LINE SIDE HARMONIC ANALYSIS OF SINGLE-PHASE DIODE BRIDGE RECTIFIER WITH A DC FILTER CAPACITOR

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ABSTRACT—Single-phase diode bridge rectifiers with dc filter capacitor usually operate in discontinuous mode and generate large amounts of harmonic currents. This paper presents a precise harmonic analysis of the line side current in the case that both the filter inductor and the filter capacitor are not infinite. The analytical expressions of the line side performance parameters associated with harmonics are derived. The curves that show the relationship of these parameters as the functions of circuit parameters are illustrated. By simulation the results are verified to be accurate and the conclusion clearly reveals the relations between the line side performance parameters and circuit parameters.

1. INTRODUCTION

Single-phase diode bridge rectifiers with dc filter capacitors have been widely used. In recent years they are often used to provide dc input voltage for voltage source inverters (VSI) and dc-to-dc converters, with small rating up to several kilowatts. Fig.1 shows the typical circuit. Here the capacitor C, connected in parallel at the dc side, is employed to reduce the ripple in the dc voltage, while the series inductor L, usually small in inductance value, is to suppress the peak of the input current. Hence the dc side current i_d is discontinuous and the line side current far from being sinusoidal. As VSI and dc-to-dc converters supplied by this type of circuit are increasingly used, the analysis on the line side harmonic current is therefore worthwhile.

So far most of the previous literatures dealing with the harmonic analysis of LC-filtered diode bridge rectifier

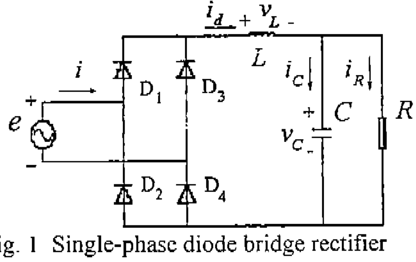


Fig. 1 Single-phase diode bridge rectifier with a filter capacitor on the dc side

assumed a near infinite filter inductance L or a near infinite filter capacitance C [1]-[4] , and laid emphasis on the continuous current mode. These assumptions made the analysis relatively simple and the results easy to be remembered, but considerably far from practical conditions. Recent published works [5] and [6] made studies in the case where both the filter inductor and the capacitor are finite, and focused on discontinuous current mode, since the rectifiers supplying VSI and dc-to-dc converters are usually operated in this mode. However these two papers, based on frequency domain method, are not so precise, as some approximations have been made in the analysis, and more over, they didn't reveal very clearly the relations between circuit parameters and the line side performance parameters associated with harmonics. Of course, time domain simulations can handle the case that both the filter inductor and capacitor are not infinite [7] [8]. This approach is very accurate, but can not exactly reveal the relations between the circuit parameters and the line side performance parameters and it requires a long computing time.

This paper makes detail analysis of the circuit in Fig.1 under steady state conditions, and obtains accurate solutions to the differential equations that describe the circuit behavior in the case that both of the filter components are finite, and more important, it clearly reveals the relations between the circuit parameters and the line side performance parameters. It is also shown in the paper that, in the discontinuous current mode I, transferring part or whole of the dc side inductor to ac side does not affect the line side current, if we keep the sum of inductance at both sides constant. Therefore the analysis presented here is certainly suitable when a ac side inductor is connected or source inductance is taken into account.

2. DEFINITIONS OF LINE SIDE PERFORMANCE PARAMETERS

Regarding a load dissipating active power P supplied by a single-phase ac source, assume that the voltage is purely sinusoidal with the RMS value E and phase angle θ, and the current with the RMS value I is distorted. If the RMS value and the phase angle of the fundamental current are I_1 and θ_1 respectively, then the phase difference between the fundamental current and voltage is given as

$$\phi_1 = \theta_1 - \theta$$

In addition, we denote the RMS value of kth order harmonics in the current as I_k. Thus, the definitions of performance parameters used in this paper, represented by the above variables are listed as follows:

Purity factor of the current $\nu = \frac{I_1}{I}$

Displacement factor $\xi = \cos\phi_1 = \cos(\theta_1 - \theta)$

Power factor

$$\lambda = \frac{P}{EI} = \frac{EI_1\cos(\theta_1 - \theta)}{EI} = \frac{I_1}{I}\cos(\theta_1 - \theta) = \nu\xi$$

Total harmonic distortion of the current

$$THD = \frac{\sqrt{I^2 - I_1^2}}{I_1}$$

kth order harmonic current content $\frac{I_k}{I_1}$

3. ANALYSIS FOR ZERO FILTER INDUCTANCE CASE

Circuit Analysis

Fig.2 shows the equivalent circuit and waveforms of the circuit in Fig.1 when the filter inductance L is zero and the circuit has already entered the steady state.

