CALCULATION OF INDUCTANCES OF TRANSFORMER EQUIVALENT NETWORK FOR HIGH FREQUENCY INTERFERENCE CONDUCTION ANALYSIS

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ABSTRACT – In order to analysis high frequency interference conduction in transformer, this paper constructs a equivalent network based on P.I.Fergested transformers physical model. A method of calculating self and mutual leakage inductance of windings sections is presented and a calculating example is given.

1. INTRODUCTION

Transformers are often connected with power electronic equipments and power line. Every kinds of high frequency interference from power line get to power electronic devices through transformers. High frequency noise produced by power electronic devices arrive at power line and disturb other electronic instrument also through transformers. In order to analysis high frequency interference conduction in transformers with digital computer, transformers equivalent circuit network must be established and the elements (L, M, C, R) in this network must be calculated.

The equivalent network for a transformer winding contains inductances, series and ground capacitances and loss resistances. This is shown in Fig. 1.

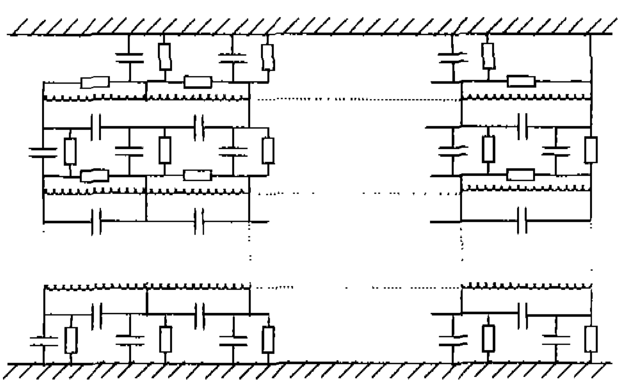

Fig. 1 Equivalent circuit

In the equivalent circuit, capacitances and loss resistances can be calculated from the geometric dimensions of the transformer. This paper presents a method of calculating self and mutual leakage inductance of sections of transformer windings. This is based on P. I. Fergested's transformers physical model.

2. CALCULATION FORMULAS OF MAGNETIC-VECTOR POTENTIAL

The model for calculation of inductions of transformer turns is shown in Fig. 2.

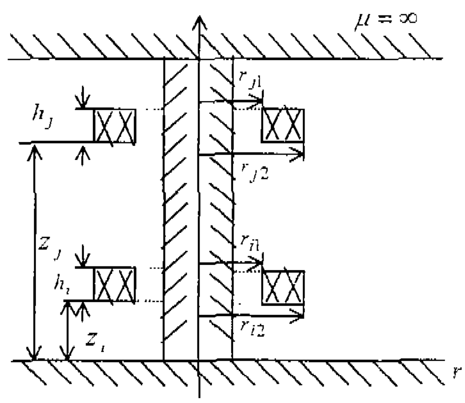

Fig.2 The model of inductions of transformer turns

When current flow in exciting turns, magnetic field in space is satisfy to Poisson equation $\nabla^2\vec{A}=-\mu\vec{\delta}$ ($\vec{A}$ is magnetic-vector potential, $\vec{\delta}$ is current density in the turns). Because the inductance model is symmetric cylindrically, (the current density and the magnetic-vector. potential in the turns) have only tangential component, so the magnetic field is expressed as following:

$$\frac{\partial^2 \vec{A}}{\partial r^2}+\frac{1}{r}\frac{\partial \vec{A}}{\partial r}+\frac{\partial^2 \vec{A}}{\partial z^2}=-\mu\vec{\delta} \tag{1}$$

The current density distribution of exciting turn along axis is shown in Fig. 3. Along axis, the exciting current density can be developed to Fourier series (where m= $\frac{n\pi}{l}$, n=1, 2, ...).

$$\begin{cases} \delta(r,z)=\delta_0(r)+\sum_{n=1}^{\infty}\delta_n(r)\cos mz & (r_{j1}\le r\le r_{j2}, \quad 0\le z\le l) \\ \delta(r,z)=0, & (r\le r_{j1} \quad or \quad r\ge r_{j2}, \quad 0\le z\le l) \end{cases}$$

where $\delta_0(r)=\frac{1}{l}\int_0^l \delta(r,z)dz=\frac{1}{l}\int_{z_j}^{z_j+h_j}\delta(r,z)dz$;

$$\delta_n(r)=\frac{2}{l}\int_0^l \delta(r,z)\cos mz dz=\frac{2}{l}\int_{z_j}^{z_j+h_j}\delta(r,z)\cos mz dz,$$

the magnetic-vector potential A can also be developed as Fourier series,

$$A(r,z)=A_0(r)+\sum_{n=1}^{\infty}A_n(r)\cos mz \tag{2}$$

Substituting from (2) in (1), we can obtain the following equations:

$$\frac{d^2A_0(r)}{dr^2}+\frac{1}{r}\frac{dA_0(r)}{dr}=-\mu\delta_0 \tag{3}$$

$$\frac{d^2A_n(r)}{dr^2}+\frac{1}{r}\frac{dA_n(r)}{dr}-m^2A_n(r)=-\mu\delta_n \tag{4}$$

Using Bassel function to solve the equation, we can obtain harmonic component $A_n(r)$ of magnetic-vector potential.

