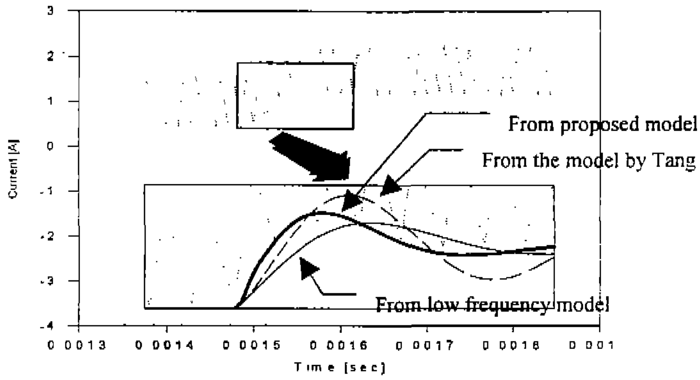


(b) Predicted results

Fig. 8. Inductor current responses with the compensation network gains of K_p=0.48 and K_i=0.0253/T_S.

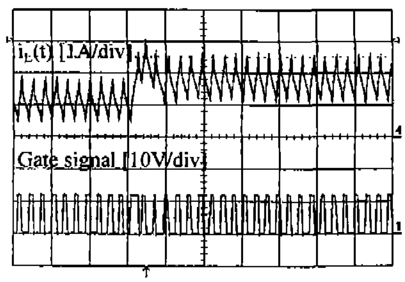


(a) Experimental results

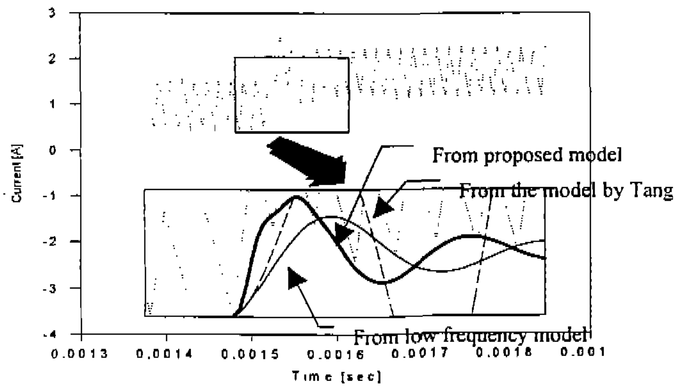


(b) Predicted results

Fig. 9. Inductor current responses with the compensation network gains of K_p=0.48 and K_i=0.5411/T_s.



(a) Experimental results



(b) Predicted results

Fig. 10. Inductor current responses with the compensation network gains of K_p=0.48 and K_i=1.1905/T_s.

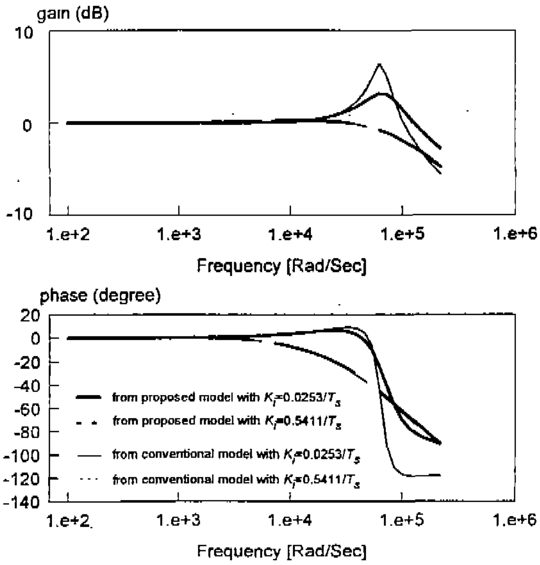


Fig. 11. Bode plot of the control-to-inductor current transfer function.

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