Assume that the source voltage is

$$e = E_m \sin(\omega t + \theta) \tag{1}$$

and diodes D1 and D4 begin to subject the forward voltage and conduct at the instant t=0, i.e., the ac source begins to recharge the dc side capacitor C, then the initial voltage of the capacitor must be

$$v_C(0) = E_m \sin\theta \tag{2}$$

Where E_m is the amplitude of the source voltage. From the equivalent circuit we get the following differential equation that holds during the conducting period of D1 and D4.

$$v_C(0) + \frac{1}{C}\int_0^t i_C dt = e \tag{3}$$

Substitute (1) and (2) to (3) and solve the equation, we

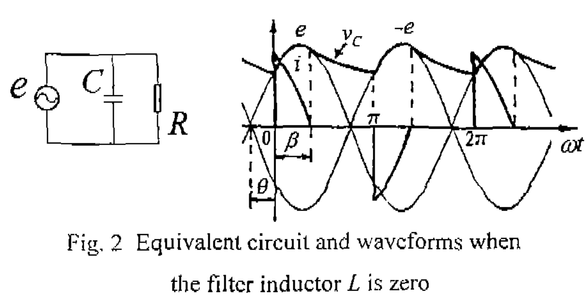


Fig. 2 Equivalent circuit and waveforms when the filter inductor L is zero

derive

$$i_C = \omega C E_m \cos(\omega t + \theta) \tag{4}$$

As for load current, it is

$$i_R = \frac{e}{R} \tag{5}$$

Substitution of (1) to (5) yields

$$i_R = \frac{E_m}{R}\sin(\omega t + \theta) \tag{6}$$

Let the conduction angle of D1 and D4 be denoted as β, we'll show how to determine the exact value of β and θ if the circuit parameters are known, and how to derive the exact expression of the dc side current i_d and the ac side current i.

When $\omega t = \beta$, i.e., i_d=0 and $v_C = e = E_m\sin(\theta + \beta)$, the diodes D1 and D4 start to block, and the capacitor C begins to discharge through the path C and R.

Substitution of (4) and (6) into $i_d = i_C + i_R$ shows that

$$i_d = \omega C E_m \cos(\omega t + \theta) + \frac{E_m}{R}\sin(\omega t + \theta) \tag{7}$$

Noticing $i_d(\beta) = 0$, we obtain the following equation through substitution of ωt by β in (7)

$$\mathrm{tg}(\theta + \beta) = -\omega RC \tag{8}$$

The capacitor discharges through R until ωt=π, when the capacitor voltage decreases to $E_m \sin\theta$, exactly the same value as what it was at the instant ωt=0, and other two diodes D2 and D3 begin to conduct. Hence the following exists

$$E_m \sin(\theta + \beta)\cdot e^{-\frac{\pi - \beta}{\omega RC}} = E_m \sin\theta \tag{9}$$

Rearrange (8) and (9) and consider that θ+β is a second quadrant angle, we obtain

$$\pi - \beta = \theta + \mathrm{arctg}(\omega RC) \tag{10}$$

$$\frac{\omega RC}{\sqrt{(\omega RC)^2 + 1}}\cdot e^{-\frac{\mathrm{arctg}(\omega RC)}{\omega RC}}\cdot e^{-\frac{\theta}{\omega RC}} = \sin\theta \tag{11}$$

Now if the product of circuit parameters ω, R and C is known, the angle θ can be determined through solving (11), and substitution of θ into (10) yields the angle β. Obviously θ and β can be determined if and only if the product ωRC is determined. In other words, θ and β are related only to ωRC. This is a very important fact, and starting from this fact we will later get the results that describes the relations between the line side performance parameters and the circuit parameters. Fig.3 (a) shows the curves of θ and β versus ωRC based on the calculation of θ and β through (11) and (10).