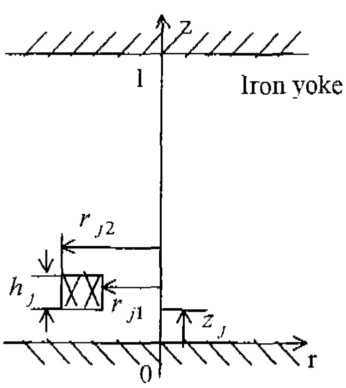

Fig. 3 The current density distribution of exciting turn along axis

3. CALCULATION FORMULAS OF INDUCTANCE

a. Inductance $(M_{ij})_1$ is decided by harmonic component δ_n. The mutual inductance of exciting turn and induction turn is M_{ij},

$$M_{ij}=\frac{1}{s_i s_j}\int_{r_{i1}}^{r_{i2}}\int_{z_i}^{z_i+h_i}\varphi(r,z)drdz \tag{5}$$

$$\varphi(r,z)=2\pi r\sum_{n=1}^{\infty}A_n(r)\cos\frac{n\pi}{l}z \tag{6}$$

Substituting from (6) in (5), we can obtain the following:

$(M_{ij})_1 =$

$$\frac{2h_i}{s_j s_i}\sum_{n=1}^{\infty}\left\{\frac{1}{n}\left[\sin\frac{n\pi(z_i+h_i)}{l_i}-\sin\frac{n\pi z_i}{l_i}\right]\int_{r_{i1}}^{r_{i2}}rA_N(r)dr\right\} \tag{7}$$

b. Inductance $(M_{ij})_0$ is decided by constant component δ_0.

$\varphi_0(r) =$

$$\pi\delta_0\left[r_0^2(\mu_r r_{j2}-\mu_r r_{j1}-r_{j2}+r_{j1})+r^2(r_{j2}-\frac{2}{3}r)-\frac{1}{3}r_{j1}^3\right]\mu_0 \tag{8}$$

$$(M_{ij})_0=\frac{2h_i}{s_j}\int_{r_{i1}}^{r_{i2}}\varphi_0(r)dr \tag{9}$$

From (7) and (9), we can obtain

$$M_{ij}=(M_{ij})_0+(M_{ij})_1 \tag{10}$$

4. CALCULATION EXAMPLE

The structure of a typical transformer is shown in Fig.4. The transformer's turns ratio is 10:1. The primary of the transformer has 1600 turns. The secondary of the transformer has 160 turns. As a calculation example, the primary is divided to eight sections (A1-A8), each section has 200 turns. The secondary is also divided to eight sections (B1-B8), each section has 20 turns.

The self and mutual inductance of the typical transformer are calculated. The Table 1 and Table 2 show the calculation results.

(unit: mm)

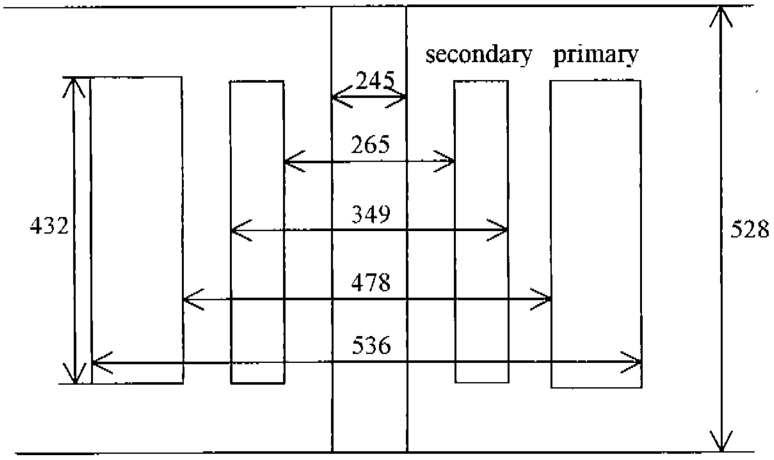

Fig. 4 The structure of a typical transformer

Table 1 The self and mutual inductance of the primary (unit: μH)

	A1	A2	A3	A4	A5	A6	A7	A8
A1	3.19	2.56	1.97	1.44	0.97	0.56	0.07	0
A2	2.56	3.19	2.56	1.97	1.44	0.97	0.56	0.07
A3	1.97	2.56	3.19	2.56	1.97	1.44	0.97	0.56
A4	1.44	1.97	2.56	3.19	2.56	1.97	1.44	0.97
A5	0.97	1.44	1.97	2.56	3.19	2.56	1.97	1.44
A6	0.56	0.97	1.44	1.97	2.56	3.19	2.56	1.97
A7	0.07	0.56	0.97	1.44	1.97	2.56	3.19	2.56
A8	0	0.07	0.56	0.97	1.44	1.97	2.56	3.19

Table 2 The self and mutual inductance of the secondary (unit: μH)

	B1	B2	B3	B4	B5	B6	B7	B8
B1	4.42	3.72	2.92	2.16	1.45	0.83	0.34	0
B2	3.72	4.42	3.72	2.92	2.16	1.45	0.83	0.34
B3	2.92	3.72	4.42	3.72	2.92	2.16	1.45	0.83
B4	2.16	2.92	3.72	4.42	3.72	2.92	2.16	1.45
B5	1.45	2.16	2.92	3.72	4.42	3.72	2.92	2.16
B6	0.83	1.45	2.16	2.92	3.72	4.42	3.72	2.92
B7	0.34	0.83	1.45	2.16	2.92	3.72	4.42	3.72
B8	0	0.34	0.83	1.45	2.16	2.92	3.72	4.42

5. REFERENCES

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