The expression of the dc side current can subsequently be determined after the determinations of θ and β. That is

$$i_d = \begin{cases} \omega C E_m[\cos(\omega t + \theta) + \frac{1}{\omega RC}\sin(\omega t + \theta)] & n\pi \le \omega t < n\pi + \beta \\ 0 & n\pi + \beta \le \omega t < (n+1)\pi \end{cases}$$
$$n = 0,1,2,\cdots\cdots \tag{12}$$

The ac side current i has a half-wave symmetric waveform and is identical with i_d in the positive half cycle. Inspection of (12) shows that the shape of the current i is related only to ωRC while the magnitude is proportional to the product ωCE_m.

Calculation of Line Side Performance Parameters Associated with Harmonics

Now that the expression of the line side current i is known, the Fourier series of i can consequently be derived, i.e.,

$$\begin{aligned} i &= \sum_{k=1,3,5,\cdots} (I_{kma}\cos k\omega t + I_{kmb}\sin k\omega t) \\ &= \sum_{k=1,3,5,\cdots} \sqrt{2} I_k \sin(k\omega t + \theta_k) \end{aligned} \tag{13}$$

The analytical expressions of I_{1ma}, I_{1mb}, I_1, θ_1, I_{kma}, I_{kmb}, and I_k are listed below.

$$\begin{aligned} I_{1ma} = \frac{\omega CE_m}{\pi}[&\beta(\cos\theta + \frac{1}{\omega RC}\sin\theta) + \frac{1}{2}\sin(2\beta+\theta) - \frac{1}{2}\sin\theta \\ &- \frac{1}{2\omega RC}\cos(2\beta+\theta) + \frac{1}{2\omega RC}\cos\theta] \end{aligned}$$

$$\begin{aligned} I_{1mb} = \frac{\omega CE_m}{\pi}[&\beta(-\sin\theta + \frac{1}{\omega RC}\cos\theta) - \frac{1}{2}\cos(2\beta+\theta) + \frac{1}{2}\cos\theta \\ &- \frac{1}{2\omega RC}\sin(2\beta+\theta) + \frac{1}{2\omega RC}\sin\theta] \end{aligned}$$

$$I_1 = \frac{1}{\sqrt{2}}\sqrt{I_{1ma}^2 + I_{1mb}^2}$$

$$\theta_1 = \mathrm{arctg}\frac{I_{1ma}}{I_{1mb}}$$

$$\begin{aligned} I_{kma} = \frac{\omega CE_m}{\pi}\{ &\frac{1}{k+1}[\sin(k\beta+\beta+\theta) - \sin\theta] \\ &+ \frac{1}{k-1}[\sin(k\beta-\beta-\theta) + \sin\theta] \\ &- \frac{1}{\omega RC(k+1)}[\cos(k\beta+\beta+\theta) - \cos\theta] \\ &+ \frac{1}{\omega RC(k-1)}[\cos(k\beta-\beta-\theta) - \cos\theta] \} \end{aligned}$$

$$\begin{aligned} I_{kmb} = \frac{\omega CE_m}{\pi}\{ &-\frac{1}{k+1}[\cos(k\beta+\beta+\theta) - \cos\theta] \\ &- \frac{1}{k-1}[\cos(k\beta-\beta-\theta) - \cos\theta] \\ &- \frac{1}{\omega RC(k+1)}[\sin(k\beta+\beta+\theta) - \sin\theta] \\ &+ \frac{1}{\omega RC(k-1)}[\sin(k\beta-\beta-\theta) + \sin\theta] \} \end{aligned}$$

$$I_k = \frac{1}{\sqrt{2}}\sqrt{I_{kma}^2 + I_{kmb}^2} \qquad k = 3,5,7\cdots\cdots$$

Keep in mind that i is half-wave symmetric, i.e., there is no even order harmonics in current i.

The RMS value of the line side current is given as

$$\begin{aligned} I &= \sqrt{\frac{1}{2\pi}\int_0^{2\pi} i^2 d(\omega t)} \\ &= \frac{\omega CE_m}{\sqrt{\pi}}\{ \frac{\beta}{2} + \frac{\beta}{2(\omega RC)^2} - \frac{1-(\omega RC)^2}{4(\omega RC)^2}[\sin(2\beta+2\theta) - \sin 2\theta] \\ &\quad - \frac{1}{2\omega RC}[\cos(2\beta+2\theta) - \cos 2\theta] \}^{1/2} \end{aligned}$$

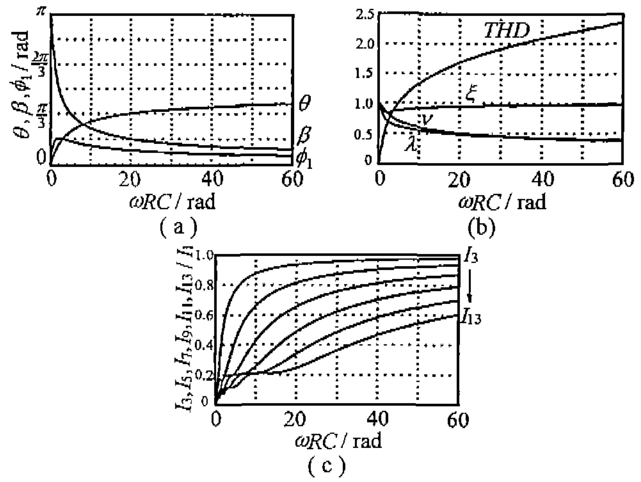


Fig. 3 Line side performance parameters versus ωRC in the case of zero filter inductance

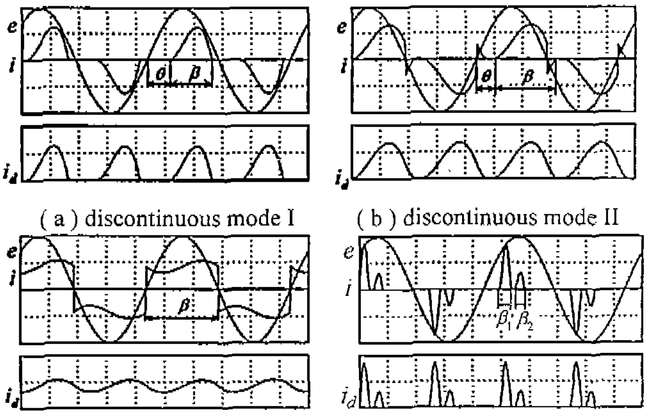
Substitute I_1, θ_1, I_k and I into the definitions listed in Section II, we get the expressions of the line side performance parameters.

Regarding these expressions and reminding that θ and β are determined only by ωRC, we can see that the line side performance parameters are determined, again, only by the product ωRC, though I_1, I_k and I are proportional to ωCE_m. This conclusion clearly reveals the relations between the line side performance parameters and the circuit parameters. Based on the calculations of the above expressions, the curves of line side performance parameters versus the product ωRC, such as purity factor ν, displacement factor ξ, power factor λ, total harmonic distortion and harmonic content in each order of the line side current, are drawn in Fig.3 (b) and (c). The phase difference between the fundamental current and voltage is drawn in Fig.3 (a). Further study on these curves results in more detail conclusions which will be shown soon afterwards in Section VI.

4. ANALYSIS FOR NONZERO FILTER INDUCTANCE CASE

Modes of Operation

Based on [2], the rectifier in Fig.1 has three possible modes of operation. Assuming the load resistance R is constant, if the inductance L varies from small to large value while the capacitance from large to small, the whole load of the rectifier will change from capacitive to inductive, and the mode in which the rectifier operates will vary from discontinuous mode I to discontinuous mode II and then continuous mode, as shown in Fig.4 (a), (b) and (c). In discontinuous mode I, i_d goes to zero before the source voltage crosses zero, so there is no commutation. On the contrary in discontinuous mode II and continuous



(a) discontinuous mode I   (b) discontinuous mode II

(c) continuous mode   (d) the fourth mode

Fig. 4 Modes of operation in the case of none zero filter inductance

mode, diodes D1 and D4 are commutated, and D2 and D3 go into conduction at the instant the source voltage crosses zero.

The authors supplement a fourth mode, shown in Fig.4 (d). It is a discontinuous mode, but different to the discontinuous mode I and II. In this mode the current i_d has two separated ripples in a half cycle of the source voltage, i.e., the corresponding diodes conduct and block two times and no commutation happens. This may occur when both the filter inductance and capacitance are very small but only in a very limited range of circuit parameters variations. Further discussion is neglected and we just supplement it for the integrity of theory.

For the sake of practical conditions, we dwell on the discontinuous mode I in the following analysis. And it should be pointed out that, as no commutation procedure happens in discontinuous mode I, transferring part or whole of the dc side inductor to the ac side does not affect the line side current, if the sum of inductance at both sides is maintained to be constant.

Circuit Analysis

Fig.5 shows the equivalent circuit and waveforms of the circuit in Fig.1 when the filter inductance is nonzero, and the rectifier operates in discontinuous mode I and has already entered steady state.

With the source voltage expressed as (1) and assume that the diodes D1 and D4 begin to conduct at the instant t=0, the following equations hold during the conduction interval.

$$\begin{cases} v_l = L\dfrac{di_d}{dt} \\ e = v_l + v_C \\ i_d = i_C + i_R = C\dfrac{dv_C}{dt} + \dfrac{v_C}{R} \\ v_C(0) = E_m \sin\theta \\ i_d(0) = 0 \end{cases} \tag{14}$$

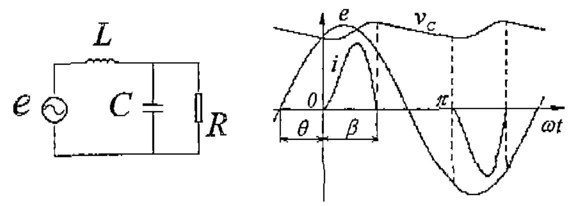


Fig. 5 Equivalent circuit and waveforms when the filter inductance L is none zero

Solve (1) and (14), and notice that current i_d is in discontinuous mode I, we obtain

$$i_d(\omega t) = \omega C E_m[\zeta_1 \sin(\omega_d \omega t + \alpha_1)e^{-\frac{\omega t}{\tau}} + \zeta_2 \sin(\omega t + \alpha_2)] \tag{15}$$

$$v_C(\omega t) = E_m[A\sin(\omega_d \omega t + \phi)e^{-\frac{\omega t}{\tau}} + B\sin(\omega t - \psi + \theta)] \tag{16}$$

The expressions of the symbols used here, ω_d, τ, B, ψ, ζ_2, A, ϕ, α_1, α_2, and ζ_1 are listed in Appendix.

As in section III, assume that the conduction angle of D1 and D4 is β; and when ωt=β, the capacitor begins to discharge through the path C and R; and that the discharge maintains until ωt=π, the capacitor voltage lowers down to $E_m\sin\theta$ and the other two diodes D2 and D3 start to conduct.

In order to determine the exact value of θ and β, we substitute (15) and (16) into $i_d(\beta)$=0 and $v_c(\pi)$= $E_m\sin\theta$, and obtain

$$\begin{cases} \zeta_1 \sin(\omega_d\beta + \alpha_1)\cdot e^{-\frac{\beta}{\tau}} + \zeta_2 \sin(\beta + \alpha_2) = 0 \\ \left[A\sin(\omega_d\beta + \varphi)\cdot e^{-\frac{\beta}{\tau}} + B\sin(\beta - \psi + \theta)\right]e^{-\frac{\pi-\beta}{\omega RC}} = \sin\theta \end{cases} \tag{17}$$

Naturally we want to know whether the angle θ and β here are only determined by the product ωRC. A more detailed inspection on the expressions listed in Appendix B shows that ω_d, τ, B, ψ, and ζ_2 are related only to two items, i.e., ωRC and $\omega\sqrt{LC}$, while A, ϕ, α_1, α_2, and ζ_1 related only to ωRC, $\omega\sqrt{LC}$ and θ. Hence from (17), it may be seen that the angles θ and β are determined only by ωRC and $\omega\sqrt{LC}$, similar to the case of zero filter inductance. Based on the calculations through (17), Fig.6 gives the curves of θ and β versus ωRC under several values of $\omega\sqrt{LC}$. The dashed lines in the figures are the boundaries between the discontinuous mode I and discontinuous mode II. The boundaries can be determined by (17) and taking the equation θ+β=π into account.

The expressions of dc side current is determined by (15) after the determination of θ and β, and similar to section III, the ac side current i is half-wave symmetric and has the same waveform as i_d during the positive half cycle.

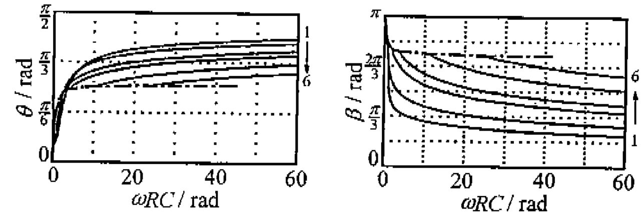


Fig. 6 Variation of θ and β versus ωRC

1—$\omega\sqrt{LC}$ =0.25rad 2—$\omega\sqrt{LC}$ =0.4rad

3—$\omega\sqrt{LC}$ =0.7rad 4—$\omega\sqrt{LC}$ =0.9rad

5—$\omega\sqrt{LC}$ =1.4rad 6—$\omega\sqrt{LC}$ =2.0rad

Reviewing (15), we also confirm that the shape of current i is related only to ωRC and $\omega\sqrt{LC}$, while its magnitude is proportional to ωCE_m.

Calculation of Line Side Performance Parameters Associated with Harmonics

Fourier series expansion on the line side current i leads to expressions identical with (13) in form, but the expressions of I_{1ma}, I_{1mb}, I_1, θ_1, I_{kma}, I_{kmb}, and I_k are somewhat more complex. The expression of the RMS value of i is complex too. To limit the length of the paper, these expressions are not listed. Substitutions of these expressions into the definitions listed in section II will result in the expressions of the line side performance parameters. Similar conclusions as in section III may be obtained: I_1, I_k and I are proportional to ωCE_m, and the line side performance parameters associated with harmonics are related only to ωRC and $\omega\sqrt{LC}$. Fig.7 shows the results calculated through these expressions, i.e., the curves of purity factor ν, phase difference angle ϕ_1, displacement factor ξ, power factor λ, total harmonic distortion and content of 3rd and 5th order harmonics in the line side current, versus the product ωRC for several values of $\omega\sqrt{LC}$. Again the dashed lines here are the boundaries between the discontinuous mode I and discontinuous mode II. More detail conclusions concerning these figures that describe the relations mentioned before will be given in section VI.

5. SIMULATION VERIFICATIONS

Fig.8 shows two examples of time domain simulations for the circuit in Fig.1, one for zero filter inductance with ωRC=50rad and the other for nonzero filter inductance with $\omega\sqrt{LC}$=0.7rad and ωRC=40rad. The simulation results extremely approach the analysis results given above, concerning the line side performance parameters.

6. CONCLUSIONS

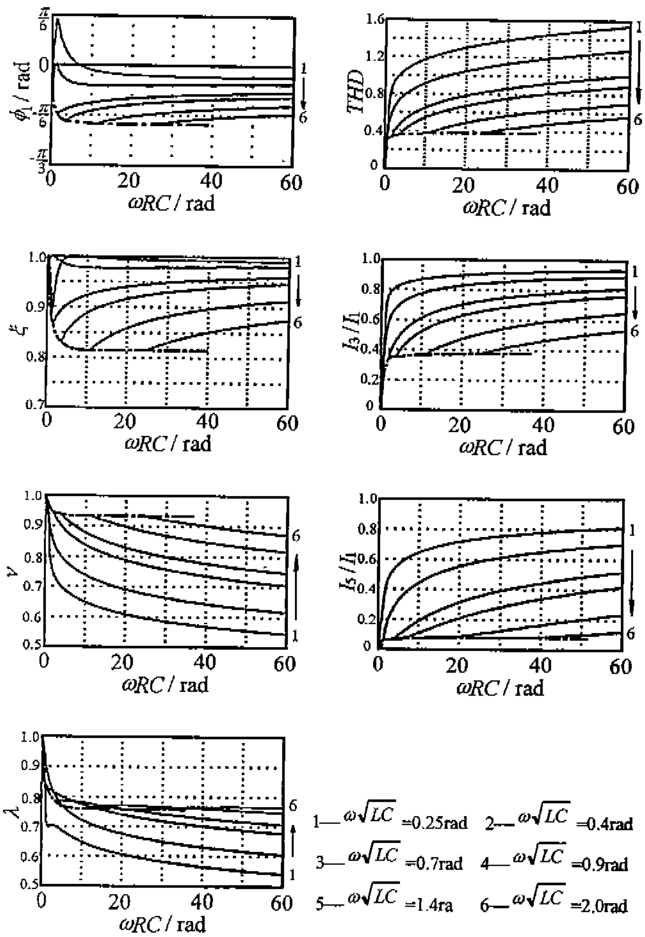


Fig. 7 Line side performance parameters in the case of none zero filter inductance

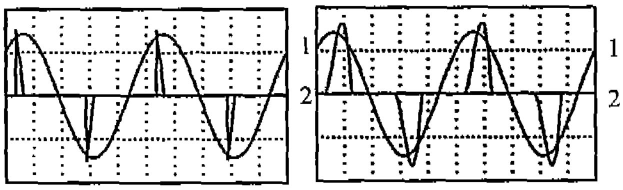
If the case of zero filter inductance is taken as the case that $\omega\sqrt{LC}$=0, then based on the obtained expressions of the line side performance parameters and the relationship curves shown in Fig.3 and Fig.7, the following observations can be made on the circuit in Fig.1:

i). The line side performance parameters associated with harmonics are related only to two items of circuit parameters, i.e., ωRC and $\omega\sqrt{LC}$.

ii). The fundamental line side current may have a leading phase to the source voltage when $\omega\sqrt{LC}$ is very small, particularly when L is zero. As $\omega\sqrt{LC}$ gets larger, the phase of line side current becomes lagging to the voltage, and the lagging angle becomes large when $\omega\sqrt{LC}$ is large or ωRC is small.

iii). The purity factor of the line side current increases when $\omega\sqrt{LC}$ becomes large or ωRC becomes small. This shows that the distortion of the current is caused mainly by the filter capacitor and is suppressed by the filter inductor.

iv). As to the power factor, it is a bit more complex as it is the product of purity factor and displacement factor. In the case of small $\omega\sqrt{LC}$, the displacement factor



t, 4ms/div t, 4ms/div

ζ=0.98 ν=0.44 λ=0.43 THD=2.08 ζ=0.95 ν=0.74 λ=0.70 THD=0.92

(a) (b)

Fig. 8 Simulation results

1—source voltage 2—line side current

(a) L=0, ωRC=50rad (b) $\omega\sqrt{LC}$ =0.7rad, wRC=40rad

approaches unity, so the power factor is determined by the purity factor and increases when ωRC becomes small. In the case of large $\omega\sqrt{LC}$, the displacement factor can no longer be taken as unity but becomes small when ωRC is small. Thus the effects of the displacement factor and the purity factor contradict each other, and the result is that the power factor varies little when ωRC varies. Keep ωRC constant and let $\omega\sqrt{LC}$ increase, the increase of the purity factor will be greater than the decrease of displacement factor, so the power factor also increases. And this becomes more obvious at a larger ωRC.

v). There are only odd order harmonics in the line side current. The harmonic content in each order and the total harmonic distortion increase when ωRC increases, and decrease when $\omega\sqrt{LC}$ increases. The higher the harmonic order, the smaller the harmonic content.

In summary, this paper analyze the harmonic currents generated by the single-phase diode bridge rectifier with a dc capacitor filter, operating in the discontinuous mode. The analysis verified by computer simulation is shown to be accurate and the conclusions drawn from the analysis clearly reveal the relations between the line side performance parameters associated with harmonics and the circuit parameters.

7. ACKNOWLEDGMENT

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APPENDIX

Expressions of the Variables Used in (15) and (16)

$$\omega_d = \sqrt{\frac{1}{(\omega\sqrt{LC})^2} - \frac{1}{(2\omega RC)^2}}$$

$$\tau = 2\omega RC$$

$$B = \left[(\omega\sqrt{LC})^2 \sqrt{\left(\frac{1}{(\omega\sqrt{LC})^2} - 1\right)^2 + \frac{1}{(\omega RC)^2}} \right]^{-1}$$

$$\psi = \mathrm{arctg}\left(\frac{\omega RC}{(\omega\sqrt{LC})^2} - \omega RC\right)^{-1}$$

$$\zeta_2 = B\sqrt{1 + \frac{1}{(\omega RC)^2}}$$

$$C_1 = \sin\theta - B\sin(\theta - \psi)$$

$$C_2 = \frac{1}{\omega_d}\left[\frac{C_1}{2\omega RC} - B\cos(\theta - \psi) - \frac{\sin\theta}{\omega RC}\right]$$

$$A = \sqrt{C_1^2 + C_2^2}$$

$$\phi = \mathrm{arctg}\frac{C_1}{C_2}$$

$$\alpha_1 = \phi + \mathrm{arctg}(2\omega RC \cdot \omega_d)$$

$$\alpha_2 = \theta - \psi + \mathrm{arctg}(\omega RC)$$

$$\zeta_1 = \frac{A}{\omega\sqrt{LC}}$$